Tuneable defect-curvature coupling and topological transitions in active shells Supplementary Information

Ludwig A. Hoffmann[1], Livio Nicola Carenza[1], and Luca Giomi[1]*
[1] *Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands*
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I. NUMERICAL METHOD AND SIMULATION DETAILS

The dynamical equations, Eqs. (A2) in the appendix of the main text, have been integrated by means of a hybrid lattice Boltzmann (LB) method, where the hydrodynamics is solved through a *predictor-corrector* LB algorithm [34], while the dynamics of the order parameter has been treated with a finite-difference approach implementing a first-order upwind scheme and fourth-order accurate stencil for the computation of spacial derivatives. The hydrodynamics was integrated in a box of size L^3, with $L = 128$ the linear size of the system and periodic boundary conditions. The radius of the shell $R = 18$ for all results presented in the main text. More radii, $R = 15, 24, 32$ have been also simulated to check consistency of results. We report no qualitative difference with the cases presented in the main text.

The numerical code has been parallelized by means of Message Passage Interface (MPI) by dividing the computational domain in slices and by implementing the ghost-cell method to compute derivatives on the boundary of the computational subdomains. Runs have been performed using 64 CPUs for at least 10^6 lattice Boltzmann iterations (corresponding to $\sim 35d$ of CPU-time on Intel Xeon 8160 processors).

The model parameters in lattice units used for simulations are $a = 0.01, k_\phi = 0.015, \phi_0 = 2.0, \mu = 0.1, \Gamma = 0.2, \eta = 5/3$. For polar liquid crystals we used $A_0 = 0.1, W_1 = 0.03, \lambda = 1.1, \kappa_F = 0.02$ and we varied the activity in the range $-0.015, 0.015$ as reported in the main text. For nematics we used $A_0 = 0.1, \Phi_0 = 2.45, \Phi_s = 1.0, W_2 = 0.01, \lambda = 1.1$. We varied the activity in the range $-0.015, 0.015$ and the Frank constant in the range $0.008, 0.04$.

II. MOVIES

Movie 1: Flattening of polar shell in presence of extensile activity. The extensile active stresses ($\alpha = -0.002$) lead to a flattening of the initially spherical shell. The two $+1$ defects have a spiral geometry and move away from the poles and the shell becomes motile. The vectors denote the polarization field, while the color code refers to the local magnitude of the polarization according to the color bar at the bottom.

Movie 2: Genus transition of polar shell. For large extensile activity ($\alpha = -0.012$) the a chaotic regime is entered and the shell is deformed significantly. Small vesicles separate from the original shell and flatten, eventually leading to a genus transition from a spherical topology to a toroidal topology.

Movie 3: Flattening of polar shell in presence of contractile activity. The contractile active stresses ($\alpha = 0.004$) lead to a flattening of the initially spherical shell albeit less than in the presence of comparable extensile stresses. The two $+1$ defects have an aster geometry and near the center of the defects we observe a buckling of the shell. The defects move away from the poles and the shell becomes motile.

Movie 4: Periodic deformation of nematic shell. In the case of small activity ($\alpha = -0.002$ in this movie) the periodic movement of the four active, motile $+1/2$ defects couples to the elastic shell resulting in a periodic deformation of the shell.

Movie 5: Tentacle formation. In the presence of intermediate extensile activity ($\alpha = -0.004$ in this movie) we observe the periodic deformation of the shell as well as the creation of tentacles which are formed by two $+1/2$ defects approaching each other and in the process creating a protrusion.

Movie 6: Chaotic deformation of nematic shell for extensile activity. For large extensile activity ($\alpha = -0.007$ in this movie) the original shell is elongated and eventually the shell rips apart creating several smaller snail-like surfaces that eventually dissolve due to Oswald ripening.

Movie 7: Spindle-like shape of nematic shell in presence of contractile activity. For intermediate contractile activity two $+1/2$ defects move towards the poles and the sphere is deformed into a spindle-like shape.

* giomi@lorentz.leidenuniv.nl