

Department of Education, Ontario

Annual Examinations, 1956

Friday, 22nd June: 9.00-11.30 am

GRADE XIII

ALGEBRA

NOTE. A book of mathematical tables may be obtained from the Presiding Officer.

1. (a) Find from first principles the sum of n terms of the geometric progression

$$a + ar + ar^2 + \dots .$$

- (b) The second term of a geometric progression is -10 . Its sum to infinity is 9 . Find the first and third terms of the series.
2. The sum of the first eight terms of an arithmetic progression is 136 . The sum of its third and fourth terms is 26 . Find the first term and the common difference.
3. (a) Solve the system of equations

$$x^2 - 2y^2 = 17 ,$$

$$xy + 3y^2 = 2 .$$

- (b) The roots of $3x^2 - 5x + 1 = 0$ are m and n . Find the quadratic equation whose roots are

$$\frac{1}{m^2} + 1 \quad \text{and} \quad \frac{1}{n^2} + 1 .$$

4. (a) The number of oscillations made by a pendulum in a given time varies inversely as the square root of its length. A pendulum 49 inches long makes 2000 oscillations in a certain period of time. Find, to *two* significant figures, the amount by which the length of the pendulum should be altered so that it will make 1995 oscillations in the same period of time.

- (b) If $f(x) = \frac{8^x - 8^{-x}}{x}$, find $f(-2/3)$.

5. (a) The roots of $ax^2 + bx + c = 0$ are real and unequal. Prove that the function $ax^2 + bx + c$ has values opposite in sign from that of a if and only if x lies between the roots.

- (b) Use the Factor Theorem to find values for k so that $3x^3 + k^2x^2 - 6kx - 16$ will have the factor $x + 2$.
6. (a) For real values of x , find the minimum value of $5 - 3x + 2x^2$ and the corresponding value of x .
- (b) For what values of x does $5 - 3x + 2x^2$ have the value 7?
- (c) Use a sketch, appropriately labelled, to illustrate the results of (a) and (b).
7. In how many ways can an athletic board of 4 persons be chosen from 3 teachers and 12 boys, if the board must include at least one teacher?
8. Use the Binomial Theorem to find the first 4 terms of the expansion of

$$\frac{\sqrt{1 - 2x^2}}{\sqrt[3]{1 + 3x}}$$

as a series in ascending powers of x .

9. A bond for \$1000 matures 10 years hence and carries semi-annual coupons of \$17.50, the first of which is due 6 months hence. Find the price of this bond if the purchaser is to receive 5% per annum compounded semi-annually on his investment.