Department of Education, Ontario

Annual Examinations, 1956

Tuesday, 12th June: 9.00-11.30

GRADE XIII

GEOMETRY

NOTE. *Squared paper may be obtained from the Presiding Officer*

1. A triangle has vertices at $A(2,1)$, $B(-1,-2)$, and $C(-2,5)$.

   (a) Show the position of these points on a diagram drawn on squared paper.

   (b) Find the equation of AB.

   (c) Find the equation of the line through A perpendicular to BC.

2. (a) Give a definition, sufficiently comprehensive to be applied to any curve, of the tangent to a curve at a given point.

   (b) Derive the equation of the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

   at the point (x_1, y_1) on the curve.

   (c) Find the equations of the two tangents from the point $(-1,1)$ to the curve $x^2 + 2y^2 = 2$.

3. (a) Write the equation for the family of circles each of which has with the circle $x^2 + y^2 - 2x = 3$ the line $x + y = 1$ as radical axis.

   (b) Among the circles referred to in (a) find the one which passes through the point $(1,1)$.

4. (a) Define a hyperbola.

   (b) Let $P(x,y)$ be a point on the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \ .$$

Find the distance from P to the nearer asymptote, and prove that this distance becomes very small as x becomes very large.

5. The directrix of a parabola is the line $x+1=0$ and the focus is the point $(1,0)$. Prove that the equation of the parabola is $y^2=4x$ by showing

   (i) that the coordinates of every point on the curve satisfy the equation, and

   (ii) that every point whose coordinates satisfy the equation is actually on the curve.

6. (a) The line passing through the points $A(2,5)$ and $B(1,1)$ is cut by the circle $x^2+y^2=9$ at the points P and Q. Find the ratios $AP:PB$, $AQ:QB$.

   (b) Suppose that the line $Ax+By+C=0$ divides the line segment joining (x_1,y_1) and (x_2,y_2) *internally* in the ratio $k:1$. Find an expression for k and deduce that Ax_1+By_1+C and Ax_2+By_2+C must have opposite signs.

7. A symmetrical arch is constructed in the form of one part of a rectangular hyperbola. The height of the arch is 12 feet and the base of the arch is 36 feet. Find the height of the arch at a distance 10 feet from the midpoint of the base.

8. The points $A(-1,0)$ and $B(1,0)$ are the endpoints of a diameter of the circle $x^2+y^2=1$, and M is a variable point on this circle. The line through B perpendicular to the tangent to the circle at M meets AM at P. Find the equation of the locus of P.