Department of Education, Ontario

Annual Examinations, 1956

Thursday, 14th June: 9.00-11.30 am

GRADE XIII

TRIGONOMETRY AND STATICS

NOTE. *The candidate should obtain a set of mathematical tables from the Presiding Officer*

1. (a) Define *radian*.

   (b) Find, correct to *four* significant figures, the radian measure of the supplement of $72°$. ($\pi = 3.1416$)

   (c) Assuming that $\log_{10} \pi = 0.49715$, express $\sqrt{\pi}$ as a power of 10.

2. If θ is an angle of the second quadrant, such that $\cos\theta = -3/5$, find, without using tables, the value of

   (i) $\tan\theta$,
   (ii) $\csc 2\theta$,
   (iii) $\tan(\theta - 45°)$,
   (iv) $\sin(\theta + 90°)$.

3. Three towns A, B, and C are located with respect to each other as follows: B is 36 miles due north of A; the bearings of C from A and B are, respectively, $30°$ east of north and $40°$ east of north. Calculate, correct to the nearest mile, the distance of C east of the line AB.

4. (a) For any triangle, ABC, prove that

$$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}\ ,$$

   where $2s = a + b + c$.

   (b) A triangle ABC is such that $b = 42$, $c = 63$, $B = 27°$, and the angle C is obtuse. Calculate the angle A, correct to the nearest degree.

5. A tunnel is to be constructed through a mountain from a point A to a point B. Both A and B are visible from a third point C. The distances AC and BC are 384.8 feet and 555.6 feet, respectively, and the angle $ACB = 35°\ 24'$. *Using logarithms and*

formulas adapted to logarithms, find the angles ABC and CAB, correct to the nearest minute.

6. (a) For any triangle, prove the formula

$$r = \frac{\Delta}{s}$$

for the radius of the inscribed circle of the triangle.

(b) If the permimeter of a right-angled triangle is 70 inches and the radius of the inscribed triangle is 6 inches, find the lengths of the sides of the triangle.

7. (a) State and prove a formula which transforms

$$\sin A - \sin B$$

into a product.

(b) Find, correct to the nearest minute, all the positive values of x, less than $360°$, which satisfy the equation

$$3\cos 2x + \cos x + 1 = 0 \ .$$

8. A wheel of radius 10 inches has its centre at O. Four points A, B, C, D are selected on the rim of the wheel so that angle $AOB = 60°$, angle $BOC = 90°$, and angle $COD = 90°$. Three forces, $F_1 = 30$ pounds, $F_2 = 20$ pounds, $F_3 = 40$ pounds, act along AB, AC, and AD, respectively, in the directions indicated by the letters.

(a) Find, to the nearest degree, the angle between AO and the direction of the resultant of F_1, F_2, F_3.

(b) Calculate, correct to *two* significant figures, the algebraic sum of the moments of F_1, F_2, and F_3 about O.

9. A tapering beam, 15 feet long and weighting 20 pounds, rests against a smooth vertical wall. The upper end is 9 feet from the ground and the lower end is prevented from slipping by a peg driven into the ground. If the centre of gravity of the beam is 6 feet from the lower end, calculate, correct to the nearest pound,

(i) the reaction of the wall,

(ii) the resultant reaction at the peg.