Index to Mathematical Problems 1975-1979
By Stanley Rabinowitz and Mark Bowron (editors).
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Anyone wanting to learn about an area of mathematics or check on a major theorem has ready recourse to reviewing journals, textbooks and the advice of known specialists in the field. In contrast, finding out the history of problems and what their status is with respect to solution can be daunting. Problems come in and out of fashion. They may be passed around by word of mouth, appear on contests in different places at different times, or be posed in any number of journals with problems sections. They may be picked up and abandoned, and their solutions (if any) communicated in various ways. Accordingly it is difficult to discover their provenance, study their development and assess their novelty. Many collections of problems have appeared in recent decades, particularly in Eastern Europe, but often these are collections from particular contests or reflect the idiosyncracies of the compilers. One such collection is *500 Challenging Problems* by Murray Klamkin, Willy Moser and Ed Barbeau (published originally by the CMS and later by the MAA). Another is the *Otto Dunkel Memorial Problem Book* that, in 1957, collected the 400 best problems from the 1918 to 1950 issues of the *American Mathematical Monthly*. A reliable guide to recent problems that have become notorious over the past fifty years is given by the essays of Martin Gardner originally published in *Scientific American*. However, few of these provide a thematic index to problems and none are comprehensive enough to serve as a definitive guide to the literature. We have had to rely on a few individuals with a lot of experience and prodigious memories to impose any kind of order on the corpus of problems.

One such person is Stanley Rabinowitz, who received his doctorate in convexity, combinatorics and number theory from the Polytechnic University of New York and professionally is a software engineer and computer consultant. In 1989, he founded *MathPro Press* to produce problem indexes, as well as collections of problems, compendia of mathematical results and books on the use of computers in the solution of mathematical problems. Rabinowitz became convinced of the necessity of a source work to which problemists could turn for interesting material and information on the origins and status of problems they come across. But how to deal with the chaotic wealth of material? His solution was to list all those problems that appeared in a selection of popular journals and contests and provide tools for searchers find their way around. In 1992, he broke new ground in his publication of the *Index to Mathematical Problems 1980-1984* which listed and classified all the problems published from standard anglophone sources during those years. This achievement was well received by the community, which eagerly awaited its successors. Finally, in 1999, the present volume was published with the collaboration of Mark Bowron, whose day job is driving an 18-wheeler transport truck.

The core of the book is the **Subject Index** (SI), 256 pages with the texts of about 4000 problems, listed by subject according to 17 classifications with many subheadings. Each problem appears with its author and source identification; it is an essentially complete collection of problems from 23 journals, mostly North American, and 6 competitions (Olympiads from Canada, USA, Australia, along with the International Mathematical Olympiad and the Putnam and Kurchak contests). Canada is represented by the *Canadian Mathematical Bulletin* (which used to have a problem section), *Crux Mathematicorum*, the *Ontario Mathematics Gazette* and the *Ontario Secondary School Mathematics Bulletin.*

This is supplemented by other indexes:

• **Subject classification scheme**: list of titles in the SI with page references to the SI;

• **Problem locator**: list of journals and problem numbers with page references to the SI;

• **Problem chronology**: list of journals and problems, along with references to external sources with solutions and comments;

• **Author index**, with a key to pseudonyms;

• **Title index**, based on titles or topics assigned by journal editors;

• **Journal issue checklist**: details about publishers, problems editors, along with lists of problems according to issue;

• **Unsolved problems**: eight pages with complete statements;

• **Citation index**: references made in the period 1975-1979 to current and previously published problems in the journals;

• **Bibliography**

• **Keyword index**

Readers of this journal will be familiar with Canadian contributors who are well represented in the compendium: **Fred Maskell** and **Leo Sauvé**, the founding editors of *Crux*, **Murray S. Klamkin**, the problems editor for *SIAM Review*, **W.J. Blundon** and **Viktors Linis**, prolific contributors to *Crux* respectively from Memorial University and the University of Ottawa, **Kenneth J. Williams**, **Edward T.H. Wang** and **Andy Liu**.

Many readers will be drawn to the unsolved problems. Some of these are wellknown, such as the Collatz or $3x+1$ problem (CRUX 133); some look tedious, technical or deep (*e.g.*, *Prove or disprove that for each natural number $n \geq 2$, one can arrange the numbers $1, 2, \cdots, n$ in a sequence such that the sum of any two adjacent terms is a prime.* E.T.H. Wang, AMM 6189). Others are more enticing: *Suppose that each square of an $n \times n$ chessboard is colored either black or white. A square, formed by horizontal and vertical lines of the board, will be called chromatic if its four distinct corner squares are all of the same color. Find the smallest n such that, with any such coloring, every $n \times n$ board must contain a chromatic square*, (AMM 6211); *Let A and B be the unique nondecreasing sequences of odd integers and even integers, respectively, such that, for all $n \geq 1$, the number of integers i satisfying $A_i = 2n-1$ is A_n and the number of integers i satisfying $B_i = 2n$ is B_n. That is, $A = \{1, 3, 3, 3, 5, 5, 5, 7, 7, 7, 9, 9, 9, 9, 9, \cdots\}$ and $B = \{2, 2, 4, 4, 6, 6, 6, 6, 8, 8, 8, 8, \cdots\}$. Is the difference $|A_n - B_n|$ bounded?* (MM 1073). Then there is this one from Paul Erdös: *Find a sequence of positive integers $1 \leq a_1 < a_2 < a_3 < \cdots$ that omits infinitely many integers from each arithmetic progression (in fact it has density 0) but which contains all but a finite number of terms of every geometric progression. Prove also that there is a set S of real numbers which contains infinitely many terms of any arithmetic progression but contains every geometric progression (disregarding a finite number of terms),* (PME 389).

The scholarship underpinning this volume is impressive. The editors have performed the difficult task of classifying the problems, matched up various versions of names and pseudonyms, tracked down if and where solutions may be found, and checked the accuracy of the statement of problems and the information provided about them. However, the collection scoops up indiscriminately from its sources, so there is quite a mixed bag of problems. They can be fascinating and ingenious or banal, very difficult or trivially easy, significant challenges or dull exercises, novel or trite, narrowly specialized or broadly appealing, memorable or ephemeral, singular or typecast. They can be solved by systematic techniques or through the right perspective and strokes of inspiration. Of course, the book does not offer guidance and the reader just has to dive in and try the problems. However, a teacher on the lookout for material suitable for various situations will find her efforts rewarded by problems of the right level of difficulty.

However, it does put into question the long term viability of the project. Problems are pouring into the literature regularly, many of which are neither novel nor particularly inventive. It is clear that the influx of new material is bound to outrun the capacity of the most energetic publisher to collect and classify it. Only the net can begin to keep on top of this task, and it may be that future books will have to focus on following the evolution of the most popular, interesting or elegant problems. However, journals with problem sections can help out with regular indexes of their own collections.

Rabinowitz has provided many resources on the net. A visit to *http://www.problemcorner.org* will provide you with 20000 problems that can be searched by keyword or author, and for which references to solutions in the literature are given. His main website *http://www.mathpropress.com* contains lists of problem books published during the last two centuries, of all problem books published recently, of web sites with problems from local, regional and international competitions, of 32 books with solutions to IMO problems, of journals with problems sections and of mathematical dictionaries. MathPro publications, including this book, can be ordered through the website at *http://www.mathpropress.com/orderinginfo.html*. One can also order by telephone: 1-800-247-6553.