Frog Jump

Each year, there are tours in which the local artisans can display their wares. For the woodworkers among them, one possibility is to create games and puzzles that have a mathematical flair. One such game is the solitaire *Frog Jump*. It is played on a block of wood into which eleven holes are sunk along a straight line; there are also ten pegs that will fit into the holes. To set it up, put five white pegs into the five leftmost holes and five black pegs into the five rightmost holes, leaving the middle hole blank:

$$W \;\; W \;\; W \;\; W \;\; W \;\; O \;\; B \;\; B \;\; B \;\; B \;\; B.$$

The object of the game is to perform a series of moves that will interchange the positions of the black and white pegs. You are permitted to move any peg into an adjacent unoccupied hole, and to allow any peg to jump over one of its neighbours into an unoccupied hole.

Thus, your first move can be only to move either the white or the black peg next to the centre into the central hole. Suppose that you have moved the white peg. Then one possibility for the second move is to allow the black peg next to it to jump over it into the vacant hole. No peg is ever removed from the board.

What makes this puzzle of mathematical interest? At its root, mathematics is the systematic analysis of structure. There are mathematical questions to be asked, foremost among them whether there exists a solution to this solitaire puzzle. If so, what is a solution that requires the fewest moves? How do we know this? Can we detect any patterns that seem to be developing as we solve this problem? In particular, what happens if we pose the puzzle with some different equal number of pegs on both sides of the middle hole? Can we describe an algorithm that will allow us to solve the problem with a million white and a million black pegs, or at least program a computer to do the job for us? Finally, can we still solve the problem if we impose more restrictions, for example, that the white pegs always have to move to the right and the black always to the left.

Let me turn this problem over to the reader. You might want to try it on a sheet of paper with five nickels and five dimes; you will need only seventy-five cents to get started. If you find it a little bit formidable, solve the problem with fewer pegs on each side. Start with one, then two, then three, and see if there is some routine that you can glom onto.