The RR charge of orientifolds

Oberwolfach

June 8, 2010

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arXiv:0906.0795

and work *in progress* with
…
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Outline

1. Statement of the problem & motivation
2. What is an orientifold?
3. B-field: Differential cohomology
4. RR Fields I: Twisted KR theory
5. Generalities on self-dual theories
6. RR Fields II: Quadratic form for self-duality
7. O-plane charge
8. Chern character and comparison with physics
9. Topological restrictions on the B-field
10. Precis

Statement of the Problem

What is the RR charge of
an orientifold?

That's a complicated question

a.) What is an orientifold?

b.) What is RR charge?

Motivation

- The evidence for the alleged ``landscape of string vacua'' (d=4, N=1, with moduli fixed) relies heavily on orientifold constructions.
- So we should put them on a solid mathematical foundation!
- Our question for today is a basic one, of central importance in string theory model building.
- Puzzles related to S-duality are sharpest in orientifolds
- Nontrivial application of modern geometry & topology to physics.

Question a: What is an orientifold?

Perturbative string theory is, by definition, a theory of integration over a space of maps:

$$\varphi : \Sigma \to \mathcal{X}$$

Σ: 2d Riemannian surface

$\mathcal{X}$: Spacetime endowed with geometrical structures: Riemannian,….

$$\exp[-\int_{\Sigma} \frac{1}{2} \parallel d\varphi \parallel^2 + \cdots]$$

What is an orbifold?

Let’s warm up with the idea of a string theory ***orbifold***

$$\varphi : \Sigma \to X$$

$$X \text{ is smooth with finite isometry group } \Gamma$$

Gauge the Γ-symmetry:

Principal Γ bundle

$$\begin{array}{ccc} \tilde{\Sigma} & \stackrel{\tilde{\varphi}}{\longrightarrow} & X \\ \downarrow & & \downarrow \\ \Sigma & \stackrel{\varphi}{\longrightarrow} & \mathcal{X} \end{array}$$

Physical worldsheet

Spacetime groupoid

$$\mathcal{X} = X//\Gamma$$

For ***orientifolds***, $\tilde{\Sigma}$ is oriented,

In addition: $1 \to \Gamma_0 \to \Gamma \overset{\omega}{\to} \mathbb{Z}_2 \to 1$

$$\Gamma_0\colon\ \omega(\gamma) = +1 \qquad \Gamma_1\colon\ \omega(\gamma) = -1$$

$$\text{On } \tilde{\Sigma}\colon \quad \begin{array}{l} \Gamma_0\colon \text{ Orientation preserving} \\ \Gamma_1\colon \text{ Orientation reversing} \end{array}$$

$$\begin{array}{ccc} \tilde{\Sigma} & \overset{\tilde{\varphi}}{\to} & X \\ \downarrow & & \downarrow \\ \hat{\Sigma} & \overset{\hat{\varphi}}{\to} & X//\Gamma_0 \\ \downarrow & & \downarrow \\ \Sigma & \overset{\varphi}{\to} & X//\Gamma \end{array}$$

Orientation double cover → $\hat{\Sigma}$

Unoriented → Σ

$X//\Gamma$ ← Space time

Orientifold Planes

$$\text{For orientifold spacetimes } X//\Gamma$$

a component of the fixed locus point of

$$g \in \Gamma_1$$

is called an ``***orientifold plane***.''

More generally, spacetime is an ``orbifold,''

In particular, $\mathcal{X}$ is a *groupoid*

(c.f. Adem, Leida, Ruan, *Orbifolds and Stringy Topology*)

$$\begin{array}{ccc} \hat{\Sigma} & \xrightarrow{\hat{\varphi}} & \mathcal{X}_w \\ \hat{\pi} \downarrow & & \downarrow \\ \Sigma & \xrightarrow{\varphi} & \mathcal{X} \end{array} \qquad w \in H^1(\mathcal{X}; \mathbb{Z}_2)$$

There is an isomorphism $\varphi^*(w) \cong w_1(\Sigma)$

Definition: An *orientifold* is a string theory defined by integration over such maps.

Worldsheet Measure

In string theory we integrate over ``worldsheets''

For the bosonic string, space of ``worldsheets'' is

$$S = \{(\Sigma, \varphi)\} = \mathrm{Moduli}(\Sigma) \times \mathrm{MAP}(\Sigma \to X)$$

$$\exp[-\int_{\Sigma/S} \frac{1}{2} \parallel d\varphi \parallel^2] \cdot \mathcal{A}_B$$

$$\mathcal{A}_B = \exp[2\pi i \int_{\Sigma/S} \varphi^*(B)]$$

B is locally a 2-form gauge potential…

Differential Cohomology Theory

In order to describe B we need to enter the world of differential generalized cohomology theories…

If $\mathcal{E}$ is a generalized cohomology theory, then denote the differential version $\check{\mathcal{E}}$

$$0 \to \mathcal{E}^{j-1}(M, \mathbb{R}/\mathbb{Z}) \to \check{\mathcal{E}}^j(M) \to \Omega_{\mathbb{Z}}(M; \mathcal{E}(pt) \otimes \mathbb{R})^j \to 0$$

$$0 \to [\text{top.triv.}] \to \check{\mathcal{E}}^j(M) \to \mathcal{E}^j(M) \to 0$$

Variations

We will need ***twisted*** versions on ***groupoids***

Both generalizations are nontrivial.

Main Actors

- B-field: Twisted differential cohomology
- RR-field: Twisted differential KR theory

Orientation & Integration

The orientation twisting of $\mathcal{E}(M)$,

denoted $\tau_{\mathcal{E}}(M)$

allows us to define an ``integration map''

in $\mathcal{E}$-theory:

$$\int_M^{\mathcal{E}} : \mathcal{E}^{\tau_{\mathcal{E}}(M)+j}(M) \to \mathcal{E}^j(pt)$$

Also extends to integration in differential theory

Where does the B-field live?

For the oriented bosonic string its gauge equivalence class is in $\check{H}^3(\mathcal{X})$

For a bosonic string orientifold its equivalence class is in $\check{H}^{3+w}(\mathcal{X})$

$$\mathcal{A}_B = \exp[2\pi i \int^{\check{H}}_{\Sigma/S} \varphi^*(\check{\beta})] \in \check{H}^1(S)$$

Integration makes sense because $\varphi^*(w) \cong w_1(\Sigma)$

Surprise!! For superstrings: not correct!

Superstring Orientifold B-field

Turns out that for superstring orientifolds

$$0 \to \check{H}^{3+w}(\mathcal{X}) \to \check{\mathcal{B}}^{3+w}(\mathcal{X}) \to H^0(\mathcal{X}, \mathbb{Z}) \times H^1(\mathcal{X}, \mathbb{Z}_2) \to 0$$

Necessary for worldsheet theory:
c.f. Talk at Singer85 (on my homepage)
and a paper to appear soon.

That's all for today about question (a)

Question b: What is RR Charge?

Type II strings have ``RR-fields'' – Abelian gauge fields whose fieldstrengths are forms of fixed degree in

$$\Omega^*(\mathcal{X}, \mathbb{R}[u, u^{-1}]) \qquad \deg(u) = 2$$

e.g. in IIB theory degree = -1:

$$G = u^{-1}G_1 + u^{-2}G_3 + \cdots + u^{-5}G_9$$

Naïve RR charge

In string theory there are sources of RR fields:

$$dG = j = \mathrm{RR\ current}$$

$$\mathrm{Naively:}\ [j] \in H^*_{deRham} = \mathrm{RR\ charge}$$

This notion will need to be refined…

Sources for RR Fields

Worldsheet computations show there are two sources of RR charge:

- D-branes
- Orientifold planes

$\mathrm{Recall\ that\ for\ } X//\Gamma$

a component of a fixed point locus of

$g \in \Gamma_1$ is called an ``orientifold plane.''

Our goal is to define precisely the orientifold plane charge and compute it as far as possible.

K-theory quantization

The D-brane construction implies

$$[j] \in K(\mathcal{X})$$

Minasian & Moore

So RR current naturally sits in $\check{K}(\mathcal{X})$

($X \to \mathcal{X}$ is a nontrivial generalization)

KR and Orientifolds

Action of worldsheet parity on Chan-Paton factors ➡

$$\text{For orientifolds replace}$$
$$K(\mathcal{X}) \to KR(\mathcal{X}_w)$$

(Witten; Gukov; Hori; Bergman, Gimon, Horava; Bergman,Gimon,Sugimoto; Brown&Stefanski,…)

$$\text{What is } KR(\mathcal{X}_w) \text{ ?}$$

For $\mathcal{X} = X//\Gamma$ use Fredholm model
(Atiyah, Segal, Singer)

$\mathcal{H}$: $\mathbb{Z}_2$-graded Hilbert space with stable Γ-action

Γ_0: Is linear Γ_1: Is anti-linear

$\mathcal{F}$: Skew-adjoint odd Fredholms

$$KR(\mathcal{X}_w) := [X \to \mathcal{F}]^{\Gamma}$$

This fits well with ``tachyon condensation.''

We need ***twisted*** KR-theory…

Following Witten and Bouwknegt & Mathai, we will interpret the B-field as a (differential) twisting of (differential) KR theory.

It is nontrivial that this is compatible with what we found from the worldsheet viewpoint.

As a bonus: This point of view nicely organizes the zoo of K-theories associated with various kinds of orientifolds found in the physics literature.

Twistings

- We will consider a special class of twistings with geometrical significance.

- We will consider the degree to be a twisting, and we will twist by a ``graded gerbe.''

- We now describe a simple geometric model

Double-Covering Groupoid

Spacetime $\mathcal{X}$ is a groupoid:

$$\mathcal{X}: \; X_0 \leftleftarrows X_1 \; \overset{\leftarrow}{\leftleftarrows} \; X_2 \; \overset{\leftleftarrows}{\leftleftarrows}$$

Homomorphism: $\epsilon_w : X_1 \to \mathbb{Z}_2$

$$\epsilon_w(gf) = \epsilon_w(g) + \epsilon_w(f)$$

Double cover: $X_{w,1} := \ker \epsilon_w$

Defines $\mathcal{X}_w$

Twisting KR Theory

Def: A *twisting* of $KR(\mathcal{X}_w)$ is a quadruple $\tau = (d, L, \epsilon_a, \theta)$

Degree $d : X_0 \to \mathbb{Z}$

L is a line bundle on X_1 $\mathbb{Z}_2$-grading: $\epsilon_a : X_1 \to \mathbb{Z}_2$

Cocycle: $\theta_{g,f} : {}^{\epsilon_w(f)}L_g \otimes L_f \to L_{gf}$

$$^{\epsilon}V := \begin{cases} V & \epsilon = 0 \\ \bar{V} & \epsilon = 1 \end{cases}$$

Twistings of KR

Topological classes of twistings of $KR(\mathcal{X}_w)$

$$H^0(\mathcal{X};\mathbb{Z}) \times H^1(\mathcal{X};\mathbb{Z}_2) \times H^{3+w}(\mathcal{X};\mathbb{Z})$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

$$d \qquad\qquad \epsilon_a \qquad\qquad (L,\theta)$$

Abelian group structure:

$$(d_1, a_1, h_1) + (d_2, a_2, h_2)$$
$$= (d_1 + d_2, a_1 + a_2, h_1 + h_2 + \tilde{\beta}(a_1 a_2))$$

The Orientifold B-field

So, the B-field is a geometric object whose topological class is

$$[\beta] = (d, a, h) \in H^0_{\mathbb{Z}} \times H^1_{\mathbb{Z}_2} \times H^{w+3}_{\mathbb{Z}}$$

d=0,1 mod 2: IIB vs. IIA.

a: Related to $(-1)^F$ & Scherk-Schwarz

h: is standard

Bott Periodicity

For $\mathcal{X} = \wp := pt//\mathbb{Z}_2$

$$H^0(\wp;\mathbb{Z}) \times H^1(\wp;\mathbb{Z}_2) \times H^{3+w}(\wp;\mathbb{Z}) \cong \mathbb{Z} \oplus \mathbb{Z}_4$$

We refer to these as ``universal twists''

$$B = d + \beta_\ell$$

Bott element $u \in KR^{2+\beta_1}(pt)$

Note $(\mathbb{Z} \oplus \mathbb{Z}_4)/\langle(2,1)\rangle \cong \mathbb{Z}_8$

So, there are 8 distinct universal B-fields

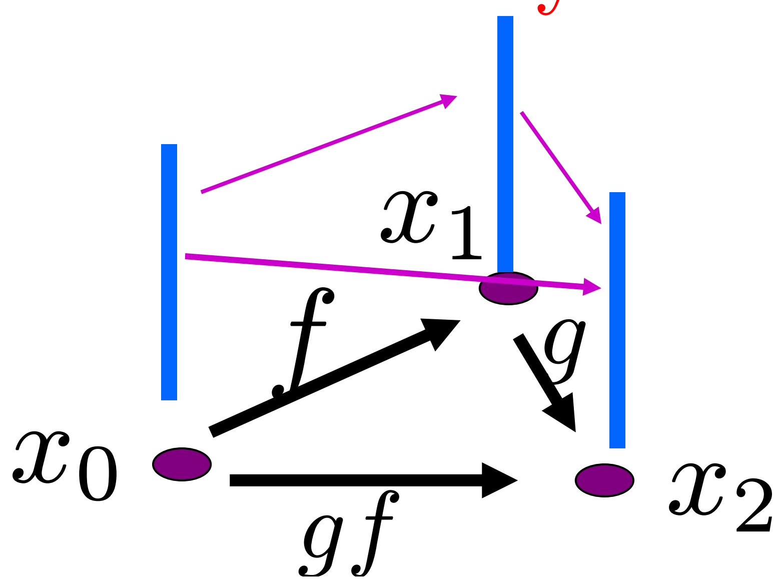
Twisted KR Class

1. $\mathbb{Z}_2$ graded $E \to X_0$ with odd skew Fredholm with graded $C\ell(d)$ action

2. On X_1 we have gluing maps:

$$f : x_0 \to x_1 \qquad \psi_f : {}^{\epsilon_w(f)}(L_f \otimes E_{x_0}) \to E_{x_1}$$

3. On X_2 we have a cocycle condition:



Where RR charge lives

The RR ***current*** is

$$[\check{j}] \in \check{K}R^{\check{\beta}}(\mathcal{X}_w)$$

The ***<u>charge</u>*** is an element of

$$KR^{\beta}(\mathcal{X}_w)$$

Next: How do we define which element it is?

The RR field is self-dual

The key to defining and computing the background charge is the fact that the RR field is a ***self-dual theory***.

How to formulate self-duality?

Generalized Maxwell Theory

(A naïve model for the RR fields)

$$\dim \mathcal{X} = n \qquad [\check{A}] \in \check{H}^{d-1}(\mathcal{X})$$

$$dF = J_m \in \Omega^d(\mathcal{X})$$

$$d * F = J_e \in \Omega^{n+2-d}(\mathcal{X})$$

Self-dual setting: $F = *F$ & $J_m = J_e$

Consideration of three examples:

1. Self-dual scalar: n=2 and d=2

2. M-theory 5-brane: n=6 and d=4

3. Type II RR fields: n=10 & GCT = K

has led to a general definition (Freed, Moore, Segal)

We need 5 pieces of data:

General Self-Dual Theory: Data

1. Poincare-Pontryagin self-dual mult. GCT

$$\mathcal{E}^{\tau}(\mathcal{M},\mathbb{R}/\mathbb{Z}) \times \mathcal{E}^{\tau_{\mathcal{E}}(\mathcal{M})-\tau-s}(\mathcal{M}) \to \mathbb{R}/\mathbb{Z}$$

$$\text{degree} \to \tau \qquad \iota : \mathcal{E}^{-s}(pt;\mathbb{R}/\mathbb{Z}) \to I^0(pt;\mathbb{R}/\mathbb{Z}) \cong \mathbb{R}/\mathbb{Z}$$

For a spacetime $\mathcal{X}$ of dimension n

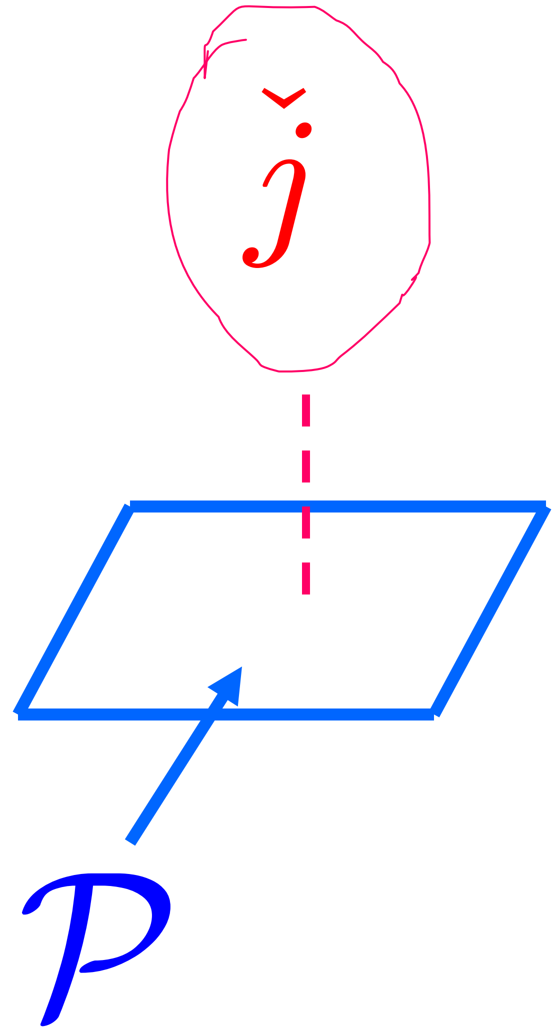
$$[\check{j}] \in \check{\mathcal{E}}^{\check{\tau}}(\mathcal{X})$$

2. Families of Spacetimes

$$\dim \mathcal{X}/\mathcal{P} = n$$

$$\dim \mathcal{Y}/\mathcal{P} = n + 1$$

$$\dim \mathcal{Z}/\mathcal{P} = n + 2$$



3. Isomorphism of Electric & Magnetic Currents

$$\theta_0 : \check{\mathcal{E}}^{\tau}(\mathcal{X}) \to \check{\mathcal{E}}^{\tau(\mathcal{X})-\tau+2-s}(\mathcal{X})$$

$$\theta_1 : \check{\mathcal{E}}^{\tau}(\mathcal{Y}) \to \check{\mathcal{E}}^{\tau(\mathcal{Y})-\tau+1-s}(\mathcal{Y})$$

$$\theta_2 : \check{\mathcal{E}}^{\tau}(\mathcal{Z}) \to \check{\mathcal{E}}^{\tau(\mathcal{Z})-\tau-s}(\mathcal{Z})$$

4. Symmetric pairing of currents:

$$b_0(\check{j}_1, \check{j}_2) = \check{\imath} \int_{\mathcal{X}}^{\mathcal{E}} \theta_0(\check{j}_1)\check{j}_2 \in \check{I}^2(\mathcal{P})$$

$$b_1(\check{j}_1, \check{j}_2) = \check{\imath} \int_{\mathcal{Y}}^{\mathcal{E}} \theta_1(\check{j}_1)\check{j}_2 \in \check{I}^1(\mathcal{P})$$

$$b_2(\check{j}_1, \check{j}_2) = \check{\imath} \int_{\mathcal{Z}}^{\mathcal{E}} \theta_2(\check{j}_1)\check{j}_2 \in \check{I}^0(\mathcal{P})$$

5. Quadratic Refinement

$$q_i(\check{j}_1 + \check{j}_2) - q_i(\check{j}_1) - q_i(\check{j}_2) + q_i(0) = b_i(\check{j}_1, \check{j}_2)$$

$$q_0(\check{j}) \in \check{I}^2(\mathcal{P}) = \text{$\mathbb{Z}_2$-graded line bundles over $\mathcal{P}$ with connection}$$

$$q_1(\check{j}) \in \check{I}^1(\mathcal{P}) = Map(\mathcal{P}, \mathbb{R}/\mathbb{Z})$$

$$q_2(\check{j}) \in \check{I}^0(\mathcal{P}) = Map(\mathcal{P}, \mathbb{Z})$$

Formulating the theory

Using these data one can formulate a self dual theory.

The topological data of q_2 and θ_2

in $(n+2)$ dimensions

determines q_1, q_0

Physical Interpretation: Holography

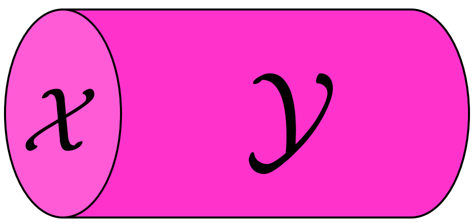
Generalizes the well-known example of the holographic duality between 3d abelian Chern-Simons theory and 2d RCFT.

See my Jan. 2009 AMS talk on my homepage for this point of view, which grows out of the work of Witten and Hopkins & Singer, and is based on my work with Belov and Freed & Segal

Holographic Formulation

$[\check{A}] \in \check{\mathcal{E}}^{\tau}(\mathcal{Y})$: Chern-Simons gauge field.

$q_1(\check{A}) \in Map(\mathcal{P}, \mathbb{R}/\mathbb{Z})$: Chern-Simons action.



Edge modes = self-dual gauge field

$$\check{A}|_{\mathcal{X}} = \check{j}$$

Chern-Simons wavefunction= Self-dual partition function

$$\Psi(\check{A}|_{\mathcal{X}}) = Z(\check{j})$$

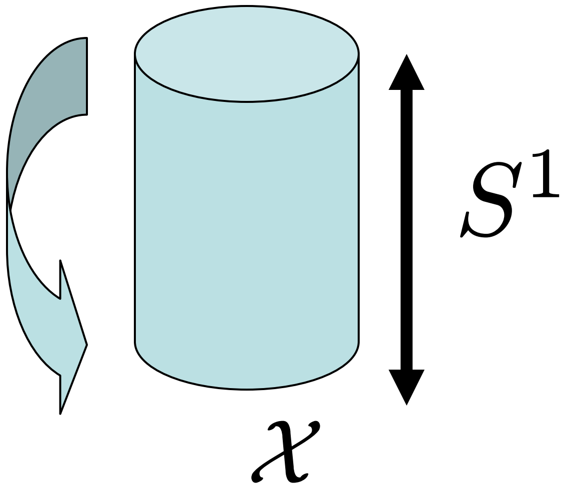
Defining the Background Charge - I

Identify automorphisms of $\check{j}$ with $\alpha \in \mathcal{E}^{\tau-2}(\mathcal{X}, \mathbb{R}/\mathbb{Z})$

Identify these with global gauge transformations

Automorphisms act on CS wavefunction

$$(\alpha \cdot \Psi)(\check{j}) = e^{2\pi i q_1(\check{j}+\check{\alpha}\check{t})}\Psi(\check{j})$$



Global gauge transformations:

$$(\alpha \cdot \Psi)(\check{j}) = e^{2\pi i \alpha \cdot \mathcal{Q}}\Psi(\check{j})$$

Defining the Background Charge - II

$$e^{2\pi i\alpha\cdot\mathcal{Q}}\Psi(\check{j}) = e^{2\pi i q_1(\check{j}+\check{\alpha}\check{t})}\Psi$$

$$q_1(\check{j}+\check{\alpha}\check{t}) = \int\theta(j)\alpha + q_1(\check{\alpha}\check{t})$$

$\alpha \to q_1(\check{\alpha}\check{t})$ is linear

Poincare duality: $q_1(\check{\alpha}\check{t}) := \iota\int_{\mathcal{X}}^{\mathcal{E}}\theta(\mu)\alpha$

$\mu \in \mathcal{E}^{\tau}(\mathcal{X})$ ``background charge''

Computing Background Charge

A simple argument shows that twice the charge is computed by

$$q_1(y) - q_1(-y) = \int_{\mathcal{X}}^{\mathcal{E}} \theta_0(-2\mu)\alpha$$

$$y = \alpha t$$

Heuristically:

$$q_1(y) = \frac{1}{2}(y-\mu)^2 + const.$$

Self-Duality for Type II RR Field

$$\text{Now } [\check{j}] \in K(\mathcal{Z}) \text{ and } \dim \mathcal{Z}/\mathcal{P} = 12$$

It turns out that

$$q_2(j) = \int_{\mathcal{Z}}^{KO} \bar{j} j \in \mathbb{Z}$$

correctly reproduces many known facts in string theory and M-theory

Witten 99, Moore & Witten 99, Diaconescu, Moore & Witten, 2000, Freed & Hopkins, 2000, Freed 2001

Self-duality for Orientifold RR field

Now $j \in KR^{\beta}(\mathcal{Z})$ and $\dim \mathcal{Z}/\mathcal{P} = 12$

We want to make sense of a formula like

$$q_2(j) = \int_{\mathcal{Z}}^{KO} \bar{j} j \in \mathbb{Z}$$

But $\bar{j} j \in KR^{\bar{\beta}+\beta}(\mathcal{Z})$, not in KO

And, we need a KO density!

The real lift

Lemma: There exists maps

$$\Re : \mathrm{Twist}_{KR}(\mathcal{M}_w) \to \mathrm{Twist}_{KO}(\mathcal{M})$$

$$\rho : KR^{\beta}(\mathcal{M}_w) \to KO^{\Re(\beta)}(\mathcal{M})$$

So that under complexification:

$$\rho(j) \to u^{-d}\bar{j}j$$

$$\Re(\beta) \to \beta + \bar{\beta} - d\tau(u)$$

Twisted Spin Structure - I

In order to integrate in KO,

$$\rho(j) \in KO^{\Re(\beta)}(\mathcal{M})$$

Must be an appropriately twisted density

For simplicity now take $\mathcal{M} = M//\mathbb{Z}_2$

$$\int_M^{KO_{Z_2}} : KO_{Z_2}^{\tau_{KO_{Z_2}}(M)+j} \to KO^j_{\mathbb{Z}_2}(pt)$$

Twisted Spin Structure- II

Definition: A twisted spin structure on $\mathcal{M}$ is

$$\kappa : \Re(\beta) \cong \tau_{KO}(T\mathcal{M} - \dim \mathcal{M})$$

Note: A spin structure on M allows us to integrate in KO. It is an isomorphism

$$0 \cong \tau_{KO}(TM - \dim M)$$

Existence of tss $\Longrightarrow$ Topological conditions on B

Orientifold Quadratic Refinement

$$\int_{\mathcal{Z}}^{KO} \kappa\rho(j) \in KO_{\mathbb{Z}_2}^{-12}(pt)$$

$$KO_{\mathbb{Z}_2}^{-12}(pt) \cong KO^{-4}(pt) \otimes (\mathbb{Z} \oplus \mathbb{Z}\varepsilon)$$

$$\iota : KO^{-4}(pt) \to I^0(pt) \cong \mathbb{Z}$$

Definition: $q_2(j) := \left[\iota \int_{\mathcal{Z}}^{KO} \kappa\rho(j)\right]_{\varepsilon}$

At this point we have defined the

``background RR charge''

of an orientifold spacetime.

How about computing it?

Localization of the charge on $X//\mathbb{Z}_2$

$$q_1(y) - q_1(-y) = \int_{\mathcal{X}}^{\mathcal{E}} \theta_0(-2\mu)\alpha$$

$$\left\{\iota \int_Y^{KO_{Z_2}} [\rho(y)) - \rho(-y)]\right\}_{\varepsilon} = \iota \int_Y^{KR} \theta(-2\mu) y$$

Localize wrt $S = \{(1-\varepsilon)^n\} \subset R(\mathbb{Z}_2)$

Atiyah-Segal localization theorem →

Background charge with 2 inverted
localizes on the O-planes.

K-theoretic O-plane charge

$$\mu = i_*(\Lambda) \in KR^{\beta}[\tfrac{1}{2}](X)$$

$$i : F \hookrightarrow X \quad \nu = \text{Normal bundle}$$

$$\Psi(\Lambda) = 2^d \frac{C(F)}{\text{Euler}(\nu)}$$

``Adams Operator" $\Psi : KR^{\beta}[\tfrac{1}{2}](Y) \to S^{-1} KO_{\mathbb{Z}_2}^{\mathrm{Re}(\beta)}(Y)$

$C(F)$ KR-theoretic Wu class generalizing Bott's cannibilistic class

Special case: Type I String

Freed & Hopkins (2001)

Type I theory: $\mathcal{X} = X // \mathbb{Z}_2$

With $\mathbb{Z}_2$ acting trivially and $\beta = 0$

$$2\mu = -\Xi(X)$$

$\Xi(F)$: KO-theoretic Wu class

$$\int_F^{KO} \psi_2(x) = \int_F^{KO} \Xi(F) x$$

$$-\mu = TX + 22 + Filt(\geq 8)$$

The physicists' formula

Taking Chern characters we get the physicist's (Morales-Scrucca-Serone) formula for the charge in de Rham cohomology:

$$-\sqrt{\hat{A}(TX)}\,\mathrm{ch}(\mu) = \pm 2^k \iota_* \sqrt{\frac{\tilde{L}(TF)}{\tilde{L}(\nu)}}$$

$\tilde{L}(V) := \prod_i \frac{x_i/4}{\tanh(x_i/4)}$ $\qquad k = \dim F - 5$

Topological Restrictions on the B-field

One corollary of the existence of a twisted spin structure is a constraint relating the topological class of the B-field to the topology of X

$$w_1(\mathcal{X}) = dw \qquad w_2(\mathcal{X}) = \frac{d(d+1)}{2} w^2 + aw$$

$$[\beta] = (d, a, h)$$

This general result unifies scattered older observations in special cases.

Examples

Zero B-field

If [β]=0 then we must have IIB theory on $\mathcal{X}$ which is orientable and spin.

Op-planes

$$\mathcal{X} = \mathbb{R}^{p+1} \times \mathbb{R}^{r} // \mathbb{Z}_2 \qquad p + r = 9$$

Compute: $w_1(\mathcal{X}) = rw \qquad w_2(\mathcal{X}) = \frac{r(r-1)}{2} w^2$

⇒ $d = r \bmod 2 \quad a = \begin{cases} 0 & r = 0,3 \mod 4 \\ w & r = 1,2 \mod 4 \end{cases}$

Pinvolutions

$$\mathcal{X} = X /\!/ \mathbb{Z}_2$$

Deck transformation σ on X lifts to Pin^- bundle.

$r = 0$	$KR^{0}(\mathcal{X}_w)$	$KR^{\beta_2}(\mathcal{X}_w)$
$r = 1$	$KR^{1+\beta_1}(\mathcal{X}_w)$	$KR^{1+\beta_3}(\mathcal{X}_w)$
$r = 2$	$KR^{\beta_1}(\mathcal{X}_w)$	$KR^{\beta_3}(\mathcal{X}_w)$
$r = 3$	$KR^{1}(\mathcal{X}_w)$	$KR^{1+\beta_2}(\mathcal{X}_w)$

$r =$ cod. mod 4 of orientifold planes

Older Classification

Op^-	K-group
$O0^-$	$KR_\pm(S^{9,0}) = \mathbb{Z} \oplus \mathbb{Z}_2$
$O1^-$	$KR^{-1}(S^{8,0}) = \mathbb{Z} \oplus \mathbb{Z}_2$
$O2^-$	$KR(S^{7,0}) = \mathbb{Z} \oplus \mathbb{Z}$
$O3^-$	$KH_\pm^{-1}(S^{6,0}) = \mathbb{Z}$
$O4^-$	$KH_\pm(S^{5,0}) = \mathbb{Z}$
$O5^-$	$KH^{-1}(S^{4,0}) = \mathbb{Z}$
$O6^-$	$KH(S^{3,0}) = \mathbb{Z} \oplus \mathbb{Z}$
$O7^-$	$KR_\pm^{-1}(S^{2,0}) = \mathbb{Z}$
$O8^-$	$KR_\pm(S^{1,0}) = \mathbb{Z}$

Op^+	K-group
$O0^+$	$KH_\pm(S^{9,0}) = \mathbb{Z}$
$O1^+$	$KH^{-1}(S^{8,0}) = \mathbb{Z}$
$O2^+$	$KH(S^{7,0}) = \mathbb{Z} \oplus \mathbb{Z}$
$O3^+$	$KR_\pm^{-1}(S^{6,0}) = \mathbb{Z}$
$O4^+$	$KR_\pm(S^{5,0}) = \mathbb{Z} \oplus \mathbb{Z}_2$
$O5^+$	$KR^{-1}(S^{4,0}) = \mathbb{Z} \oplus \mathbb{Z}_2$
$O6^+$	$KR(S^{3,0}) = \mathbb{Z} \oplus \mathbb{Z}$
$O7^+$	$KH_\pm^{-1}(S^{2,0}) = \mathbb{Z}$
$O8^+$	$KH_\pm(S^{1,0}) = \mathbb{Z}$

Table 2: Orientifold K-theory groups for RR fields.

(Bergman, Gimon, Sugimoto, 2001)

Orientifold Précis : NSNS Spacetime

1. $\mathcal{X}$: 10-dimensional Riemannian orbifold with dilaton.

2. Orientifold double cover $\mathcal{X}_w$, $w \in H^1(\mathcal{X}, \mathbb{Z}_2)$.

3. B: Differential twisting of $\check{K}R(\mathcal{X}_w)$

4. Twisted spin structure:

$$\kappa : \Re(\beta) \cong \tau_{KO}(T\mathcal{X} - \dim \mathcal{X})$$

Orientifold Précis: Consequences

1. Well-defined worldsheet measure.

2. K-theoretic definition of the RR charge of an orientifold spacetime.

3. RR charge localizes on O-planes after inverting two, and $\mu = i_*(\Lambda) \quad \Psi(\Lambda) = \frac{2^d C(F)}{\mathrm{Euler}(\nu)}$

4. Well-defined spacetime fermions and couplings to RR fields.

5. Possibly, new NSNS solitons.

Conclusion

The main future direction is in applications

- Destructive String Theory?
- Tadpole constraints (Gauss law)
- Spacetime anomaly cancellation
- S-Duality Puzzles