This column is for your mathematical amusement. Its author is very happy to correspond with readers about mathematical matters, and hopes that the column will turn out to be a dialogue with readers of the Frontenac News. His email address is barbeau@math.utoronto.ca.

Pythagorean triples

Many of you will be familiar with the equation $3^2 + 4^2 = 5^2$. The set $(3, 4, 5)$ of three whole numbers is called a pythagorean triple, because the square of the largest number is equal to the sum of the squares of the two smaller numbers. Chidren usually find out about these triples when they are in middle school or junior high school. There are lots of triples like this around, and it can be an absorbing task to see how many you can discover. One way to do this is to check for possible patterns, and experiment with them. Two other pythagorean triples are $(5, 12, 13)$ and $(20, 21, 29)$, because $13^2 = 5^2 + 12^2$ and $29^2 = 20^2 + 21^2$. (The square of any number is the result of multiplying the number by itself, and is denoted by putting a superscript 2 after the number.)

Here are a few lines of investigation for you:

(1) Find other pythagorean triples besides $(3, 4, 5)$ and $(5, 12, 13)$ where the largest two numbers differ by 1. (Hint: Add the two largest numbers together.)

(2) Find other pythagorean triples besides $(3, 4, 5)$ and $(20, 21, 29)$ where the smallest two numbers differ by 1. This is a harder task and the pattern is not so easy to find. It may be worth pulling out a computer or a pocket calculator and seeing if you can find any more. You can email me for more hints.

(3) See if you can find other pythagorean triples besides those described in (1) and (2). There is a general formula that gives them, but do not go to a search engine before you have given yourself a chance to explore the situation a bit.

There are other ways to generalize the relationship $3^2 + 4^2 = 5^2$. There is another sequence of equations with this one at the top. The next equation is the sequence is this:

$$10^2 + 11^2 + 12^2 = 13^2 + 14^2.$$

(Writing out the squares, we see that this asserts

$$100 + 121 + 144 = 365 = 169 + 196.)$$

Can you find other equations that continue this pattern?

There is another tantalizing equation that seems to generalize $3^2 + 4^2 = 5^2$ that involves cubes. The cube of a number is the product of the number taken three times, and is indicated by a superscript 3 to the right of the number. You might want to check that

$$3^3 + 4^3 + 5^3 = 6^3.$$

Both sides of this equation equal 216. There seems to be no natural continuation of this pattern to fourth and higher powers. However, there are lots of cubes that are equal to the sum of three other cubes. For example,

$$1^3 + 6^3 + 8^3 = 9^3$$

and

$$(-1)^3 + 7^3 + 10^3 = 12^3,$$

See how many more you can find.