

## A pair of geometry problems

### §1. The problem of the square and triangle.

Suppose that we have a square  $BCDE$ , all of whose sides have length 1. Atop this square and sharing a side with it is an equilateral triangle  $ABE$  whose sides also all have length 1. The three points  $A$ ,  $C$  and  $D$  do not lie on a straight line, so there is exactly one circle that passes through the three of them. What is the radius of this circle?

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This is an elementary geometry problem that can cause more or less difficulty, depending on how you look at it. When I have given this to secondary students, they often come up with quite involved ways to solve it. However, with a problem like this, it is often a disadvantage to know too much. You might try to fit it into some scheme rather than simply address it on its own terms. In fact, a reader who has either not learned or forgotten his high school geometry may have an advantage.

The key is to visualize the situation in a productive way. Imagine that triangle  $ABE$  is like a piston or plunger on top of the “cylinder”  $BCDE$  and let the piston drop to the bottom, so that  $BE$  comes to rest on  $CD$  and  $A$  descends to some point  $F$ . The dimensions of the triangle  $ABE$  and the same as those of the triangle  $FCD$ , so that the lengths of  $FC$  and  $FD$  are both 1. Now how far did our piston descend to take  $A$  to  $F$ ? Why, the length 1 of the sides  $BC$  and  $ED$ .

So  $F$  is a point that is distant 1 from each of  $A$ ,  $C$  and  $D$ , so that it is the centre of a circle of radius 1 that passes through these three points.

### §2. A radius problem.

The foregoing problem reminded Kerry Skipper of a similar find-the-radius problem that he found in the book *Games for the Super-intelligent* by James Foxx (the famous jogger who died of a heart attack in 1984). A rectangle  $ABCD$  with two sides  $AB$  and  $AC$  along diameters of a circle with centre  $A$  is drawn. The distance from  $C$  to the circumference of the circle is 2 inches. The length of the diagonal  $BC$  is 8 inches. What is the radius of the circle? Foxx noted that “the trained mind is lured into concentrating on an irrelevant aspect of the problem”.

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The radius of the circle is 8 inches. The two diagonals  $AD$  and  $BC$  of the rectangle have the same length, and the former is a radius of the circle. The 2 inch dimension is irrelevant.