# Better Bunching, Nicer Notching

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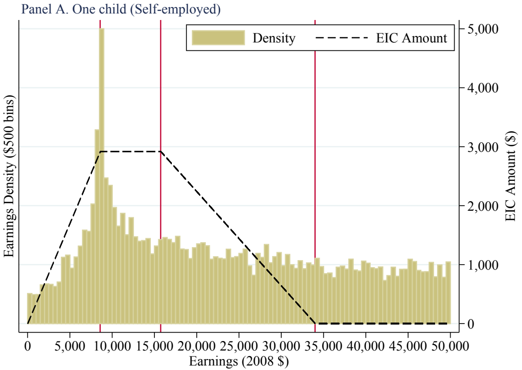
# Goal: Estimate the Tax Elasticity of Reported Income.

Saez [2010] and Kleven and Waseem [2013]'s insight is that the elasticity relates to amount of bunchers.

More bunchers = larger elasticity; *ceteris paribus*.



## The Bunching Estimator

- Ideally, you would like to observe the same distribution of individuals facing different tax changes
  - There are not that many quasi experiments.

- The bunching estimator is an appealing identification strategy because it only takes one population of individuals and one (piece-wise linear) budget set to estimate the elasticity.

- Many examples of applications with piece-wise linear budget sets: Prescription drug insurance [Einav et al., 2017], pensions systems [Brown and Laschever, 2012], welfare programs [Camacho and Conover, 2011], education policy [Dee et al., 2011], labor regulations [Garicano et al., 2016], minimum wages [Dube et al., 2017], fuel economy policy [Sallee and Slemrod, 2012], real estate taxes [Kopczuk and Munroe, 2015], and pricing schemes in electricity [Ito, 2014], cellular service [Huang, 2008], and water markets [Olmstead et al., 2007].

- Our focus is on one population facing one budget set.
  - There are other estimators using variation from piece-wise linear budget constraints e.g., Blomquist and Newey [2002].

# What Do We Learn From Bunching?

- This paper clarifies that non-parametric identification of the elasticity is impossible in the case of kinks.

- We propose identifying conditions that are weaker than those used before.

Solution 1 (Bounds)
- non-parametric shape restriction: partial identification

Solution 2 (Truncated Tobit)
- covariates and semi-parametric distribution restriction: point identification

# What Do We Learn From Bunching?

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weakest assumptions

Solution 1 (Bounds)
- non-parametric shape restriction: partial identification

Solution 2 (Truncated Tobit)
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strongest assumptions

# Utility Maximization Problem

- Agent type $N^*$ maximizes utility $U(C, Y; N^*)$ choosing consumption $C$ and labor income $Y$

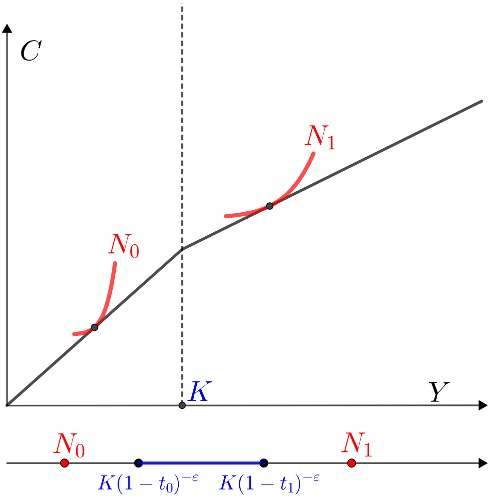
$$\max_{C,Y} \quad C - \frac{N^*}{1+1/\varepsilon}\left(\frac{Y}{N^*}\right)^{1+\frac{1}{\varepsilon}}$$

$$s.t. \quad C = \mathbb{I}\{Y \leq K\}[I_0 + (1-t_0)Y] + \mathbb{I}\{Y > K\}\left[I_1 + (1-t_1)(Y-K)\right]$$

- Piece-wise linear budget with intercept $I_j$ and slope $1 - t_j$

- There is a tax rate change $t_0 < t_1$ at $Y = K$ (kink)

# Solution to Utility Maximization Problem



# Goal: Invert Solution Mapping to Get $\varepsilon$

The solution in logs is:

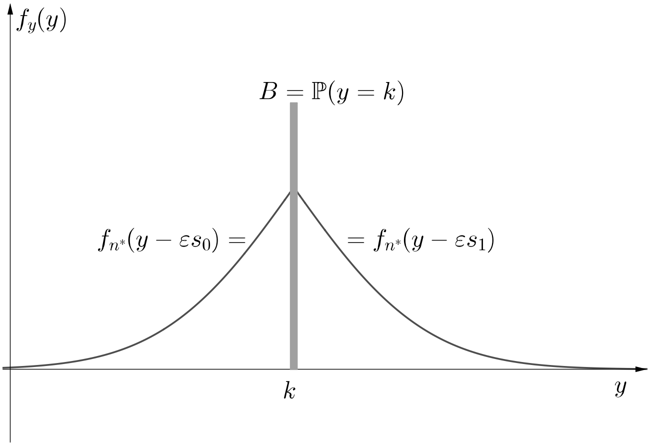
$$y = \begin{cases} n^* + \varepsilon s_0 & \text{, if } 0 < n^* < k - \varepsilon s_0 \\ k & \text{, if } k - \varepsilon s_0 \leq n^* \leq k - \varepsilon s_1 \\ n^* + \varepsilon s_1 & \text{, if } k - \varepsilon s_1 < n^* < \infty \end{cases} \tag{1}$$

where:
$y = \log(Y)$ , $n^* = \log(N^*)$, $k = \log(K)$, $s_j = \log(1 - t_j)$,

- It is helpful to think in terms of the counterfactual scenario of no tax change: $t_0 = t_1$
- Counterfactual income : $y_0 = n^* + \varepsilon s_0$

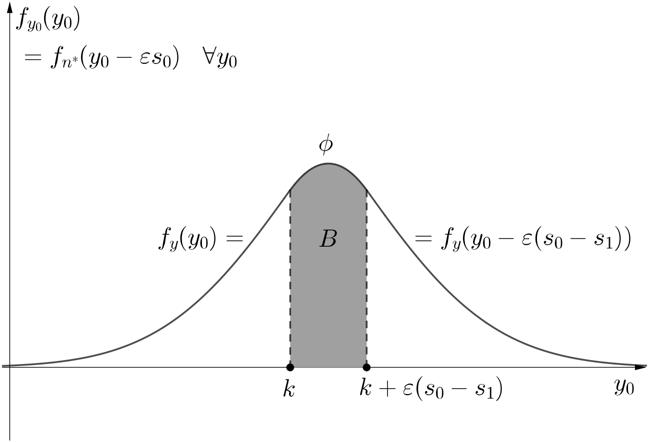
# Distribution of Income

## Observed Data



# Distribution of Income

## Unobserved Counterfactual



# How Does the Literature Identify the Elasticity?

- The 'bunching estimator' of Saez [2010] uses the definition of $B$

$$B = \int_{k}^{k+\varepsilon(s_0 - s_1)} f_{y_0}(y) \; dy,$$

  - makes a trapezoidal approximation,

$$\cong 0.5 \left[ f_{y_0}\left(k + \varepsilon\left(s_0 - s_1\right)\right) + f_{y_0}\left(k\right) \right] \varepsilon\left(s_0 - s_1\right),$$

  - replaces $f_{y_0}$ by $f_y$,

$$= 0.5 \left[ f_y(k^+) + f_y(k^-) \right] \varepsilon\left(s_0 - s_1\right),$$

  - and solves for the elasticity

$$\varepsilon \cong B / \left\{ \left(s_0 - s_1\right) 0.5 \left[ f_y(k^+) + f_y(k^-) \right] \right\}$$

- This restricts the PDF $f_{n^*}$ to be "linear" for $n^* \in [k - \varepsilon s_0, k - \varepsilon s_1]$.

# Literature Relies on Strong Functional Form Assumptions

- "*But isn't linearity approximately true if the bunching interval is small?*"

- There is no way to know because the length of the interval depends on the unknown $\varepsilon$.

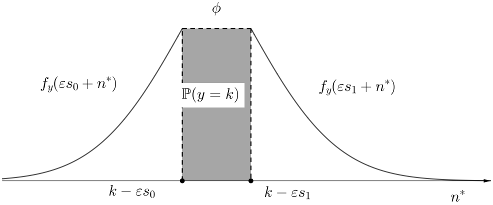
- Chetty et al. [2011] make a stronger functional form assumption:
  - $n^*$ has a flat PDF inside the bunching interval.

- These are not non-parametric identification strategies.

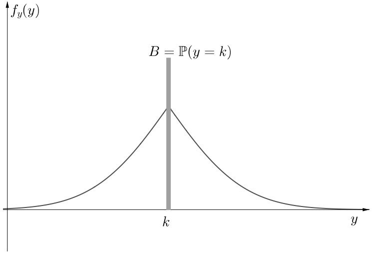
- We find $\varepsilon$ is very sensitive to the shape of $f_{n^*}$.

# Identification without Restrictions on $f_{n^*}$ is Impossible

Unobserved ability distribution $f_{n^*}$ and $\varepsilon$



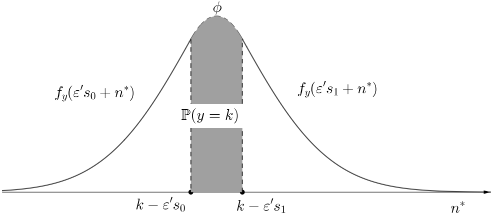

$\Downarrow$ $\Downarrow$ Solution in Equation (1) $\Downarrow$ $\Downarrow$



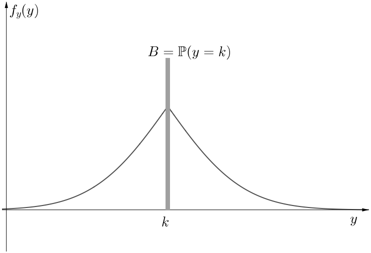

Observed income distribution $f_y$

# Identification without Restrictions on $f_{n^*}$ is Impossible

Unobserved ability distribution $f_{n^*}$ and $\varepsilon'$




⇓ ⇓ Solution in Equation (1) ⇓ ⇓




Observed income distribution $f_y$

# Data Justifies Any Elasticity

**Lemma:** Assume $f_{n^*}$ is an unobserved continuous PDF with support $(-\infty, +\infty)$, and that we observe $f_y$.

Then, for every elasticity $\varepsilon > 0$, there exists a $f_{n^*}$ such that Equation (1) maps the distribution of $n^*$ into the distribution of $y$.

Therefore, it is impossible to identify the elasticity.

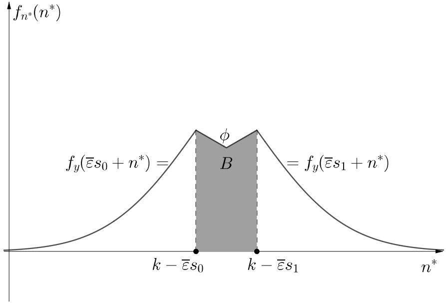
- There is an old literature on impossible inference.
  - For a review see Bertanha and Moreira (2019).
  - Here we have a worse problem: not even identification.

- This result also appears in Blomquist and Newey [2017].

# Shape Restriction Yields Partial Identification

Assume $f_{n^*}$ is Lipschitz with constant $M$

Unobserved ability distribution, $f_{n^*}$

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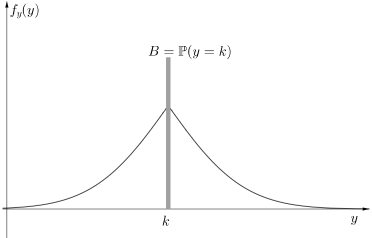

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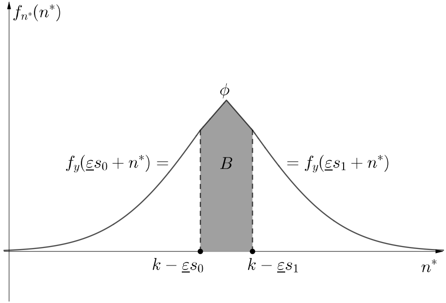


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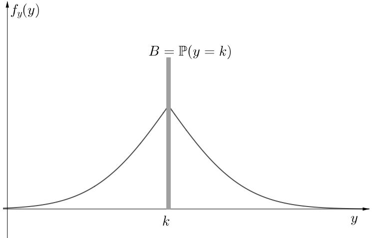


# Partial Identification of $\varepsilon$

**Theorem:** Assume $f_{n^*}$ is Lipschitz with known constant $M$.

Then,

$$\varepsilon \in \Upsilon = \begin{cases} \emptyset & \text{, if } B < \frac{\left|f_y(k^+) - f_y(k^-)\right| \left[f_y(k^+) + f_y(k^-)\right]}{2M} \\ [\underline{\varepsilon}, \overline{\varepsilon}] & \text{, if } \frac{\left|f_y(k^+) - f_y(k^-)\right| \left[f_y(k^+) + f_y(k^-)\right]}{2M} \leq B < \frac{f_y(k^+)^2 + f_y(k^-)^2}{2M} \\ [\underline{\varepsilon}, \infty) & \text{, if } \frac{f_y(k^+)^2 + f_y(k^-)^2}{2M} \leq B \end{cases}$$

where

$$\underline{\varepsilon} = \frac{2\left[f_y(k^+)^2/2 + f_y(k^-)^2/2 + M\ B\right]^{1/2} - \left(f_y(k^+) + f_y(k^-)\right)}{M(s_0 - s_1)}$$

$$\overline{\varepsilon} = \frac{-2\left[f_y(k^+)^2/2 + f_y(k^-)^2/2 - M\ B\right]^{1/2} + \left(f_y(k^+) + f_y(k^-)\right)}{M(s_0 - s_1)}$$

**Solution 1 (Bounds) :** We recommend a sensitivity analysis for various choices of $M$.

# Bunching is a Censored Regression Model

- **Classic Tobit**

  Suppose $n^* \sim N(\mu, \sigma^2)$.

  Left censored at $k$: $y = \max\{k; n^*\}$, or

  right censored at $k$: $y = \min\{k; n^*\}$.

- **Mid-censored Model**: Equation (1) rewrites as

  $$y = \min\{\varepsilon s_0 + n^* \ , \ \max\{k \ , \ \varepsilon s_1 + n^*\}\}.$$

  - The elasticity is the difference of intercepts divided by $(s_1 - s_0)$

# Use Covariates to Predict Ability with a Tobit

- **Mid-censored Tobit**
  -There is a vector of covariates $X$

- **Idea:** instead of polynomial or linear extrapolations, use the relationship between $n^*$ and $X$ for non-bunching people to predict the distribution of $n^*$ for bunching people.

Assumption: $n^* \mid X \sim N(X\beta; \sigma^2)$

- **Question:** what if conditional normality is incorrect?

  - MLE is inconsistent ...

# Tobit is Robust to Non-normality

Assumption: there exists an unique $(\beta, \sigma)$ such that $F_{n^*}(n) = \mathbb{E}\left[\Phi\left(\frac{n - X\beta}{\sigma}\right)\right]$, plus some regularity conditions.

- Semi-parametric class that does NOT require conditional normality.

**Lemma:** Let $\theta^* = (\beta^*, \sigma^*, \varepsilon^*)$ be the probability limit of the Tobit MLE, and suppose the Assumption above holds.

If the true distribution $F_y(y)$ matches the Tobit fitted distribution $G_y(y; \theta^*)$, then $\varepsilon^*$ is equal to the true elasticity.

**Solution 2 (Truncated Tobit)**:

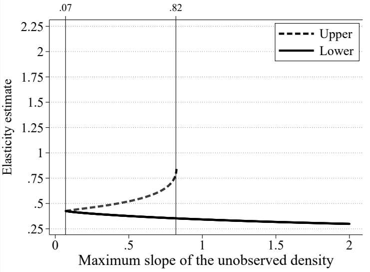
- truncate the sample of $y$ to a neighborhood of $k$;
- vary the size of the neighborhood, from full to smallest sample;
- examine how the fit and estimates vary.

# Application

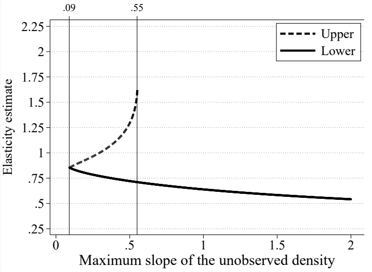
- Employ data originally studied by Saez [2010]
  - – Individual Public Use Tax Files constructed by the IRS
  - – Annual cross-sections 1995-2004
- Focus on $8,580 kink in the EITC schedule:
  - – tax rate changes from -34 % to 0%
- We constrast trapezoid-based estimates with
  - \- Solutions 1 (Bounds)
  - \- Solutions 2 (Truncated Tobit)
    - – dummy covariates: year effects, types of deductions, received social security benefits, used a tax prep software, etc.

# Partial Identification

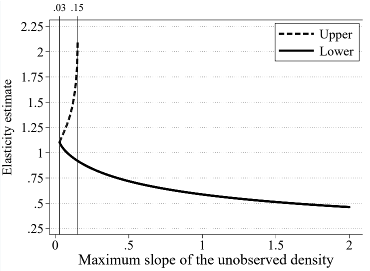
(a) All Filers



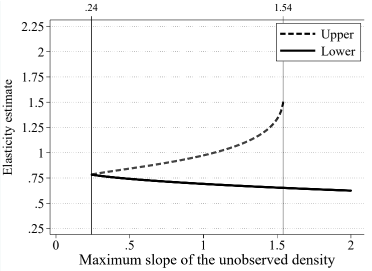

(b) Self-Employed Filers



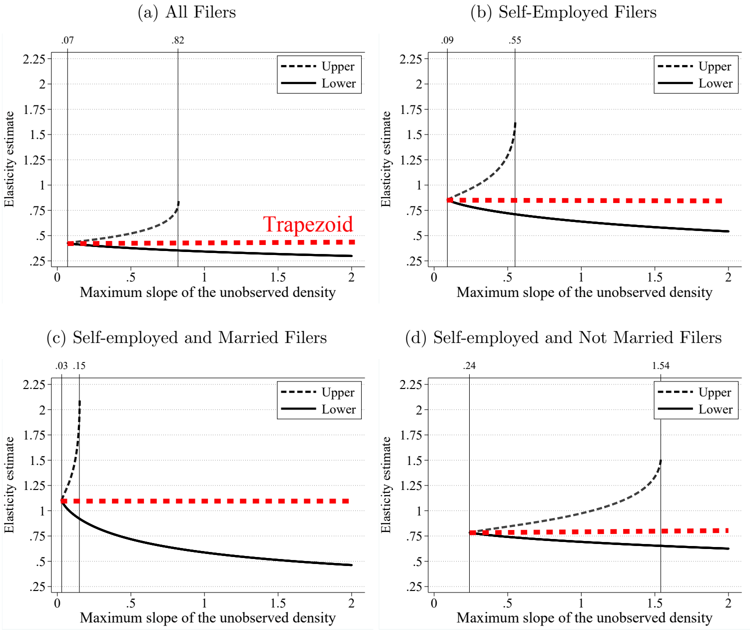

(c) Self-employed and Married Filers




(d) Self-employed and Not Married Filers



# Partial Identification



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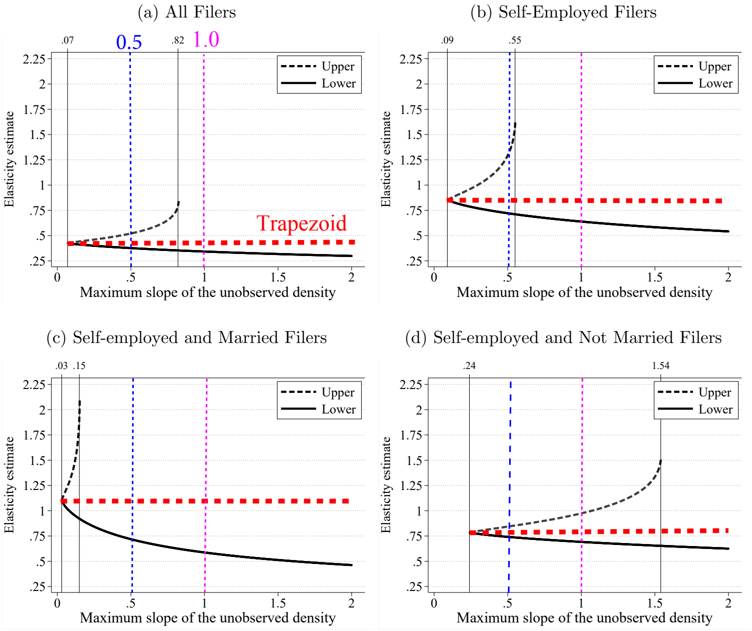
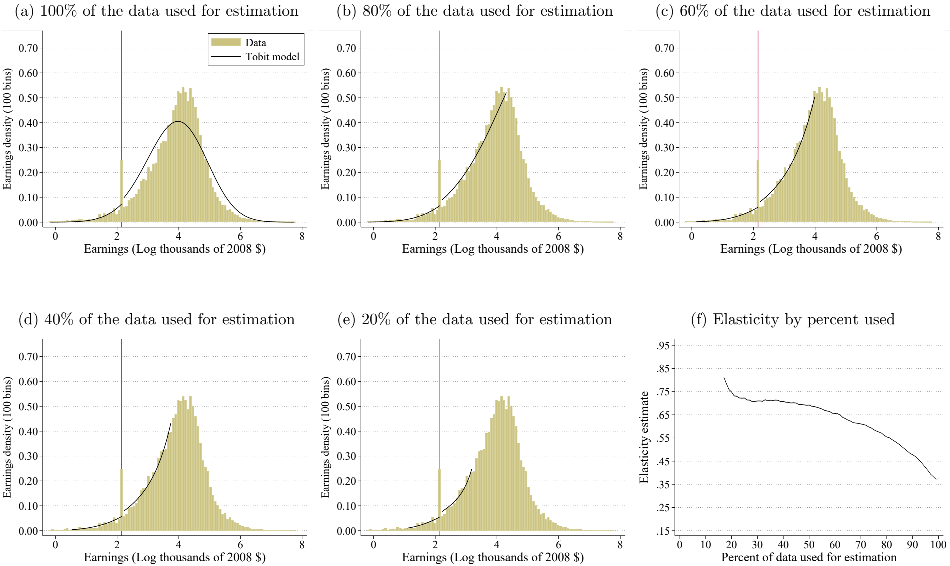


# Truncated Tobit

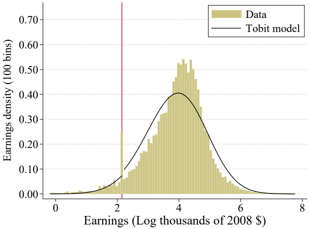
## Self-employed Married



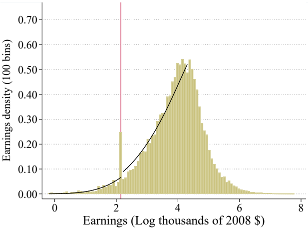
# Truncated Tobit

## Self-employed Married

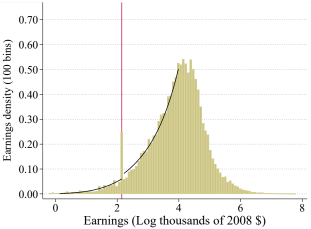
(a) 100% of the data used for estimation



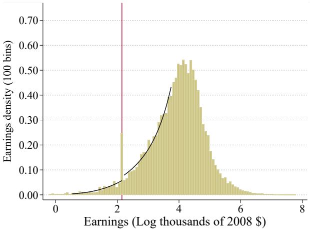

(b) 80% of the data used for estimation



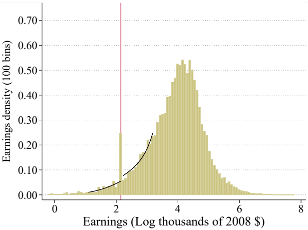

(c) 60% of the data used for estimation




(d) 40% of the data used for estimation



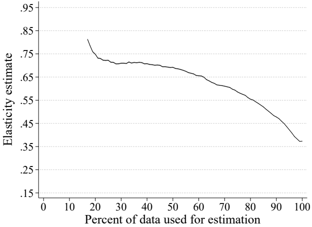

(e) 20% of the data used for estimation




Trapezoid

1.102

(f) Elasticity by percent used



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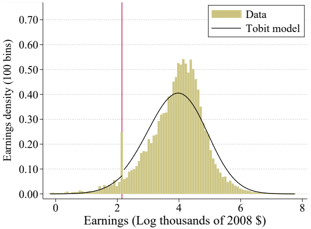


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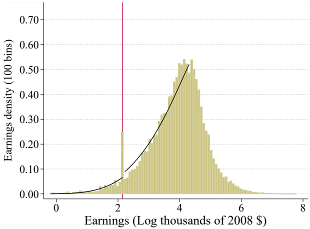


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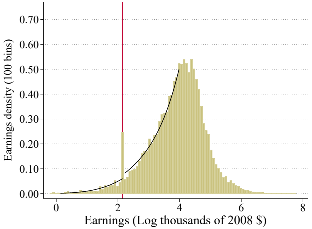


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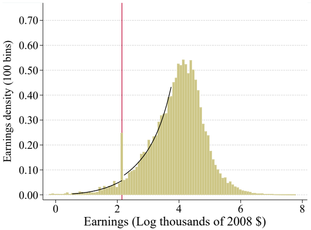


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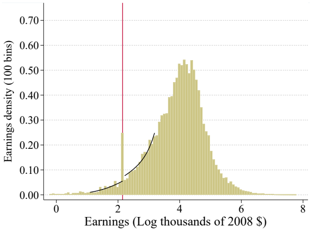


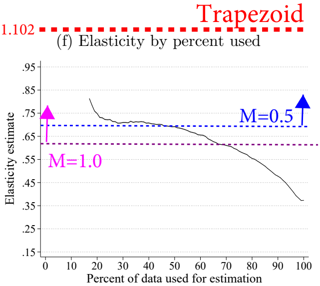


(f) Elasticity by percent used

# Conclusions

- Minimal restrictions partially identify the elasticity
- Connection between bunching and censored regressions allows for:
  - Covariates: more meaningful extrapolation
  - Semi-parametric restrictions on the distribution of $n^*$

**Future of Bunching:**

- examine sensitivity of estimates using Bounds and Truncated Tobit
- STATA package coming soon!

**More on the Paper, Skipped Today:**

- failure of the widely used "polynomial strategy"
- multiple kinks and notches
- non-parametric identification is possible in the case of notches
- point indentification with censored quantile regressions