Optimal Tax Policy Under Uncertainty Over Tax Revenues

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Abstract

This paper investigates optimal tax policy with uncertainty in private and public consumption. Tax policy affects volatility through two channels; spreading risk between public and private consumption and hedging idiosyncratic tax source risk. This paper develops an excessive risk index to quantify the amount of unnecessary risk governments accept. For US state governments, I find the cost of inefficient risk is roughly 40% of their expected tax revenues. This cost is mostly due to states being overdependent on a given tax source, e.g., income or sales taxes. In 2014, 36 states are found to be overdependent in either the income or sales tax.

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1 Introduction

The principles of optimal taxation have been built up from a large literature that has considered efficiency, redistribution, agency costs, among others. However, there are still open questions about how optimal tax principles differ when uncertainty is added. In part this is due to the fact that uncertainty can arise from, and impact, many different facets of an optimal tax model. Most of the literature on optimal taxation and uncertainty has focused on uncertainty of private consumption through uncertainty about production or wages (see, e.g., Eaton and Rosen, 1980; Diamond and Mirrlees, 1978). The purpose of this paper is to analyze optimal taxation when there is uncertainty over private and *public* consumption.[1]

The investigation of optimal taxation with uncertainty about public consumption was motivated by large shocks to state government expenditures in the 2000s. For example, California, in the middle of the 2012 fiscal year, made a billion dollars of cuts to core services, such as K-12 education. In 2002 Indiana faced a $1.3 billion deficit that led the governor to freeze hiring and a series of capital projects and to call for a 7 percent reduction in expenditures across the board. Previous studies have shown these shocks in expenditures are linked to shocks in tax revenues because states typically do not smooth their expenditures but spend revenues received.[2]

The optimal tax policy when uncertainty is added to private and public consumption in a standard model characterizes the tradeoff between tax revenue volatility and deadweight loss. This tradeoff produces several novel implications. First, lump sum taxes are no longer optimal because they concentrate risk in private consumption. Second, optimal tax policy reduces to optimal portfolio weights without deadweight loss. This motivates using an op-

[1]The author is unaware of any other study that has incorporated public consumption uncertainty into a model of optimal taxation.

[2]There are several potential reasons states have difficulty smoothing expenditures. First, 49 states have balanced budget constraints, which limit their ability to smooth. Second, there are political economy and moral hazard problems. Third, even when states try to smooth revenues through the use of rainy day funds, these funds have been found to be inadequate Sobel and Holcombe (1996a).

timal portfolio approach to better understand tax revenue risk. In particular, different tax portfolios are shown to expose governments to different amounts of risk. Some of these portfolios, however, are found to be mean-volatility efficient. That is, there are a set of portfolios such that a government cannot increase its expected revenues without accepting more risk. This set is a government's minimum volatility frontier.

I develop an excessive risk index to distinguish between efficient and inefficient risk. This index is comparable across governments and across time and can be interpreted as the percent decrease in expected tax revenue associated with holding a tax portfolio that has excessive risk.

I use data from 1970–2014 on all US states to demonstrate how to calculate the excessive risk index. I find that tax portfolios were on average 7 percent riskier in the 2000s than in the 1970s.[3] Despite the substantial increase in tax risk, excessive risk experienced only a slight increase. Even so, states are accepting a large amount of excessive risk. On average, states could increase their expected tax revenues by roughly 40 percent without having to increase their tax revenue volatility.

The optimal portfolio approach to understand optimal tax policy in the face of uncertainty in tax revenues adds to a literature that considers the costs of suboptimal policy in the face of uncertainty (Golosov and Tsyvinski, 2007). These dynamic Mirrleesian models focus on the misallocation of capital and labor due to uncertainty in privately observable heterogeneous skills (Golosov et al., 2003; Werning, 2007).[4] I use a representative-agent Ramsey model,

[3]The result that the riskiness of tax portfolios increased in the 2000s adds to a literature that discusses the increase in tax revenue volatility in the 2000s, (Mattoon and McGranahan, 2012; Seegert, 2012; Kodrzycki, 2014). These papers all build off of a seminal paper on tax revenue volatility by Groves and Kahn (1952). Papers building off of Groves and Kahn (1952) consider the volatility of different tax revenue sources (Dye and McGuire, 1991; Sobel and Holcombe, 1996b; Bruce et al., 2006; Giertz, 2006).

[4]The seminal papers by Mirrlees (1971); Diamond and Mirrlees (1978); Atkinson and Stiglitz (1976); Rogerson (1985); Stiglitz (1987) have been extended by the new dynamic public finance literature to understand when it is optimal to distort savings, capital, and labor Golosov et al. (2003); Kocherlakota (2005). Recent papers by Shimer and Werning (2008), Albanesi and Sleet (2006), Abraham and Pavoni (2008), Da Costa and Werning (2008), and Golosov and Tsyvinski (2007) consider optimal policy when uncertainty creates a wedge in capital allocation. Other papers considers optimal policy when there is a wedge in labor allocation, including but not limited to Battaglini and Coate (2008a,b), Rothschild and Scheuer (2011), and

which abstracts from this type of uncertainty, to focus on optimal policy in response to aggregate shocks following papers by Lucas and Stokey (1983); Judd (1999); Kingston (1991); Judd (1992); Zhu (1992), and Chari et al. (1994).[5] I extend this model by using an optimal portfolio approach that also takes into account the ways taxation can insure production risk across public and private consumption.[6]

The rest of the paper is organized as follows: in Section 2, I develop a model of optimal taxation with uncertainty in private and public consumption. The implications of this model are derived in section 3. Section 4 describes the data and methods used to demonstrate the theoretical results empirically. The excessive risk index for all states in 1970 and 2014 is reported in section 5. Section 6 concludes.

2 Optimal Tax Policy With Volatility

The following model adds volatility to a traditional model of optimal taxation of J goods with a representative agent.[7] Income taxation can be modeled by including labor as a consumption good with a price equal to negative the wage.[8] Appendix B provides a microfounded model that focuses on income and sales tax revenue, which is also used as an example in 3.2.3.

Farhi and Werning (2013).

[5]Previous research suggests in response to aggregate shocks the optimal policy smooths taxes; for example, the expected capital gains tax should be zero and there should be a relatively constant tax on labor (Aiyagari et al., 2002; Barro, 1979; Lucas and Stokey, 1983).

[6]The use of automatic stabilizers, particularly in the tax code, is gaining increasing attention (e.g., Krueger and Perri (2006); Chetty and Saez (2010); Huggett and Parra (2010); Krueger and Perri (2011); Ramey (2011); Mankiw and Weinzierl (2011); Auerbach and Gorodnichenko (2012); Clemens and Miran (2012); Michaillat (2014); Michaillat and Saez (2015); Shoaa (2013); Dauchy and Seegert (2015)).

[7]The model can be extended to include heterogeneous agents following Diamond and Mirrlees (1971).

[8]Labor equals a time endowment minus leisure and can be defined as good $x_J = L \equiv \bar{L} - l$ with $p_J = w$. This implies that leisure is a consumption good with a price equal to the wage.

2.1 Adding Volatility to a Traditional Optimal Tax Model

The model consists of a representative agent, a government, and nature. First, the government chooses its tax policy. Second, nature chooses the state of the world, captured by income m. Finally, the representative agent chooses his consumption after observing tax policy and income.

The representative agent chooses his consumption of J goods, $x = (x_1, ..., x_J)$, subject to the budget constraint $px \leq m$ where m is exogenous income. There is no uncertainty for the representative agent because at the time he chooses x, he observes income m and the government's tax policy choice.[9] The representative agent is risk averse and maximizes his utility, $u(x; g)$, which is increasing and concave in private consumption, x, and the composite public good g. For expositional ease, I make the common assumption that the cross-price elasticities are zero.[10]

Uncertainty in the model is due to supply-side shocks to income m that enter utility only through uncertainty in consumption. The government does not know the realization of income m when it makes its decision, but it knows the distribution of income, $F(m, \sigma_m^2)$.[11] The government anticipates that the representative agent will optimally choose consumption in response to the realization of income and the government's tax policy. The government makes its decision based on the mean and variance-covariance matrix of the representative agent's equilibrium consumption bundle, conditional on the government's tax policy defined as the $1 \times J$ vector of means, $\mu_x^* = (\mu_1^*, ..., \mu_J^*)$, and the $J \times J$ variance-covariance matrix, W_x^*.

The government moves first and sets linear taxes on the J consumption goods, $\tau = (\tau_1, \tau_2, ..., \tau_J)$. By choice of units, I can set the vector of consumer prices to be $p = (1 +$

[9] This model abstracts from uncertainty to individuals, which has been the focus of most of the rest of the extant literature, to focus on tax revenue volatility.

[10] Formally, this assumption is that $\partial x_j / \partial p_k = 0$ for all $j \neq k$. This common assumption is made to write the optimal tax rule in a way that is consistent with past derivations of optimal tax rules.

[11] For expositional ease, I focus on a distribution of income fully characterized by its mean and variance. The analysis can be extended to allow for higher-order moments.

$\tau_1, ..., 1+\tau_J)$. The composite public good, g, is equal to the tax revenues the government collects, with expected value $g = \tau'\mu_x$ and variance $\sigma_g^2 = \tau' W_x \tau$.

The expected utility of the representative agent can be written as a function of prices, expected income, expected public good consumption, the variance of income, and the variance of the public good,

$$V(p, \bar{m}, \bar{g}, \sigma_m^2, \sigma_g^2) = \int v(p, m, g) F(\bar{m}, \sigma_m^2),$$

where $v(p, m, g)$ is the representative agent's indirect utility function.

The government maximizes the expected utility of the representative agent by setting linear taxes τ,

$$max_\tau \quad V(p, \bar{m}, \bar{g}, \sigma_m^2, \sigma_g^2),$$

where prices, p, and the mean and variance of the public good, g and σ_g^2, are functions of the tax rates the government sets.

The government's problem can be rewritten from choosing tax rates τ on all goods x, to choosing a lump sum tax T, a risk sharing tax rate t, and a good specific tax $\hat{\tau}$, where $\sum_J \hat{\tau}_j = 0$. This transforms representative agent's budget constraint from

$$\sum_J (1+\tau_j) x_j = m \quad \text{to} \quad \sum (1+\hat{\tau}_j) x_j = (1-t)m - T.$$

For expositional ease, I write the expected utility function as a function of after-tax income $\tilde{m} = (1-t)m - T$, instead of income, $V(p, \tilde{m}, g, \sigma_{\tilde{m}}^2, \sigma_g^2)$. Public and private consumption risk can now be written as a function of the risk sharing tax rate t and the good specific tax $\hat{\tau}_j$,

$$\sigma_{\tilde{m}}^2 = (1-t)^2 \sigma_m^2 \qquad \sigma_g^2 = t^2 \sigma_m^2 + \hat{\tau}' W_x \hat{\tau}.$$

From this equation it is clear that t determines the share of risk between public and private consumption and $\hat{\tau}$ affects the hedging of tax revenues.

The government is sensitive to the variances of private and public consumption because the government maximizes the expected utility of a risk-averse representative agent. The government's attitude toward risk encompasses the representative agent's risk preferences and, implicitly, the ability of the representative agent and government to smooth shocks: $V_{\sigma_m^2} = \partial V/\partial\sigma_m^2 \leq 0$ and $V_{\sigma_g^2} = \partial V/\partial\sigma_g^2 \leq 0$. When the government and representative agent can perfectly smooth income shocks, $V_{\sigma_m^2} = V_{\sigma_g^2} = 0$, and the model reduces to a traditional optimal tax model without uncertainty.

The government balances costs due to deadweight loss and volatility, which can be summarized by two elasticities. The disutility from deadweight loss is captured by the elasticity of good x_k with respect to the tax rate τ_k: $\varepsilon_{x_k,\tau_k} = (\partial x_k/\partial\tau_k)(\tau_k/x_k)$. The disultility from volatility is captured by the elasticity of the variance of tax revenue with respect to the tax rate τ_k, scaled by tax revenue generated by tax rate τ_k,: $e_{\sigma_g,\tau_k} = (\partial\sigma_g/\partial\tau_k)(\tau_k/\sigma_g)(1/\tau_k x_k)$. The optimally set tax rates balance these costs subject to their importance to utility, captured by the welfare weight, $\omega = (\partial V/\partial\sigma_g^2)(1/(\partial V/\partial g))\sigma_g^2$. The welfare weight depends on the specific utility function and is the same across all tax rates.

2.2 Optimal Tax Rule With Volatility

Theorem 1 *Optimal tax policy with volatility is characterized by the rule for any j, k pair,*

$$\varepsilon_{x_j,\tau_j} + \omega e_{\sigma_g,\tau_j} = \varepsilon_{x_k,\tau_k} + \omega e_{\sigma_g,\tau_k}, \tag{1}$$

where ω is a weight that depends on the utility function, ε_{x_k,τ_k} is the elasticity of good k with respect to the tax rate τ_k, and e_{σ_g,τ_j} is the elasticity of tax revenue volatility with respect to the tax rate τ_k, scaled by tax revenues from good k.

The proof of Theorem 1 is provided in Appendix A.

The optimal tax rule with volatility reflects the tradeoff between deadweight loss and volatility. Each revenue source the government uses balances the costs from distorting consumption behavior, captured by the elasticity, ε_{x_k,τ_k}, and the cost of volatility, captured by e_{σ_g,τ_k}. The optimal tax rule implies that governments are willing to accept more distortions to a given good if it decreases the cost of volatility. The following corollaries explore this tradeoff and its implications for optimal tax policy. The proofs for all of the corollaries are given in Appendix A.

3 Implications of the Optimal Tax Rule With Volatility

3.1 Comparing Optimal Tax Rules With and Without Volatility

First, consider the optimal tax rule without volatility, when the government knows the state of the world before making its decision.

Corollary 1 *When the government knows the state of the world before making its decisions, the optimal tax rule with volatility reduces to the traditional optimal tax rule, which can be written as the inverse elasticity rule,*

$$\frac{\tau_k}{p_k} = \frac{V_m - V_g}{V_g} \frac{1}{\varepsilon_{x_k,p_k}}. \tag{2}$$

Corollary 1 restates the traditional optimal tax rule that all goods should have an approximately proportional reduction in compensated demand.[12] This conclusion, however, holds only in the absence of volatility as this conclusion does not follow from the optimal tax rule in equation (1).

[12]The inverse elasticity rule can be rewritten to look similar to the optimal tax rule derived in Theorem 1. Combining the inverse elasticity rule for any pair j, k gives the relationship $(\tau_j/p_j)\varepsilon_{x_j,p_j} = (\tau_k/p_k)\varepsilon_{x_k,p_k}$. This relationship can be reduced to $\varepsilon_{x_j,\tau_j} = \varepsilon_{x_k,\tau_k}$ given the relationship $\varepsilon_{x_k,\tau_k} = (\tau_k/p_k)\varepsilon_{x_k,p_k}$. This relationship is the optimal tax rule derived in Theorem 1 when the cost of volatility is zero: $\varepsilon_{\sigma,j} = 0$ for all j.

Second, consider whether lump sum taxes are optimal with and without volatility.

Corollary 2 *When the government knows the state of the world before making its decisions, lump sum taxes are optimal, but when the government does not know the state of the world before making its decision, this no longer holds.*

Corollary 2 captures the tradeoff between private and public volatility embedded in the optimal tax rule with volatility. Lump sum taxes minimize deadweight loss. Without volatility, minimizing deadweight loss is optimal; therefore, lump sum taxes are optimal. With volatility, lump sum taxes provide certain revenue to the government and concentrate risk in private consumption. In this case, welfare can often be improved by spreading risk between public and private consumption; therefore, lump sum taxes are not optimal with volatility.

This result is related to but distinct from the result that lump sum taxes are inefficient with uncertainty in private consumption. Eaton and Rosen (1980) notes that proportional taxation of earnings provides insurance to individuals because the government shares in both losses and gains. In the case with volatility, this insurance is also true but comes with a cost of increased volatility of public consumption. In the case with volatility, therefore, proportional taxation is not solely insurance in private consumption but also about sharing risk between public and private consumption. For example, in the case with exogenous income and uncertainty a 100% earnings tax with lump sum payments of the expected value of earnings would be optimal, as in the case considered by Eaton and Rosen (1980), but is not optimal in the case of volatility because it concentrates risk in public consumption (the opposite case of lump sum taxes).

Finally, corrollary 3 derives a sufficient condition to determine whether a government's tax policy deviates from the optimal rule, without specifying a utility function.

Corollary 3 *A tax system does not satisfy the optimal tax rule with volatility if for some good j the elasticities of good j and the volatility of good j are both larger than for any other* *good* $k \neq j$.

$$\varepsilon_{x_j,\tau_j} > \varepsilon_{x_k,\tau_k} \quad \text{and} \quad e_{\sigma_g,\tau_j} > e_{\sigma_g,\tau_k}.$$

Corollary 3 can be used to empirically test whether states deviate from the optimal tax rule in Theorem 1. This empirical test is demonstrated in section 5.4. To implement this test, I first note that the elasticity of good j with respect to tax rate j equals the elasticity of tax revenues of good j with respect to tax rate j minus one. The condition in corollary 3 reduces to comparing the elasticity of tax revenue and the elasticity of revenue volatility scaled by tax revenues.

3.2 Deriving an Optimal Tax Portfolio

3.2.1 Principles of Optimal Taxation from Optimal Portfolio Theory

An alternative approach to understanding the optimal tax rule with volatility is to consider optimal tax rates as akin to an optimal portfolio of assets a government holds. For example, consider the optimal tax rule in the absence of deadweight loss and private consumption volatility. This is the case when the representative agent can perfectly smooth shocks to private consumption, and taxes do not cause a substitution between consumption goods (e.g., perfect complements). In this case, the government's objective function reduces to minimizing the variance of public good consumption, $\tau' W_x \tau$, subject to the expected revenue constraint, $\bar{g} = \tau' \mu_x$. With this objective function, tax rates τ can be thought of as weights on different assets x, as in the classic portfolio selection model of Markowitz (1952).

Corollary 4 *The optimal tax rule reduces to the formula for optimal portfolio weights when the representative agent can perfectly smooth shocks to private consumption, and taxes do not cause a substitution of consumption goods (e.g., perfect complements).*

Corollary 4 captures the tradeoff between taxing different sources to hedge revenue-specific risk. Several aspects of optimal taxation can be illuminated by appealing to the literature on optimal portfolio weights. First, uniform taxation is optimal if each revenue source is independently and identically distributed and non-negative (Samuelson, 1967). Uniform taxation, therefore, is not generally optimal if revenue sources are not independently and identically distributed and non-negative. Second, if revenue stream j is independently distributed from the other revenue streams, then the optimal tax system must have a positive tax on revenue stream j.[13] These characterizations highlight that hedging revenue-specific risk should be added to the set of optimal tax principles—especially for governments that find it difficult to smooth revenues.

3.2.2 Characterizing the Optimal Tax Portfolio

To understand the effects of tax rates on tax revenue volatility, consider the variance of tax revenue when a government taxes two goods. For concreteness call the two goods wage income $I = wl$, which is a function of a wage and labor supply l, and taxable consumption $c = \beta C$, which is a function of total consumption C and the fraction of consumption in the consumption tax base. Labor supply and the fraction of consumption in the consumption tax base are chosen by the representative agent and depend on the wage and consumption tax rates, τ_w and τ_c. For exposition, assume wages and total consumption are random variables with variances σ_w^2 and σ_c^2 and covariance $\sigma_{w,c}$. In this example, the variance of public consumption is,

$$\sigma_g^2 = \tau_w^2 l^2 \sigma_w^2 + \tau_c^2 \beta^2 \sigma_c^2 + 2\tau_c \beta \tau_w l \sigma_{w,c}. \tag{3}$$

See Appendix B for a microfounded model with the same variance of public consumption.

The total derivative of the variance of tax revenue, allowing the tax rate on wage income

[13] This result relies on 1) a choice of units such that each revenue source has a common mean, and 2) the assumption that each revenue source has a finite but nonzero variance.

to vary is

$$\frac{d\sigma_g^2}{d\tau_w} = \underbrace{(2\tau_w l^2 \sigma_w^2 + 2l\tau_c\beta\sigma_{c,w})}_{\text{Direct}}(1+\varepsilon_{l,\tau_w}) + \underbrace{(2\beta^2\frac{\tau_c^2}{\tau_w}\sigma_c^2 + 2l\tau_c\beta\sigma_{c,w})\varepsilon_{\beta,\tau_w}}_{\text{Horizontal}}$$
$$+ \underbrace{\frac{\tau_c^2}{\tau_w}\beta^2\sigma_c^2\varepsilon_{\sigma_c^2,\tau_w} + \tau_w l^2\sigma_w^2\varepsilon_{\sigma_w^2,\tau_w} + 2l\tau_c\beta\sigma_{c,w}\varepsilon_{\sigma_{c,w},\tau_w}}_{\text{Higher Moments}}. \tag{4}$$

Equation (4) can be broken into three parts: (1) the direct effect, which captures the change in income tax revenue volatility and its covariance with consumption tax revenue, (2) the horizontal effect, which captures the change in the volatility of consumption tax revenues from a change in the wage income tax, and (3) the higher moments effect, which captures the effect of changing the wage income tax on the variance and covariance of wage income and consumption.

The direct effect is damped due to behavioral responses of the representative individual, defined as leakage. The leakage is given by the elasticity of labor supply, which increasingly damps the direct effect as the elasticity of labor supply becomes more negative. The sign on the horizontal effect depends on whether the change in the wage income tax causes individuals to consume more or less of the private goods in the consumption tax base, captured by the elasticity of β with respect to the wage income tax rate.

Traditional optimal portfolio problems only consider the direct effect, without leakage. However, because the government is a large player, its optimal portfolio problem must consider the horizontal and higher moment effects. Said differently, actions of the government have broader implications for the economy as a whole than a single trader optimizing their personal portfolio.

3.2.3 Minimum Variance Frontier

Similar to traditional optimal portfolio problems, the government is constrained by a minimum volatility frontier. On this frontier, tax portfolios have the minimum volatility for a given expected level of tax revenue. Portfolios on the minimum volatility frontier are characterized by the condition

$$\frac{\varepsilon_{\sigma_g^2,\tau_c}}{\varepsilon_{g,\tau_c}} = \frac{\varepsilon_{\sigma_g^2,\tau_w}}{\varepsilon_{g,\tau_w}}. \tag{5}$$

Corollary 5 *The set of feasible tax portfolios a government can hold is constrained by a minimum volatility frontier that includes leakage, horizontal effects, and higher order effects, in addition to the direct effect considered in traditional optimal portfolio problems.*

Figure 1 draws the minimum volatility frontier with volatility on the horizontal axis and expected revenue on the vertical axis. The underlying economic uncertainty is taken as exogenous in the minimum volatility frontier, e.g., the variance of wages and total consumption. Differences in risk between tax portfolios are due to differences in tax policy. For example, a portfolio consisting solely of a highly progressive income tax is riskier than a balanced portfolio that includes and consumption taxes.

The frontier is increasing and concave. The shape of the frontier implies that the amount of volatility a government needs to accept increases as the level of expected revenues increases. This has implications for how much of the public good the government should provide. It is well understood that the cost of taxation due to deadweight loss is convex. The addition of volatility, as highlighted by the minimum volatility frontier, adds to this convexity because the cost of volatility also increases as the mean increases.

The minimum variance frontier demonstrates that tax revenues may be volatile because governments are holding portfolios with high expected tax revenues or because governments are holding portfolios far off of the minimum variance frontier. Knowing why tax revenues are volatile is important for policymakers. To separate these two components, the following section decomposes risk into necessary and unnecessary risk and derives a volatility index.

3.3 Excessive Risk Index

This section derives an excessive risk index that characterizes the necessary and unnecessary risk of a tax portfolio. The riskiness of a tax portfolio can be characterized in many different ways—each with their own implications. The purpose of the excessive risk index is to provide a measure of mean-volatility efficiency, which is distinct from volatility and other such measures.

Corollary 6 *Risk from tax policy can be decomposed into economic uncertainty and necessary and unnecessary tax policy risk.*

To develop intuition for different measures of riskiness consider the four tax portfolios and the minimum volatility frontier drawn in Figure 1. Volatility of each tax portfolio is given on the horizontal axis, such that portfolios C and D are more volatile than portfolio A.[14] The change in volatility (holding economic conditions fixed) represents the tax effect; the amount of additional volatility due to changes in a state's tax rates. For example, a state changing its tax rate to move from portfolio A to D would be accepting more volatility.

The tax effect, however, does not distinguish between efficient and inefficient risk. Whether a state is efficiently accepting more risk or not depends on whether it is moving closer or further away from its minimum volatility frontier. In the example drawn in Figure 1, moving from portfolio A to D moves the government closer to its frontier. This government, therefore, increases its volatility (as seen on the horizontal axis) but does so in a relatively efficient manner.

The goal of the excessive risk index is to provide a measure that captures the effi-

[14] The riskiness of a portfolio can also be measured relative to its mean by the coefficient of variation (or inverse Sharpe ratio), defined as the volatility divided by the mean. With this measure of riskiness, portfolio C is less risky than portfolio A, even though it has a higher volatility because portfolio C's mean is sufficiently larger than portfolio A's mean. In contrast, the coefficient of variation still characterizes portfolio D as being riskier than portfolio A because portfolio D's mean is not sufficiently larger than portfolio A's.

cient/inefficient acceptance of risk.

The excessive risk index for a portfolio with expected tax revenue μ_A and volatility σ_A is defined as

$$\frac{\text{Excessive Risk Index}}{100} = \frac{\sigma_A/\mu_A - \sigma_{MV}/\mu_{MV}}{\sigma_A/\mu_A} = \frac{\mu_{MV} - \mu_A}{\mu_{MV}} \tag{6}$$

where μ_{MV} is the expected tax revenue on the minimum volatility frontier for the level of volatility $\sigma_{MV} = \sigma_A$. The excessive risk index is bounded by 0 and 100.

A portfolio with an excessive risk index of zero indicates the tax portfolio does not accept any unnecessary risk because the portfolio is on the minimum volatility frontier, that is $\mu_A = \mu_{MV}$. The excessive risk index can be interpreted as the cost of unnecessary risk measured as the percentage of tax revenues lost by not being on the frontier.

To better understand the excessive risk index, consider two tax policy changes, one from portfolio C to portfolio B and one from portfolio C to portfolio A. Both led to the same decrease in volatility, as measured on the horizontal axis. However, the change from C to B had a substantially smaller decrease in revenues than the change from C to A. The excessive risk index for portfolio C is zero, because it is on the minimum volatility frontier. Similarly, the excessive risk index for portfolio B is zero. The move from portfolio C to B is, therefore, a move between two efficient portfolios. In contrast, the move from C to A led to a larger loss in expected tax revenues. This loss is captured by the difference in expected tax revenues between portfolio B and portfolio A.

Section 4 demonstrates empirically how to calculate the excessive risk index for different states.

4 Tax Revenue Volatility

4.1 Data

I collect data on tax revenues, tax rates, and economic conditions from all US states for the years 1964–2013. I collect tax revenue and rate data from the *Book of States* and cross-check it with the Tax Foundation and the Advisory Commission on Intergovernmental Relations biannual report "Significant Features in Fiscal Federalism." Tax revenues include those from personal income, sales (both general and selective), and corporate taxes. These revenues include both state and local government collections.[15] Tax rates are the statutory rates imposed by the state and include the top and bottom income tax rate, the sales tax rate, and the corporate income tax rate. State-level economic conditions such as state-level GDP (broken into industries) and personal income are collected from the Bureau of Economic Analysis.[16]

4.2 Measuring Volatility

The empirical analysis defines the volatility of a variable $y_{i,t}$ as the log of the absolute value of the residual, $\varepsilon_{i,t}(y_{i,t})$, from its nonparametrically estimated time trend,

$$\text{Vol}_{i,t}(y_{i,t}) = log(|\varepsilon_{i,t}(y_{i,t})|). \tag{7}$$

where i denotes state and t denotes year. This measure of volatility represents year-over-year variability and abstracts from differences in time trends across states.[17]

[15]The results are robust to using only state level collections.

[16]Appendix C provides additional descriptive statistics on how the tax bases states rely on have changed and the correlation across tax bases.

[17]For example, tax revenue increase through time. The variance of tax revenue, therefore, captures both the year-over-year variability and the how much tax revenues have increased. To abstract from the general time trend, previous studies have used a similar detrending method to Seegert (2012) or have used the variance over four years surrounding a given year similar to Poterba (1994).

4.3 Tax Revenue Accounting Identity and Econometric Model

4.3.1 Deriving Tax Revenues and Tax Revenue Volatility

Tax revenues from a given source is the product of a tax rate times a tax base:

$$R_j = \tau_j B_j(\tau, x), \tag{8}$$

where R_j is the revenue from source j, τ is a vector of tax rates, x is a vector of economic conditions, and $B_j(\tau, x)$ is defined as the tax base for revenue source j. For example, sales tax revenues equal the sales tax rate times the sales tax base, which is the part of personal consumption that is taxable. Changes in the sales tax base represent changes in government policy (e.g., a change in which goods are taxed) and changes in individual behavior (e.g., consuming more online). These changes affect the mapping from tax rates and economic conditions to the size of the tax base and therefore are captured by changes in the tax base function $B_j(\cdot)$.

Tax revenues are uncertain due to uncertainty about the tax base (tax rates are set by the government ex ante). The variance of tax revenues, therefore, can be written as,

$$\sigma_j^2 = \tau_j^2 \sigma_{B_j}^2(\tau, \sigma_x^2). \tag{9}$$

The log of tax revenue and the variance of tax revenues can be written as,

$$log(R_j) = log(\tau_j) + log(B_j(\tau, x)) \quad \text{and} \tag{10}$$

$$log(\sigma_j^2) = 2log(\tau_j) + log(\sigma_{B_j}^2(\tau, \sigma_x^2)). \tag{11}$$

The tax base can be approximated by a function of economic conditions and tax rates such that the log of the tax base and the variance of the tax base can be written as

$$log(B_j(\tau, x)) = \alpha_1 + (\gamma_1 - 2)log(\tau_j) + log(\tau_{-j})\gamma_2 + log(x)\delta_1 + \varepsilon. \tag{12}$$

$$log(\sigma^2_{B_j}(\tau, \sigma^2_x)) = \alpha_2 + (\gamma_3 - 2)log(\tau_j) + log(\tau_{-j})\gamma_4 + log(\sigma^2_x)\delta_2 + \varepsilon. \tag{13}$$

Combining this approximation with the accounting identity creates an econometric model for tax revenues and tax revenue volatility that can be used in estimation, simulations, and decompositions.

$$log(R_j) = \alpha_1 + log(\tau)\beta_1 + log(x)\delta_1 + \varepsilon. \tag{14}$$

$$log(\sigma^2_j) = \alpha_2 + log(\tau)\beta_2 + log(\sigma^2_x)\delta_2 + \varepsilon. \tag{15}$$

4.3.2 Estimating Elasticities for the Optimal Tax Rule

To demonstrate the optimal tax rule, I compare the income and sales tax–two major sources of revenues for most states. I use data from 1970-2014 to estimate the elasticity of income and sales tax revenue and the elasticity of income and sales tax revenue volatility.[18] I estimate these four elasticities using equations (14) and (15).

I estimate these four regressions separately for each state and exclude states that do not collect both income and sales taxes. The average elasticity over all years is given by the coefficient on the log of the specific tax rate, β. Year-specific elasticities are given by

$$\varepsilon_{Y_{j,t}} = \beta_1 \frac{\bar{Y}_j}{\bar{\tau}_j} \frac{\tau_{j,t}}{Y_{j,t}} = \frac{\partial Y_{j,t}}{\partial \tau_{j,t}} \frac{\tau_{j,t}}{Y_{j,t}}.$$

[18] The four elasticities that I estimate are with respect to the tax rate for the given tax base. For example, the elasticity of income tax revenue is with respect to the income tax rate, and the elasticity of sales tax revenue volatility is with respect to the sales tax rate. These elasticities characterize how tax revenue (and thus deadweight loss) and tax revenue volatility change with a given tax rate.

4.3.3 Simulating Tax Revenues and Tax Revenue Volatility

I estimate tax revenues and tax revenue volatility separately for each state using a weighted least squares specification. To estimate tax revenues and tax revenue volatility for state i, each observation (state k year t) receives a separate weight. An observation receives a larger weight if there is a larger probability the observation could have been observed in state i. For example, to estimate Minnesota's tax revenues and tax revenue volatility, the 1990 observation in Wisconsin has a large weight and the 2002 observation in Rhode Island has a small weight. These probabilities are estimated using a logit model with a dependent variable equal to one for state i and zero otherwise. The shares of GDP in each industry are used as independent variables to capture the similarity in economic conditions.[19]

Using weighted least squares, I estimate equations (14) and (15) separately for each state i and tax base j (sales, income, and corporate). The set of tax rates includes the top and bottom income tax rate, the sales tax rate, and the corporate income tax rate. I use the coincident index calculated by the Federal Reserve Bank of Philadelphia to summarize economic conditions. The coincident index combines state-level indicators into a single statistic and includes nonfarm payroll employment, average hours worked in manufacturing by production workers, the unemployment rate, and wage and salary disbursements deflated by the consumer price index (U.S. city average).[20] For robustness, I report estimates without using weighted least squares in the appendix.

Each regression in equations (14) and (15) produce 150 separate sets of coefficients, one for each state-tax base pair. Most coefficients are statistically different from zero. The coincident index consistently has a t-statistics above four. The coefficients on tax rates and

[19]The share of GDP in each industry provides a measure of the economic activity that determines the response of tax revenues and tax revenue volatility to different tax rates and economic shocks. The results are robust to using additional independent variables in the logit model including tax rates, personal income, population, and land area.

[20]The coincident indexes are created using a dynamic single-factor model. For more information, see Crone and Clayton-Matthews (2005) and https://www.philadelphiafed.org/research-and-data/regional-economy/indexes/coincident/.

squared tax rates consistently have t-statistics above three.

The identifying assumption is that the conditional expectation of the error term is zero. Some concern over misspecification of the model is alleviated by noting that the regressions in equations (14) and (15) are derived from an accounting identity. The identifying assumption could still be violated due to omitted variable bias or the inclusion of endogenous variables. The results are qualitatively similar when using state and year fixed effects, regional dummy variables, and regional dummy variables interacted with time dummy variables that control for time-varying regional omitted variables. Endogenous variables may be a concern if state governments can predict, and respond to, tax revenue projections by changing their tax rates. This does not appear to be a major concern as the model is robust to using two-year lags of the tax rates.

4.3.4 Minimum Volatility Frontier Estimation

To estimate each state's minimum volatility frontier, expected tax revenue and tax revenue volatility are simulated for nearly 200,000 different tax portfolios using estimates from equations (14) and (15). Each tax portfolio is characterized by the sales, corporate, and top and bottom income tax rates. Each tax rate takes on 21 different values between zero and the larger of either four or 115 percent of the largest tax rate the specific state has levied.[21] Each value of a given tax rate is then paired with every other value of each of the other three tax rates, creating nearly 200,000 different tax portfolios.

The simulation estimates the mean and volatility of the tax revenues using the actual economic conditions of the state to decompose changes into tax policy and economic conditions.[22] The minimum volatility frontier is estimated as the upper convex hull of the nearly 200,000 simulated points in volatility and expected revenue space. Figure A.1 graphs the

[21] The tax rate values are used to limit out-of-sample projection in the simulation. The results are robust to using different upper bounds.

[22] Using the actual economic conditions of the state abstracts from any general equilibrium effects the tax rates might have on the economic conditions.

simulated portfolios for Illinois and the convex hull used to estimate a minimum volatility frontier. This process is run separately for each state, producing state-specific minimum volatility frontiers. A similar process is used to estimate the expected tax revenue and tax revenue volatility for each of the actual tax portfolios a state levied in 1970 and 2014.

5 Empirical Results

5.1 Changes in Volatility, Tax Risk, and Excessive Risk Index

Table 1 reports the percent change in volatility, tax risk, and the excessive risk index. Volatility increased by 11.8% on average and increased in most states. This increase in volatility has caused welfare costs for states because it is difficult for states to smooth revenues. Tax risk, which measures the impact of tax policy changes on risk, increased by 7.3% on average and in 45 states. This increase in tax risk suggests a substantial portion of the increase in volatility is due to policy changes; consistent with evidence from Seegert (2012). This increase, however, does not distinguish between efficient increases in risk (in return for higher expected revenues) and inefficient increases in risk. The excessive risk index, however, does distinguish between efficient and inefficient changes in tax risk.

There are two key aggregate findings in Table 1. First, despite states accepting more risk from 1970 to 2014, they did so in a relatively efficient way. Specifically, while tax risk increased by 7.3% on average, the excessive risk index increased only slightly from 43.18 in 1970 to 43.66 in 2014 (the median excessive risk index increased slightly more from 33 to 36.5). Second, states are accepting a large amount of excessive risk. Specifically, states could generate 43% more revenue, on average, without increasing their volatility. There are several reasons states may be accepting this additional risk including, redistributive motives, administrative costs of implementing new taxes, or they are unaware of this cost.

5.2 Changes in Excessive Risk Index By State

Beyond these aggregate findings, Figure 2 demonstrates there is substantial variation across states in their excessive risk index. Specifically, the excessive risk index ranges from 5 to 116 in 1970 and 6 to 111 in 2014 (reported in Table 1). Panels A and B denote states with the largest excessive risk indexes in dark red (least efficient) and the smallest excessive risk index in white (most efficient). Despite the large variation across states, there seems to be less variation across time as the most and least efficient states stay roughly the same between 1970 and 2014. To investigate this further, Table 1 groups states into four panels, depending on whether their excessive risk index is above or below the median and whether they increased or decreased their excessive risk index between 1970 and 2014.

Panel A of Table 1 reports states that have higher than median excessive risk index in 2014 and increased their excessive risk index between 1970 and 2014. This panel includes 15 states such as Colorado, Michigan, and New York. These states all had excessive risk indexes in 2014 above 37. Said differently, these states could have changed their tax portfolios to bring in at least 37% more revenues without increasing their tax revenue volatility. Many of these states substantially increased their excessive risk index. For example, Colorado increased their excessive risk index from 39 to 56, or a 43.6% increase from 1970 to 2014.

Panel B of Table 1 reports the 10 states that have high but decreasing excessive risk indexes. These states include Nebraska, Oregon, and Virginia and span the US. Even though these states decreased their excessive risk index most of them increased their tax risk; all but Hawaii. Said differently, these states changed their tax portfolios in ways that increased volatility but in relatively efficient ways such that they moved closer to their minimum volatility frontiers.

Panel C of Table 1 reports the 11 states that have low and increasing excessive risk indexes. These states include Florida, Massachusetts, and Utah. Florida and Utah increased their excessive risk index by 10 points (20 to 30 and 22 to 32, respectively) but did so in different ways. Utah changed their tax portfolios to accept more risk and more excessive risk moving it further from its minimum volatility frontier. Florida, in contrast, decreased its tax risk but in an inefficient way that caused it to move further from its minimum volatility frontier.

Panel D of Table 1 reports the 14 states with low and decreasing excessive risk. These states include Idaho, Illinois, and New Mexico. These states increased their tax risk but in relatively efficient ways that moved them closer to their minimum volatility frontiers. For example, Idaho decreased its excessive risk index by 12 points from 32 in 1970 to 20 in 2014. Despite the decrease in excessive risk index, Idaho increased its tax risk by 9%. Idaho's experience underscores the difference between volatility (which includes economic factors), risk associated with tax rate changes (tax effect), and the excessive risk index.

5.3 New York, Virginia, Utah, and Illinois

Figure 3 graphs the minimum volatility frontier and the 1970 and 2014 tax portfolios for four states; New York, Virginia, Utah, and Illinois. New York and Virginia have high excessive risk indexes in comparison to Utah and Illinois that have low excessive risk indexes. The excessive risk index is observed by considering the vertical difference between a tax portfolio and the minimum volatility frontier. New York and Utah increased their excessive risk indexes from 1970 to 2014, which is seen by the vertical distance between the minimum volatility frontier and the 2014 portfolio being greater than the distance between the minimum volatility frontier and the 1970 portfolio. In contrast, Virginia and Illinois decreased their excessive risk index.

The excessive risk index captures the balance of a state's tax portfolio. Between 1970 and 2014 Virginia increased its tax revenues in a balanced way, increasing sales, income, and corporate taxes. This balanced move Virginia closer to its minimum volatility frontier. Similarly, Illinois moved closer to its minimum volatility frontier by increasing sales and corporate tax sources. In contrast, between 1970 and 2014 New York and Utah changed their tax portfolio to rely more on income taxes causing them to move further from its minimum volatility frontier.

The following section uses corollary 3 to test whether states are overdependent on their income or sales tax revenues.

5.4 Optimal Tax Rule

Corollary 3 provides a benchmark to test whether states are deviating from the optimal tax rule. To demonstrate how to test for deviations, I consider two representative years: 1970 and 2014. Comparing the number of states that violate the optimal tax rule provides evidence on whether tax policy became more or less efficient at trading off deadweight loss and volatility over this period. It is important to note that there are other potential reasons a state may deviate from the optimal tax rule as given in Theorem 1. For example, the optimal tax rule does not take into account redistributive motives. This analysis only provides evidence on the relative tradeoff between deadweight loss and volatility. Also, the condition in corollary 3 is a sufficient condition, which means that states that do not violate the condition may still violate the optimal tax rule.

Figure 4 indicates which states deviated from the optimal tax rule in 1970 (Panel A) and 2014 (Panel B), using estimates from equations (14) and (15). In this example, states can deviate by being overdependent on income taxes (denoted by dark red) or overdependent on sales taxes (denoted by white). States that do not deviate from the sufficient conditions in corollary 3 are shaded yellow.

The number of states that deviated from the optimal tax rule increased from 26 to 36 between 1970 and 2014. This increase is due to a shift toward income taxes. From 1970 to 2014, 12 more states became overdependent on the income tax, while 2 fewer states became overdependent on the sales tax. States that are overdependent on the income tax have an opportunity to decrease costs from deadweight loss and volatility. These states can lower these costs while holding expected revenue constant by shifting toward the sales tax.

Between 1970 and 2014 New York and Utah moved from being neutral or overdependent on the sales tax to being overdependent on the income tax, consistent with evidence from their excessive risk index. In contrast, Virginia did not violate the sufficient conditions from corollary 3 in 1970 or 2014, again consistent with changes in its excessive risk index.

6 Conclusion

This paper provides a framework for understanding tax revenue volatility. Using a standard model of optimal taxation and allowing for uncertainty demonstrates how states should optimally tradeoff the costs of tax revenue volatility and deadweight loss. From this tradeoff, several corollaries follow. First, lump sum taxes are shown to no longer be optimal with volatility. Second, sufficient conditions are derived for testing the tradeoff between volatility and deadweight loss. Third, the optimal tax rule is shown to be the traditional Ramsey rule without volatility and the optimal portfolio weights without deadweight loss. Fourth, governments are constrained by a minimum variance frontier that demonstrates governments must accept more risk to increase expected revenues. Finally, an excessive risk index is derived to distinguish between efficient and inefficient risk.

To demonstrate the theoretical framework, data from 1970 to 2014 and the 50 US states are used. States are found to be increasing their tax risk but in relatively efficient ways. Even so, states could increase their expected tax revenues by roughly 40 percent without increasing their tax revenue volatility. This suggests a large cost to states associated with their current (imbalanced) tax portfolios. From 1970 to 2014, ten additional states deviated from the optimal tax rule, bringing the total to 36. The majority of these states are overdependent on the income tax, relative to the sales tax.

Future research is needed to understand the trade-offs involved with tax revenue volatility and other optimal taxation considerations. For example, states may be able to decrease their tax revenue volatility by decreasing the progressivity of their income tax, but this comes at a cost of decreasing the benefits associated with redistribution. In addition, the model in this paper provides a framework for future research to investigate different hedging opportunities for state governments. Considering optimal taxation with uncertainty will become increasingly important as economic conditions become more uncertain.

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Table 1: Changes In Volatility and Risk: By State

State	Change in: Volatility	Change in: Tax Risk	Excessive Risk Index: 1970	Excessive Risk Index: 2014	Excessive Risk Index: Change
Panel A: High and Increasing Excessive Risk					
Alaska	-0.086	0.218	74	111	0.500
Arizona	0.311	0.065	46	64	0.391
Colorado	0.027	0.016	39	56	0.436
Delaware	0.458	0.134	84	102	0.214
Maine	0.072	0.054	81	97	0.198
Michigan	-0.123	0.070	45	55	0.222
New Jersey	0.068	-0.031	72	73	0.014
New York	0.030	0.047	26	45	0.731
North Carolina	-0.156	0.117	36	40	0.111
Ohio	0.099	0.154	68	70	0.029
Oklahoma	-0.063	0.085	43	55	0.279
Rhode Island	0.195	0.090	40	46	0.150
South Carolina	0.170	0.300	34	41	0.206
West Virginia	-0.009	-0.058	29	39	0.345
Wyoming	0.165	0.140	78	79	0.013
Panel B: High and Decreasing Excessive Risk					
Hawaii	-0.513	-0.010	100	100	0.000
Louisiana	0.197	0.048	116	81	-0.302
Maryland	0.012	0.079	68	61	-0.103
Montana	0.174	0.051	71	69	-0.028
Nebraska	0.061	0.235	70	60	-0.143
Nevada	0.092	0.358	41	41	0.000
New Hampshire	0.131	0.003	73	70	-0.041
Oregon	0.004	0.015	69	65	-0.058
Tennessee	0.148	0.136	76	68	-0.105
Virginia	0.272	0.063	46	37	-0.196
Panel C: Low and Increasing Excessive Risk					
Alabama	0.253	0.004	26	27	0.038
Arkansas	0.747	0.063	26	32	0.231
Connecticut	0.158	0.276	22	31	0.409
Florida	0.148	-0.412	20	30	0.500
Iowa	0.273	0.062	5	6	0.200
Massachusetts	0.271	0.075	22	28	0.273
Minnesota	0.044	0.138	17	18	0.059
Mississippi	0.046	0.064	31	34	0.097
North Dakota	0.198	-0.001	28	36	0.286
Utah	0.084	0.100	22	32	0.455
Wisconsin	-0.174	0.064	13	16	0.231
Panel D: Low and Decreasing Excessive Risk					
California	0.120	0.066	21	15	-0.286
Georgia	0.105	0.064	26	24	-0.077
Idaho	0.234	0.092	32	20	-0.375
Illinois	0.037	0.042	41	17	-0.585
Indiana	0.347	0.096	85	31	-0.635
Kansas	0.205	0.054	28	17	-0.393
Kentucky	0.004	0.004	30	27	-0.100
Missouri	0.093	0.037	22	22	0.000
New Mexico	0.051	0.095	30	23	-0.233
Pennsylvania	0.178	0.084	11	7	-0.364
South Dakota	0.203	0.000	7	7	0.000
Texas	0.144	0.082	24	24	0.000
Vermont	0.071	0.122	30	25	-0.167
Washington	0.307	0.016	14	9	-0.357

Notes. This table reports changes in volatility, tax risk, and excessive risk index using weighted least squares.

Figure 1: Minimum Volatility Frontier

This figure depicts a minimum volatility frontier, the tax effect, and excessive risk index for tax portfolios A and D. The minimum volatility index is given by the blue solid line. Average tax revenues in billions from income, sales, and corporate taxes is given on the vertical axis. Volatility in billions from income, sales, and corporate taxes is given on the horizontal axis.

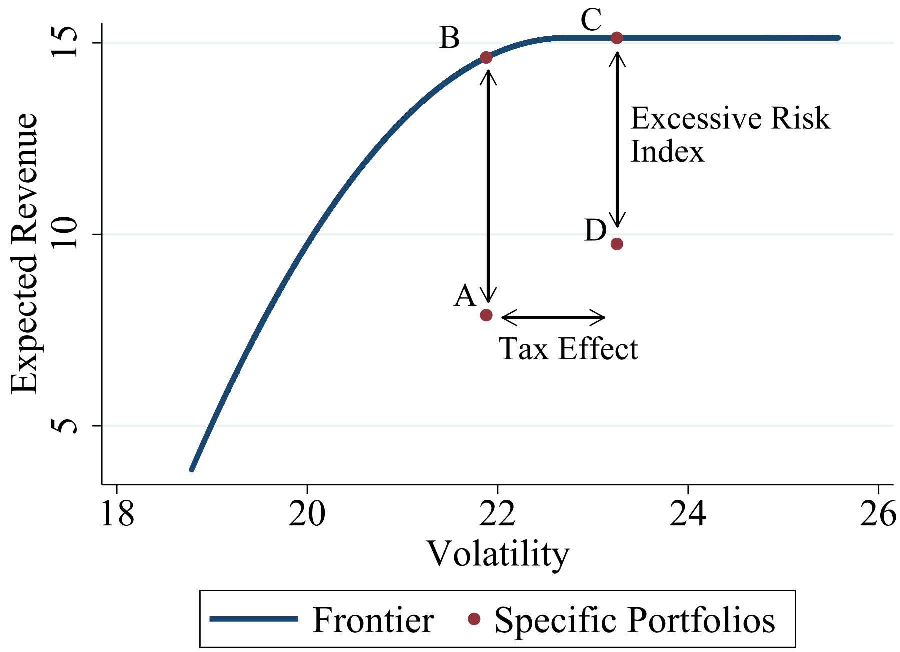
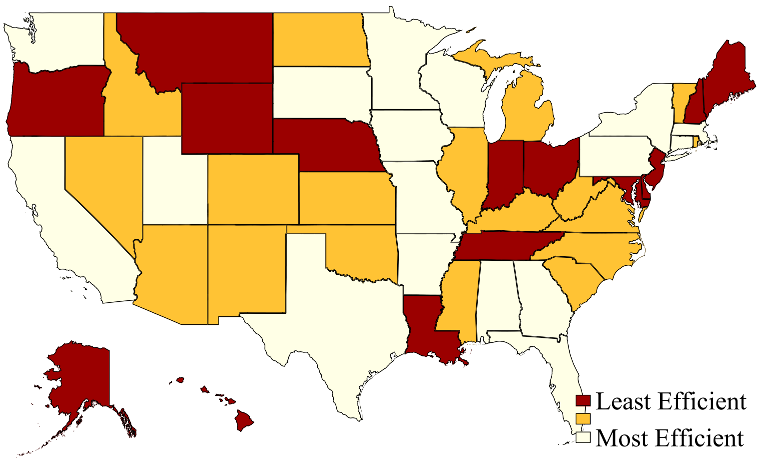


Figure 2: **Changes in Excessive Risk Index 1970 to 2014**

This figure reports a summary of the excessive risk index. States with high excessive risk index (least efficient) are shaded dark red, states with low excessive risk index (most efficient) are shaded white, and states in the middle are shaded yellow. Panel A reports estimates from 1970. Panel B reports estimates from 2014.

A) **Excessive Risk Index 1970**



B) **Excessive Risk Index 2014**

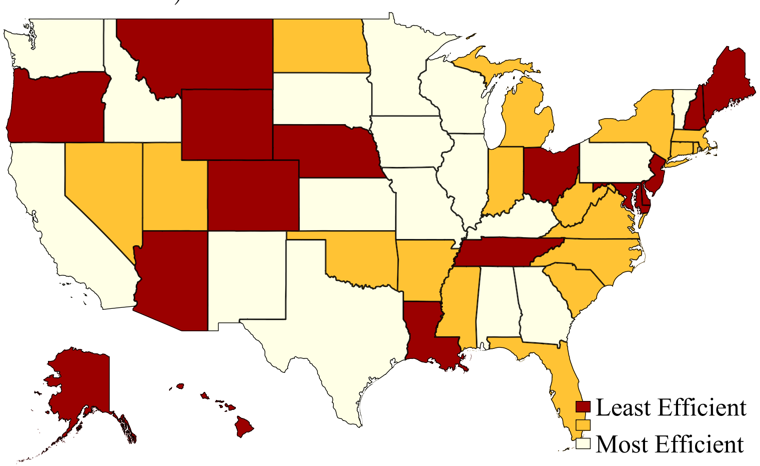
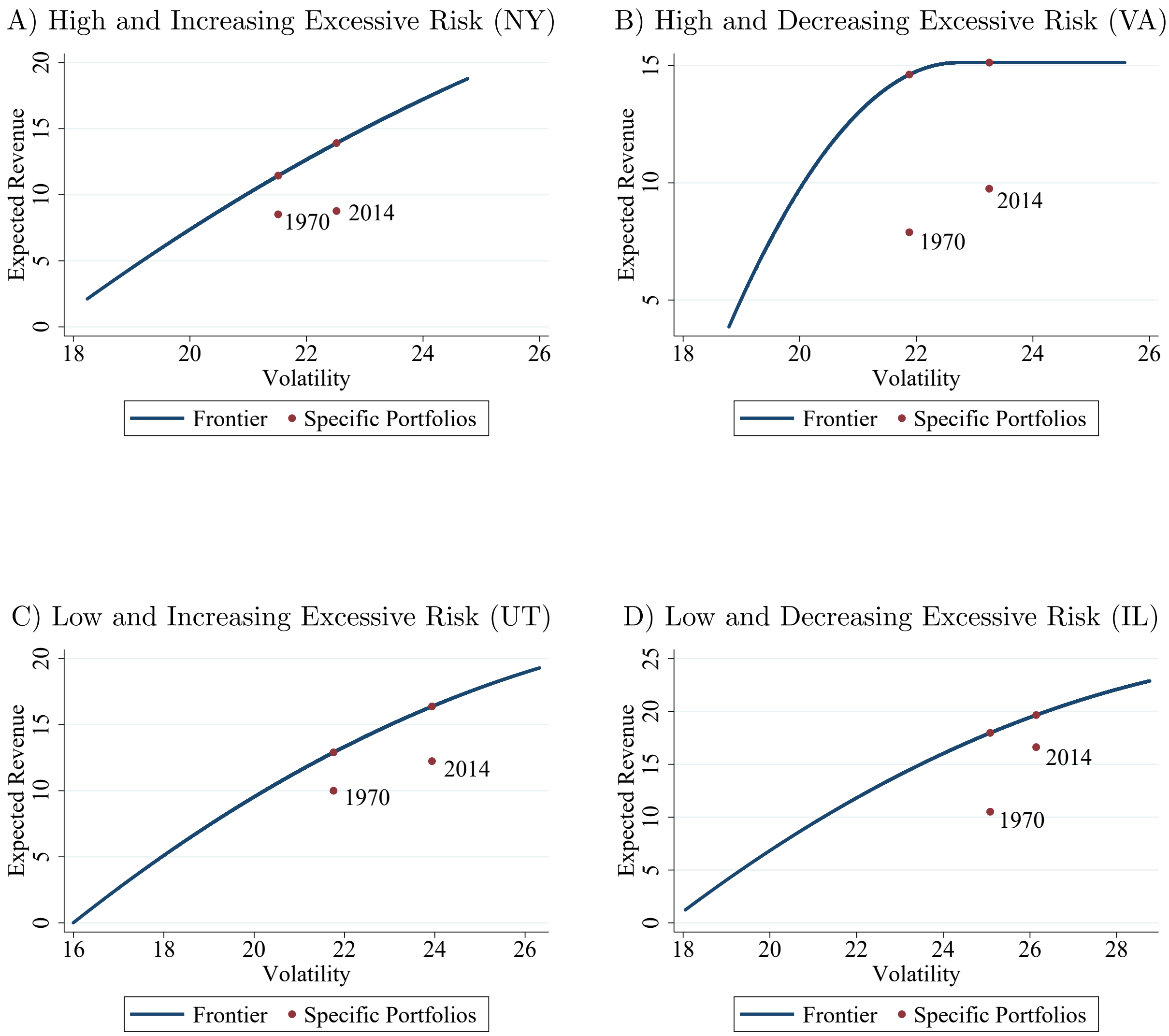


Figure 3: Minimum Volatility Frontiers

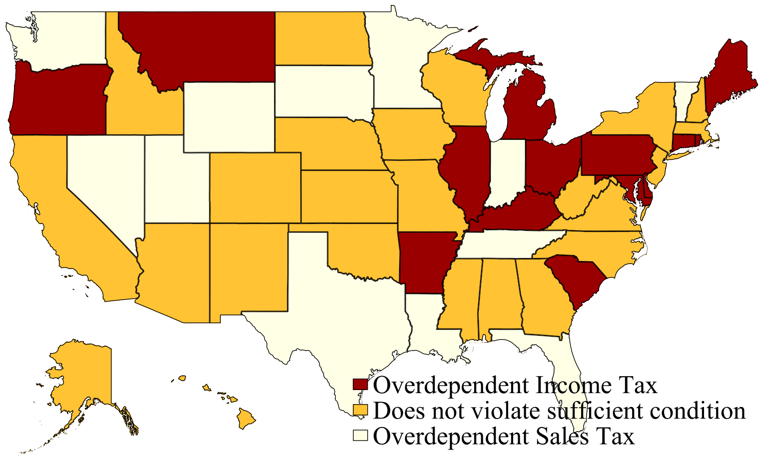


NOTE— This figure depicts a minimum volatility frontier for New York (Panel A), Virginia (Panel B), Utah (Panel C), and Illinois (Panel D). The minimum volatility index is given by the blue solid line. Average tax revenues in billions from income, sales, and corporate taxes is given on the vertical axis. Volatility in billions from income, sales, and corporate taxes is given on the horizontal axis.

Figure 4: **Changes in State Tax Policies**

This figure reports the summary of the comparison of the elasticity of tax revenue and the elasticity of tax revenue volatility. States that are overdependent on the sales tax are shaded white, states that are overdependent on the income tax are shaded dark red, and states that do not violate the sufficient condition are shaded yellow. Panel A reports estimates from 1970. Panel B reports estimates from 2014.

A) **State Tax Portfolios 1970**



B) **State Tax Portfolios 2014**

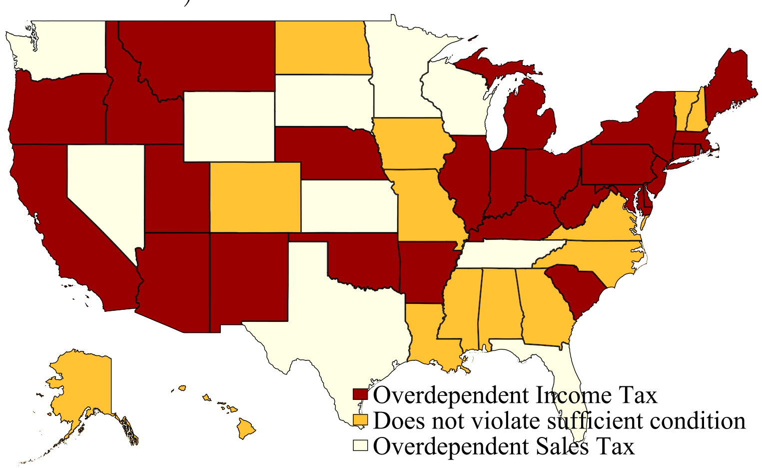


Table A.1: Changes In Volatility and Risk: By State

	Change in		Excessive Risk Index		
State	Volatility	Tax Risk	1970	2014	Change
Alabama	0.253	0.004	26	27	0.038
Alaska	-0.086	0.218	74	111	0.500
Arizona	0.311	0.065	46	64	0.391
Arkansas	0.747	0.063	26	32	0.231
California	0.120	0.066	21	15	-0.286
Colorado	0.027	0.016	39	56	0.436
Connecticut	0.158	0.276	22	31	0.409
Delaware	0.458	0.134	84	102	0.214
Florida	0.148	-0.412	20	30	0.500
Georgia	0.105	0.064	26	24	-0.077
Hawaii	-0.513	-0.010	100	100	0.000
Idaho	0.234	0.092	32	20	-0.375
Illinois	0.037	0.042	41	17	-0.585
Indiana	0.347	0.096	85	31	-0.635
Iowa	0.273	0.062	5	6	0.200
Kansas	0.205	0.054	28	17	-0.393
Kentucky	0.004	0.004	30	27	-0.100
Louisiana	0.197	0.048	116	81	-0.302
Maine	0.072	0.054	81	97	0.198
Maryland	0.012	0.079	68	61	-0.103
Massachusetts	0.271	0.075	22	28	0.273
Michigan	-0.123	0.070	45	55	0.222
Minnesota	0.044	0.138	17	18	0.059
Mississippi	0.046	0.064	31	34	0.097
Missouri	0.093	0.037	22	22	0.000
Montana	0.174	0.051	71	69	-0.028
Nebraska	0.061	0.235	70	60	-0.143
Nevada	0.092	0.358	41	41	0.000
New Hampshire	0.131	0.003	73	70	-0.041
New Jersey	0.068	-0.031	72	73	0.014
New Mexico	0.051	0.095	30	23	-0.233
New York	0.030	0.047	26	45	0.731
North Carolina	-0.156	0.117	36	40	0.111
North Dakota	0.198	-0.001	28	36	0.286
Ohio	0.099	0.154	68	70	0.029
Oklahoma	-0.063	0.085	43	55	0.279
Oregon	0.004	0.015	69	65	-0.058
Pennsylvania	0.178	0.084	11	7	-0.364
Rhode Island	0.195	0.090	40	46	0.150
South Carolina	0.170	0.300	34	41	0.206
South Dakota	0.203	0.000	7	7	0.000
Tennessee	0.148	0.136	76	68	-0.105
Texas	0.144	0.082	24	24	0.000
Utah	0.084	0.100	22	32	0.455
Vermont	0.071	0.122	30	25	-0.167
Virginia	0.272	0.063	46	37	-0.196
Washington	0.307	0.016	14	9	-0.357
West Virginia	-0.009	-0.058	29	39	0.345
Wisconsin	-0.174	0.064	13	16	0.231
Wyoming	0.165	0.140	78	79	0.013

Notes. This table reports changes in volatility and risk using weighted least squares and reporting states alphabetically.

Table A.2: Correlation Across Tax Revenues

State	Income and Sales 1964-1998	Income and Sales 1999-2007	Income and Corporate 1964-1998	Income and Corporate 1999-2007	Corporate and Sales 1964-1998	Corporate and Sales 1999-2007
Aggregate	0.19	0.39	0.21	0.55	0.20	0.43
Alabama	0.12	0.52	-0.19	-0.11	0.06	0.69
Alaska	0.00	0.00	0.00	0.00	0.00	0.00
Arizona	0.16	0.65	-0.17	0.86	0.21	0.44
Arkansas	0.25	0.68	0.49	0.64	0.50	0.78
California	0.48	0.59	0.32	0.72	0.36	0.78
Colorado	0.27	0.85	0.30	0.84	0.25	0.68
Connecticut	0.27	-0.05	0.14	0.52	0.00	0.04
Delaware	0.00	0.00	-0.06	0.52	0.00	0.00
Florida	0.00	0.00	0.00	0.00	0.35	0.58
Georgia	-0.02	0.88	0.40	0.73	0.24	0.92
Hawaii	0.42	0.66	0.35	0.76	0.60	0.34
Idaho	-0.21	0.33	0.23	0.81	-0.02	0.23
Illinois	-0.16	0.39	-0.12	0.70	0.24	0.34
Indiana	0.30	0.14	0.14	0.64	0.11	0.06
Iowa	-0.33	0.29	0.09	0.70	0.04	0.01
Kansas	0.47	0.33	0.62	0.65	0.44	0.62
Kentucky	0.26	-0.16	0.37	-0.42	-0.14	0.91
Louisiana	0.45	0.35	0.40	0.82	0.39	0.66
Maine	0.41	0.19	0.20	0.45	0.23	0.85
Maryland	0.19	0.26	0.13	0.62	0.33	-0.01
Massachusetts	0.12	0.08	-0.02	0.60	0.17	-0.47
Michigan	0.23	0.36	0.65	0.13	0.50	-0.04
Minnesota	0.52	0.71	0.24	0.69	0.22	0.60
Mississippi	0.58	0.50	0.11	0.72	0.33	0.60
Missouri	-0.09	0.67	0.38	0.55	0.17	0.67
Montana	0.00	0.00	0.11	0.82	0.00	0.00
Nebraska	0.37	-0.59	0.47	0.55	0.46	-0.26
Nevada	0.00	0.00	0.00	0.00	0.00	0.00
New Hampshire	0.00	0.00	0.44	0.60	0.00	0.00
New Jersey	-0.20	0.23	0.12	-0.04	-0.26	0.00
New Mexico	0.23	-0.02	-0.11	0.54	0.06	0.76
New York	0.03	0.50	-0.10	0.72	0.05	0.42
North Carolina	-0.01	0.21	0.33	0.57	0.56	0.44
North Dakota	0.33	0.44	0.24	0.76	0.33	0.38
Ohio	0.50	-0.06	-0.09	-0.18	0.08	0.38
Oklahoma	0.39	0.71	0.33	0.62	0.40	0.80
Oregon	0.00	0.00	0.33	0.79	0.00	0.00
Pennsylvania	0.06	0.82	0.49	0.63	0.05	0.81
Rhode Island	0.20	0.71	-0.03	0.50	0.12	0.02
South Carolina	0.07	0.33	0.23	0.45	0.27	0.88
South Dakota	0.00	0.00	0.00	0.00	-0.33	0.29
Tennessee	-0.17	0.19	0.16	0.78	0.50	0.71
Texas	0.00	0.00	0.00	0.00	0.00	0.00
Utah	-0.04	0.88	0.20	0.77	0.28	0.74
Vermont	0.17	0.48	-0.12	0.46	0.06	0.71
Virginia	0.31	0.52	0.36	0.69	0.43	0.33
Washington	0.00	0.00	0.00	0.00	0.00	0.00
West Virginia	0.44	0.29	0.13	0.19	-0.32	0.06
Wisconsin	-0.09	0.38	0.43	0.29	0.04	-0.30
Wyoming	0.00	0.00	0.00	0.00	0.00	0.00

Notes: This table reports the correlation of detrended tax revenue from various tax bases. The aggregate row reports that average correlation across all states with nonzero correlations.

Table A.3: Changes In Volatility and Risk: By State

	Change in		Excessive Risk Index		
State	Volatility	Tax Risk	1970	2014	Change
Panel A: High and Increasing Excessive Risk					
Alaska	-0.086	0.015	71	106	0.493
Colorado	0.027	0.013	32	45	0.406
Delaware	0.458	0.032	49	85	0.735
Florida	0.148	0.097	45	49	0.089
Illinois	0.037	0.073	43	65	0.512
Indiana	0.347	0.156	65	69	0.062
Kansas	0.205	0.049	35	41	0.171
Louisiana	0.197	0.080	31	52	0.677
Michigan	-0.123	0.038	50	51	0.020
Minnesota	0.044	0.091	18	45	1.500
Nebraska	0.061	0.117	31	41	0.323
Nevada	0.092	0.067	38	39	0.026
New Hampshire	0.131	0.014	53	55	0.038
New York	0.030	0.047	26	45	0.731
North Carolina	-0.156	0.055	36	48	0.333
Oregon	0.004	0.016	47	56	0.191
Tennessee	0.148	0.076	41	45	0.098
Vermont	0.071	0.077	25	45	0.800
Washington	0.307	0.031	34	39	0.147
West Virginia	-0.009	0.069	33	45	0.364
Wisconsin	-0.174	0.059	22	44	1.000
Wyoming	0.165	0.024	37	38	0.027
Panel B: High and Decreasing Excessive Risk					
Massachusetts	0.271	0.056	42	41	-0.024
Mississippi	0.046	0.031	40	40	0.000
Panel D: Low and Increasing Excessive Risk					
Alabama	0.253	0.015	31	37	0.194
Arizona	0.311	0.055	24	32	0.333
California	0.120	0.054	11	14	0.273
Idaho	0.234	0.056	19	23	0.211
Missouri	0.093	0.042	31	36	0.161
New Mexico	0.051	0.051	18	31	0.722
North Dakota	0.198	0.028	8	22	1.750
Pennsylvania	0.178	0.006	30	36	0.200
Texas	0.144	0.055	35	37	0.057
Utah	0.084	0.049	23	37	0.609
Panel C: Low and Decreasing Excessive Risk					
Arkansas	0.747	0.060	30	22	-0.267
Connecticut	0.158	0.017	34	32	-0.059
Georgia	0.105	0.023	25	24	-0.040
Hawaii	-0.513	-0.009	28	15	-0.464
Iowa	0.273	0.058	24	13	-0.458
Kentucky	0.004	0.009	30	30	0.000
Maine	0.072	0.022	25	14	-0.440
Maryland	0.012	0.034	32	28	-0.125
Montana	0.174	-0.001	31	28	-0.097
New Jersey	0.068	0.034	35	22	-0.371
Ohio	0.099	0.007	54	24	-0.556
Oklahoma	-0.063	0.075	30	26	-0.133
Rhode Island	0.195	0.036	34	34	0.000
South Carolina	0.170	0.005	29	12	-0.586
South Dakota	0.203	0.000	32	32	0.000
Virginia	0.272	0.049	37	33	-0.108

Notes. This table reports changes in volatility and risk using a one step estimation without weighted least squares.

Figure A.1: Convex Hull

This figure graphs the simulated tax portfolios and the convex hull for Illinois. This graph is produced using the stata command *conhull*.

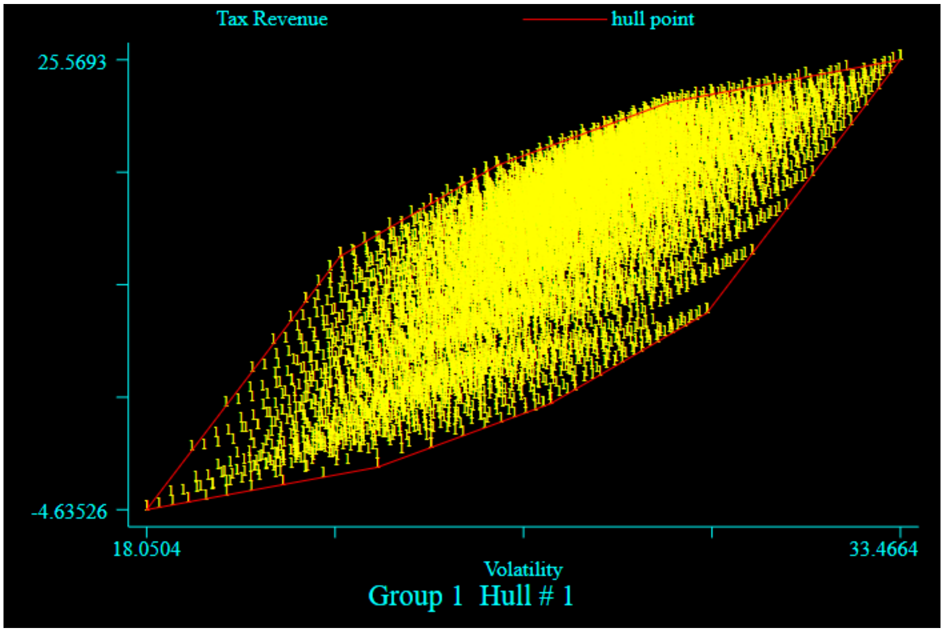
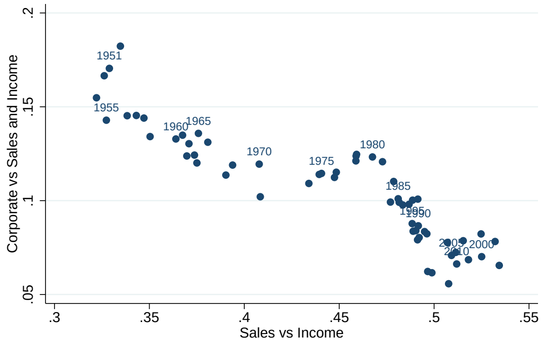


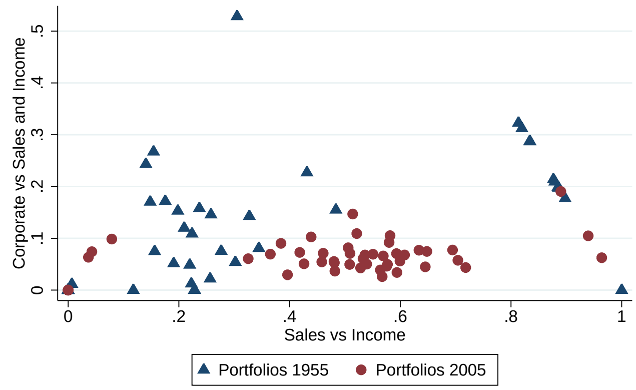
Figure A.2: Tax Portfolios Across Time

A) Aggregate State Tax Portfolios Over Time



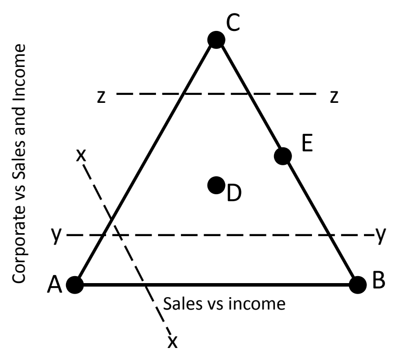
Simplex: Top node 100 percent corporate tax revenue (.5,.866),
left node 100 percent sales tax (0,0), right node 100 percent income tax (1,0).
Data: State tax revenue from the US Census.

B) State Tax Portfolios 1955 and 2005



Simplex: Top node 100 percent corporate tax revenue (.5,.866),
left node 100 percent sales tax (0,0), right node 100 percent income tax (1,0).
Alaska and New Hampshire not shown.
Data: State tax revenue from the US Census.

C) Simplex Example



NOTE— Graphing state government's tax portfolios onto a two-simplex demonstrates how much a government relies on each tax bases. A two-simplex is a triangle drawn in two-space that represents three-space. In this case, the dimensions are tax revenues collected from income, sales, and corporate tax bases as a percentage of the sum of these three tax bases. The simplex is characterized by $\Delta^2 = \{(s_{income}, s_{sales}, s_{corporate} \in R^3 | s_{income} + s_{sales} + s_{corporate} = 1)\}$, where s_{income} is the percentage of revenue collected from the income tax. Panel C is an example of a two-simplex that depicts the percentage of tax revenue from income, sales, and corporate tax bases (three-space) in two-space. The nodes of the simplex denoted by A, B, and C represent tax portfolios that rely on only one tax base. Each node of the triangle represents a portfolio made up entirely of one tax base with nodes at $(0,0), (1,0)$, and $(0.5, 0.866)$ corresponding to a portfolio entirely of sales, income, or corporate tax revenue respectively. Point A represents a tax portfolio that relies only on the sales tax, point B a tax portfolio that relies only on the income tax, and C only the corporate tax. Interior points represent mixtures of the three tax bases. Point D represents a tax portfolio that relies equally on all three. Point E represents a tax portfolio that relies fifty percent on corporate tax revenue and fifty percent on income tax revenue. Movements along the dashed lines xx, yy, and zz represent changes in the reliance on two of the three tax bases. For example, moving along the dashed line zz shifts the reliance on sales and income taxes but keeps the reliance on the corporate tax fixed. Similarly, moving along the line yy shifts the reliance on the sales and income taxes but for a a tax portfolio that relies less on the corporate tax than portfolios along the line zz. Finally, moving along the line xx represents tax portfolios shifting between the income and corporate tax, holding fixed the reliance on the sales tax.

Appendix A Theory

Appendix A.1 Proof of Theorem 1

The government maximizes the expected utility of the representative agent $V(p, \bar{m}, \bar{g}, \sigma_m^2, \sigma_g^2)$ which is a function of prices, income, public consumption, and the variances of private and public consumption. The first-order condition with respect to tax rate τ_k is

$$\frac{\partial V}{\partial p_k} + \frac{\partial V}{\partial g}\left[x_k^* + \sum_j \tau_j \frac{\partial x_j^*}{\partial \tau_k}\right] + \frac{\partial V}{\partial \sigma_g^2}\frac{\partial \sigma_g^2}{\partial \tau_k} + \frac{\partial V}{\partial \sigma_m^2}\frac{\partial \sigma_m^2}{\partial \tau_k} = 0.$$

To derive a concise expression for optimal taxation, I follow the literature and use several simplifying assumptions. Specifically, the expression is simplified by assuming that cross-price elasticities of demand and variance are zero and the income effect is approximately zero. The expression is further simplified using Roy's identity, $\frac{\partial V}{\partial \tau_j} = -\frac{\partial V}{\partial m}x_j^*$, and the Slutsky decomposition, $\frac{\partial x_j^*}{\partial \tau_j} = \frac{\partial h_j^*}{\partial \tau_j} - x_j^*\frac{\partial x_j^*}{\partial m}$, where h_j is the compensated demand. The first term in the Slutsky decomposition is the substitution effect, which by symmetry can be written, $\frac{\partial h_j^*}{\partial \tau_k} = \frac{\partial h_k^*}{\partial \tau_j}$. The second term in the Slutsky decomposition is the income effect.

$$(-V_m + V_g)x_j^* + V_g\tau_j\frac{\partial h_j^*}{\partial \tau_j} + V_{\sigma_g^2}\frac{\partial \sigma_g^2}{\partial \tau_j} + V_{\sigma_m^2}\frac{\partial \sigma_m^2}{\partial \tau_j} = 0.$$

This expression can be expressed as a function of elasticities, ε_{x_j,τ_j}, $\varepsilon_{g,j}^{\sigma}$ and $\varepsilon_{\sigma_m,\tau_j}$, with some rearranging,

$$(-V_m + V_g)/V_g + \underbrace{\frac{\partial h_j^*}{\partial \tau_j}\frac{\tau_j}{x_j^*}}_{\varepsilon_{x_j,\tau_j}} + \frac{V_{\sigma_g^2}}{V_g}\frac{\sigma_g^2}{p_j x_j^*}\underbrace{\frac{\partial \sigma_g^2}{\partial p_j}\frac{p_j}{\sigma_g^2}}_{\varepsilon_{\sigma_g,\tau_j}} + \frac{V_{\sigma_m^2}}{V_g}\frac{\sigma_m^2}{p_j x_j^*}\underbrace{\frac{\partial \sigma_m^2}{\partial p_j}\frac{p_j}{\sigma_m^2}}_{\varepsilon_{\sigma_m,\tau_j}} = 0.$$

Finally, by grouping terms that include the welfare implications of changes in public good consumption and public and private variance, the expression can be written as,[23]

$$\varepsilon_{x_j,\tau_j} + \omega\varepsilon_{\sigma,j} = V_m/V_g,$$

where the right-hand side is a constant. The parameter $\omega = 1/V_g$ collects all other terms and weights the elasticity of variance relative to the elasticity of demand depending on their impact on the utility function. The total disutility of uncertainty is given by the elasticity

[23] The elasticity of tax revenues j with respect to a tax rate j, is given by $\varepsilon_{R_j,\tau_j} = \frac{\partial R_j}{\partial \tau_j}\frac{\tau_j}{R_j} = \left(x_j + \frac{\partial x_j}{\partial \tau_j}\right)\frac{\tau_j}{\tau_j x_j} = 1 + \varepsilon_{x_j,\tau_j}$.

of volatility, $\varepsilon_{\sigma_m,\tau_j} = \omega_{\sigma_g^2,j}\varepsilon_{\sigma_g,\tau_k} + \omega_{\sigma_m^2,\tau_j}\varepsilon_{\sigma_m^2,\tau_j}$, which combines uncertainty in public and private consumption weighted by their impact on utility, $\omega_{\sigma_g^2,j} = (\partial V/\partial\sigma_g^2)(\sigma_g^2/(p_j x_j))$ and $\omega_{\sigma_m^2,j} = \partial V/\partial\sigma_m^2(\sigma_m^2/(p_j x_j))$, respectively. Combining the first-order condition with respect to tax rate τ_k and τ_j, produces the optimal tax rule with volatility,

$$\varepsilon_{x_j,\tau_j} + \omega\varepsilon_{\sigma,j} = \varepsilon_{x_k,\tau_k} + \omega\varepsilon_{\sigma,k}$$

Appendix A.2 Proof of Corollary 1: Traditional Ramsey Rule

When the expected utility of the representative agent does not depend on the variance of public and private consumption, the first-order condition reduces to,

$$(-V_m + V_g)x_k^* + V_g\tau_k\frac{\partial h_k^*}{\partial\tau_k} = 0.$$

The traditional optimal tax rule can be derived from this condition, noting that $\frac{\tau_k}{x_k}\frac{\partial h_k}{\partial\tau_k} = \frac{\tau_k}{p_k}\frac{p_k}{x_k}\frac{\partial h_k}{\partial p_k}$,

$$\frac{\tau_k}{p_k} = \frac{V_m - V_g}{V_g}\frac{1}{\varepsilon_{x_k,p_k}}.$$

Appendix A.3 Proof of Corollary 2: Lump Sum Taxes

Corollary 2 could be proven by inspection because the intuition is clear; If minimizing deadweight loss is the only objective of the government, then lump sum taxes are optimal, but if the government's objective includes public and private consumption risk then a lump sum tax concentrates risk in private consumption and there are cases where this is not optimal. To provide additional insights into the model, I prove corollary 2 by decomposing the tax system choice of the government. The decomposition delineates the tradeoff between deadweight loss, risk sharing, and hedging of tax source idiosyncratic risk.

The decomposition rewrites the government's problem from choosing tax rates $\boldsymbol{\tau}$ on all goods $\boldsymbol{x}$, to choosing a lump sum tax T, a risk sharing tax rate t, and a good specific tax $\hat{\boldsymbol{\tau}}$, where $\sum_J \hat{\tau}_j = 0$. This transforms the budget constraint from

$$\sum_J (1+\tau_j)x_j = m \quad \text{to} \quad \sum(1+\hat{\tau}_j)x_j = (1-t)m - T.$$

$$\sum_J (1+\tau_j)x_j = m$$

$$\sum_J \frac{(1+\tau_j)}{1+\bar{\tau}}x_j = \frac{m}{1+\bar{\tau}} \qquad \text{where } \bar{\tau} \equiv (1/J)\sum \tau_j$$

$$\sum_J (1+\tau_j)(1-t)x_j = (1-t)m \qquad \text{where } 1-t \equiv \tfrac{1}{1+\bar{\tau}}$$

$$\sum_J (1+\hat{\tau}_j)x_j = (1-t)m \qquad \text{where } \hat{\tau} \equiv \tau_j - t - \tau_j t$$

$$\sum_J (1+\hat{\tau}_j)x_j = (1-t)m - T$$

For expositional ease, I write the expected utility function as a function of after-tax income $\tilde{m} = (1-t)m - T$, instead of income, $V(p, \tilde{m}, g, \sigma^2_{\tilde{m}}, \sigma^2_g)$. Public and private consumption risk can now be written as a function of the risk sharing tax rate t and the good specific tax $\hat{\tau}_j$,

$$\sigma^2_{\tilde{m}} = (1-t)^2\sigma^2_m \qquad \sigma^2_g = t^2\sigma^2_m + \hat{\boldsymbol{\tau}}' W_x \hat{\boldsymbol{\tau}}.$$

From this equation it is clear that t determines the share of risk between public and private consumption and $\hat{\boldsymbol{\tau}}$ affects the hedging of tax revenues.

The government maximizes the expected utility of the representative agent, $V(p, \tilde{m}, g, \sigma^2_{\tilde{m}}, \sigma^2_g)$, by choosing the tax system $(t, \hat{\boldsymbol{\tau}}_{\boldsymbol{j}}, T)$. The first-order conditions are,

$$\frac{\partial V}{\partial t} = -\frac{\partial V}{\partial \tilde{m}}m + \frac{\partial V}{\partial g}m - \frac{\partial V}{\partial \sigma^2_{\tilde{m}}}2(1-t)\sigma^2_m + \frac{\partial V}{\partial \sigma^2_g}2t\sigma^2_m = 0$$

$$\frac{\partial V}{\partial T} = -\frac{\partial V}{\partial \tilde{m}} + \frac{\partial V}{\partial g} = 0$$

$$\frac{\partial V}{\partial \hat{\tau}_k} = \frac{\partial V}{\partial p_k} + \frac{\partial V}{\partial g}\left[x^*_k + \sum_j \hat{\tau}_j \frac{\partial x^*_j}{\partial \hat{\tau}_k}\right] + \frac{\partial V}{\partial \sigma^2_g}\frac{\partial \sigma^2_g}{\partial \hat{\tau}_k} = 0.$$

First, consider the case when the government knows the state of the world before making its tax system choices, such that $V_{\sigma^2_{\tilde{m}}} = V_{\sigma^2_g} = 0$. In this case, a lump sum tax, $(0, \mathbf{0}, T^*)$, is a solution to the first-order conditions, where T^* ensures $V_m = V_g$.

Second, consider the case when the government does *not* know the state of the world before making its tax system choices. In this case, it is straightforward to show that the first-order condition $\frac{\partial V}{\partial t} = 0$, is not satisfied by a lump sum tax. In particular, evaluate the first order conditions at $(0, \mathbf{0}, T^*)$,

$$\frac{\partial V}{\partial t} = \underbrace{-\frac{\partial V}{\partial \tilde{m}} m + \frac{\partial V}{\partial g} m}_{=0} - \frac{\partial V}{\partial \sigma^2_{\tilde{m}}} 2(1-t)\sigma^2_m + \frac{\partial V}{\partial \sigma^2_g} 2 \underbrace{t}_{=0} \sigma^2_m = 0$$

$$\frac{\partial V}{\partial t} = -\frac{\partial V}{\partial \sigma^2_{\tilde{m}}} 2(1-t)\sigma^2_m \neq 0.$$

When the government's objective includes minimizing the cost of risk lump sum taxes are not optimal because lump sum taxes concentrate risk in private consumption.

Appendix A.4 Proof of Corollary 4: Optimal Portfolio Weights

When taxes do not cause a substitution between consumption goods, the maximization problem reduces to,

$$V(\cdot, \cdot, \tau' x, \cdot, \tau' W t)$$

where public consumption can be written as $\tau' x$, and the variance of public consumption can be written as $\tau' W \tau$. In this maximization, tax rates are weights on different revenue sources. The first-order condition reduces to,

$$V_g x + V_{\sigma^2_g} W \tau = 0$$

$$\tau = \frac{V_g}{-V_{\sigma^2_g}} W^{-1} x.$$

Appendix B Micro Founded Model

Appendix B.0.1 Players, Strategies, and Payoffs

The model consists of three players: a representative individual, a government, and nature, which captures the uncertainty in the model. First, the government decides the rates at which to tax consumption, t_c, and wage income, t_w. The revenues from these taxes go to provide a public good, g. Second, nature chooses the wage and profit income (w, π) received by the representative individual. Third, the individual chooses her supply of labor and her consumption of goods that are taxed and untaxed (l, c_1, c_2) after observing nature's choice of wage and profit and the government's choice of tax rates. Finally, given these choices, production and utility are realized. In this model, only the government experiences uncertainty about the realization of the wage and profits because it must make its decisions before the uncertainty is realized.[24]

[24]The model focuses on the uncertainty for the government by assuming that the individual makes her decisions after nature, abstracting from additional concerns of uncertainty from the individual's perspective.

Order of Decisions	*Choices*
1st - Government	t_c, t_w
2nd - Nature	w, π
3rd - Individual	l, c_1, c_2
4th - Production occurs	
5th - Utility realized	

A. Individual Behavior

A representative individual provides l units of labor to the market for which she gets paid a wage w. Total net income, $y = (1 - t_w)wl + \pi$, consists of wage income, net of the wage tax, and profit income. The individual knows her wage and profit income at the time of her decisions. She uses this net income for private consumption, c, that is split between goods that are taxed, $c_1 \equiv \beta c$, and goods that are untaxed, $c_2 \equiv (1 - \beta)c$. For notation purposes, it is beneficial to write the individual's choices as (l, c, β) where β is the fraction of total consumption that is in the sales tax base.

The representative individual maximizes her utility, which consists of her supply of labor, l, total private consumption, c, the fraction of consumption in the sales tax base, β, and a public good, g,

$$\begin{aligned} &max_{c,l,\beta} \quad u = U(c, l, \beta; g) \\ &\text{subject to} \quad y = (1 - \beta)c + \beta c(1 + t_c). \end{aligned}$$

Utility maximization requires that (i) the marginal disutility from supplying labor equals the marginal utility of the income it produces, given in equation (Appendix B.1), and (ii) the ratio of marginal utilities of total consumption, c, and the split of consumption, β, is equal to the consumption tax rate multiplied by consumption, given in equation (Appendix B.2). When the consumption tax rate is zero, there is no distortion between consumption goods, and the marginal utility with respect to β is zero. Composing utility in terms of total consumption, c, and the fraction of consumption that is taxed, β, simplifies the composition of deadweight loss because β encompasses all behavioral responses between goods.

$$-U_2 = (1 - t_w)wU_1 \qquad \text{(Appendix B.1)}$$

$$\frac{U_3}{U_1} = t_c c \qquad \text{(Appendix B.2)}$$

B. Nature

Nature chooses the wage and profit income $\mathbf{I} = (w, \pi)$ from the distribution $h(\bar{\mathbf{I}}, \sigma^2_{\mathbf{I}})$, where these realizations are allowed to be correlated.[25] Private consumption depends on the realization of the wage and profit income. The mean and variance of private consumption

[25] The variance of the wage and profit are denoted by σ^2_w and σ^2_π, and the covariance is denoted by $\sigma_{w,\pi}$.

are given by[26]

$$\bar{c} = (1-\tau_c\beta)((1-t_w)\bar{w}\bar{l}+\bar{\pi}) \quad \text{and} \quad \sigma_c^2 = (1-\tau_c\beta)^2((1-t_w)^2\sigma_{wl}^2+\sigma_\pi^2+2(1-t_w)\sigma_{wl,\pi}). \tag{Appendix B.3}$$

Consumption and wage income are not perfectly correlated as long as income from wages and profits are not perfectly correlated. The ability of the government to hedge idiosyncratic risk between consumption and wage income tax bases depends on the correlation between these two variables.

C. Government

The government sets the tax rates on wage income and consumption (t_w and t_c) to maximize the expected utility of the representative individual. Tax revenues finance the production of a public good, $g = t_w wl + t_c\beta c$, that enters the representative individual's utility. The expected utility can be written as a function of the moments (e.g., mean, variance, skewness) of the state-dependent variables. This analysis restricts attention to the cases where expected utility is fully characterized by a function of the first two moments (mean and variance) but is robust to considering higher order moments. This restriction holds if the joint distribution of the state-dependent variables is fully characterized by the first two moments (e.g., normal, log-normal, or uniform).[27] The utility function is assumed to be additively separable in private and public consumption, $U_{1,4} = 0$, such that the government's objective function can be written as

$$E[u] = \int U(c,l,\beta,g)f(c,g,\sigma_g^2,\sigma_c^2) \equiv V(c,g,\sigma_c^2,\sigma_g^2,l,\beta), \tag{Appendix B.4}$$

where expected utility is increasing in private and public consumption, $V_1 > 0$ and $V_2 > 0$, and decreasing in volatility and labor supply, $V_3 < 0$, $V_4 < 0$, and $V_5 < 0$. The relative magnitudes of the marginal disutility from volatility capture the ability of the government and representative-agent to smooth consumption. For example, $V_3 = 0$ models a representative-agent that can perfectly smooth private consumption.

The variance of private consumption is given in equation (Appendix B.3), and the variance of public consumption (or tax revenue) is given by

$$\sigma_g^2 = t_c^2\beta^2\sigma_c^2 + t_w^2 l^2\sigma_w + 2t_c\beta t_w l\sigma_{w,c}.$$

Two insights about tax revenue volatility follow from this equation. First, tax revenue volatility depends on the interaction between tax rates and the underlying economic uncertainty. For example, the variance of tax revenues is zero if the government sets its tax rates to zero or if there is no underlying economic uncertainty ($\sigma_c^2 = 0$ and $\sigma_w^2 = 0$). Second, taxing only the base with least uncertainty does not necessarily minimize the variance of tax revenues because the variance of tax revenues depends on the square of the tax rate and the

[26]For ease of exhibition, I use the notation $\tau_c = t_c/(1+t_c\beta)$.

[27]Assuming the representative individual's utility function is quadratic is another example of when the expected utility function is characterized fully by the first two moments of the state-dependent variables and is frequently used in the finance literature.

covariance of the tax bases, $\sigma_{w,c}$. This provides an incentive for the government to tax both income and consumption as a hedge against idiosyncratic risk in either tax base.

Appendix B.0.2 Equilibrium

This section derives an updated Ramsey rule that characterizes the optimal tax rates with uncertainty. As intermediate steps, the analysis derives the first-order conditions and determines a minimum volatility frontier for the government. The analysis provides a framework to understand the observed increase in state tax revenue volatility in the 2000s.

Appendix B.0.3 Government's Objective

The government's problem can be thought of as an optimal portfolio problem where it chooses the portfolio weights (tax rates) on the assets it has available to it (wage income and consumption). Formally, the government's objective can be written as minimizing the negative welfare impact of volatility:

$$min_{t_w,t_c} \quad -V(c, g, \sigma_c^2, \sigma_g^2, l, \beta).$$

The first-order conditions set equal the marginal benefit and cost associated with increasing tax rate i:

$$V_1\frac{\partial c}{\partial t_i} + V_2\frac{\partial g}{\partial t_i} + V_3\frac{\partial \sigma_c^2}{\partial t_i} + V_4\frac{\partial \sigma_g^2}{\partial t_i} + V_5\frac{\partial l}{\partial t_i} + V_6\frac{\partial \beta}{\partial t_i} = 0, \qquad \text{(Appendix B.5)}$$

The marginal benefit of increasing either tax rate is the value of the additional public goods the tax rate produces, given by the second term in equation (Appendix B.5). The marginal cost due to distorting consumption decisions and labor supply is captured by the first, fifth, and sixth terms.

With uncertainty the government considers the marginal effect of changing a tax rate on the volatility of public and private consumption, which is captured by the third and fourth terms in equation (Appendix B.5). The government has an incentive to tax state-dependent tax sources to distribute risk between private and public consumption and to tax both all state-dependent tax sources to hedge idiosyncratic risk. The model presented here considers income and consumption taxes and is easily extended to include corporate taxes. The insight that the government has an incentive to tax a portfolio of different sources also extends to dynamic models which allow for property and estate taxes.

Appendix B.1 Consumption Base Decomposition

In the text, private consumption of the representative agent is decomposed into consumption that is taxed and consumption that is untaxed such that the fraction β of total consumption is taxed and $(1 - \beta)$ is untaxed. This decomposition changes the variables from two consumption goods into total consumption and the fraction spent on taxable items. This section demonstrates the change of variables and its benefits.

First, start with two goods, B, N, such that the consumption of B is taxed and the consumption of N is not taxed and the representative agent has utility $\mathbb{U}(B, N)$ over the two goods. By definition, $B = \beta c$ and $N = (1-\beta)c$. The utility function can be written as a function of β and c by substituting these equations in for B and N. If the utility function is homothetic, then the utility function can be written as $\mathbb{U}(B, N) = v(\beta)U(c)$, otherwise $\mathbb{U}(B, N) = U(c, \beta)$. The budget constraint is given below written both as a function of B and N and β and c.

$$\begin{aligned} W &= (1+\tau_c)B + N \\ &= (1+\tau_c)\beta c + (1-\beta)c \\ &= c(1+\beta\tau_c) \end{aligned}$$

Now we want to know the welfare impact of a tax change. We can separate the impact into the wealth effect and the substitution effect where the substitution effect is the deadweight loss from the behavioral responses.

$$\begin{aligned} \frac{\partial \mathbb{U}(B,N)}{\partial \tau_c} &= \mathbb{U}_1 \frac{\partial B}{\partial \tau_c} + \mathbb{U}_2 \frac{\partial N}{\partial \tau_c} \\ &= \mathbb{U}_1 (S_{B,\tau_c} - \frac{\partial B}{\partial M} B) + \mathbb{U}_2 (S_{N,\tau_c} - \frac{\partial N}{\partial M} B) \quad \text{Slutsky Decomposition} \\ &= \underbrace{\mathbb{U}_1 S_{B,\tau_c} + \mathbb{U}_2 S_{N,\tau_c}}_{\text{Substitution Effect}} - \underbrace{\left(\mathbb{U}_1 \frac{\partial B}{\partial W} B + \mathbb{U}_2 \frac{\partial N}{\partial W} B \right)}_{\text{Income Effect}} \end{aligned}$$

The benefit of writing the utility in terms of β and c is that $U_1 \frac{\partial c}{\partial \tau_c}$ captures the income effect and $U_2 \frac{\partial \beta}{\partial \tau_c}$ captures the behavioral response and deadweight loss.

Appendix C Empirical

Figure A.2 shows how state governments have changed the tax bases they rely on from 1951 to 2013. Panels A and B graph tax portfolios onto a simplex. The horizontal axis of the simplex represents the fraction of total tax revenues derived from sales and income tax revenues, and the vertical axis represents the fraction derived from corporate tax revenues. Panel C and the accompanying note provide more details on the simplex. Panel A shows that between 1951 to 2013 the aggregate tax portfolio shifted away from the sales and corporate tax and toward the income tax, depicted as movement southeast on the simplex. Panel B of Figure A.2 graphs each state's tax portfolio in 1955 and 2005. This figure demonstrates that a large number of states shifted their tax portfolios to rely more heavily on the income tax base and less heavily on the sales tax base.