

EXTRA-SENSORY PERCEPTION

Geoff and Jewel say that they can communicate by ESP, mental waves that emanate from one to the other. To demonstrate this ability, Geoff asks you to draw from a regular deck five playing cards at random and hand them to him. He gives four of them to Jewel, who then tells him what is the fifth card retained by Geoff.

You, of course, are suspicious and suspect that Geoff is somehow coding the answer in the cards he passes. You realize that he can determine the order in which Jewel receives the cards. Since there are 52 cards in the deck and Jewel has four of them, there are 48 possibilities for the fifth card. However, there are only 24 ways in which Geoff can order the four cards that he passes.

You then note that Geoff has another degree of flexibility. He can determine which four cards he passes to Jewel. So here is how the trick is done.

Given any five cards, there must be at least one pair of them with the same suit. Geoff fixes on such a pair. Adopting the convention that A = 1, J = 11, Q = 12, K = 13, he imagines the face of a 13-hour clock. He looks at the ranks of the two cards; it is always possible to count from one to another around the face of the clock taking no more than six steps in the clockwise direction.

For example, if the two cards are 3 and 9, then we can count six steps from 3 to 9; if the two cards are 3 and 10, then we can count six steps from 10 to 3, going through J, Q, K, A, 2.

Geoff will retain the end card of the two in the count and pass the other card first. This will establish the suit of the card retained. Now he needs to pass the remaining three cards in an order that codes how many steps need to be counted up from the card passed. Think of the cards of the deck linearly ordered like the letters of the alphabet. The ordering is by rank, and within each rank, by suit in alphabetical order: $A\clubsuit, A\diamond, A\heartsuit, A\spadesuit, 2\clubsuit, 2\diamond, 2\heartsuit, 2\spadesuit, \dots, K\heartsuit, K\spadesuit$. The three remaining cards can be thought as “letters” that can be made to form three-letter words in six possible ways. Let these words in dictionary order code the numbers 1 up to 6.

Suppose that Geoff passes in order the cards $4\spadesuit, Q\clubsuit, 9\clubsuit, 9\diamond$. What card has he retained? Jewel notes from the first card passed that Geoff holds a spade. The remaining three cards in “alphabetical” order are: $9\clubsuit, 9\diamond, Q\clubsuit$. The words made from these “letters” in dictionary order are: $9\clubsuit 9\diamond Q\clubsuit, 9\clubsuit, Q\clubsuit, 9\diamond, 9\diamond, 9\clubsuit, Q\clubsuit, 9\diamond, Q\clubsuit, 9\clubsuit, Q\clubsuit, 9\clubsuit, 9\diamond, Q\clubsuit, 9\diamond, 9\clubsuit$. The order in which Jewel received the cards was the fifth one of these, so she counts five places up from the four and deduces that Geoff has retained $9\spadesuit$.