The journey of Ardeth and Charlotte

It is still the case that the most important thing that elementary pupils can gain from their mathematical instruction is how to deal with ratio and proportion. This includes such topics as rates, prices, interest, conversion of units and fractions. There are a great many jobs for which this is the main mathematical requirement. Yet it is an area that many are not comfortable with. Here is a problem that yields to the right mindset.

Ardeth lives in Arden and Charlotte lives in Sharbot Lake. One morning at exactly the same hour, each set out walking to the town of the other, following exactly the same route in opposite directions. Each woman maintained the same constant speed throughout, although one was slower than the other. They passed each other at noon. Ardeth arrived at Sharbot Lake at 2 pm, while Charlotte, being less fleet of foot, did not get to Arden until 4:30 in the afternoon. What time in the morning did they set out?

The key to solving the problem is to recognize that, when you move at a constant rate, the distance covered is proportional to the time taken. If you move twice as long, you cover twice the distance for example. There are two legs of the journey. The eastern leg between Sharbot Lake and the passing point and the western leg between the passing point and Arden. The time taken for each woman on these legs is proportional to the length of the legs, and therefore the times taken by the women on these legs are proportional to each other.

Suppose that T is the length of time in hours between setting out and passing at noon. Charlotte takes T hours to cover the eastern leg and $4 + 1/2 = 9/2$ hours to cover the western leg. Ardeth takes 2 hours to cover the eastern leg and T hours to cover the western leg. Therefore

$$T \div 9/2 = 2 \div T.$$

This means that $T^2 = 2 \times 9/2 = 9$ so that $T = 3$. Thus each woman set out at 9 o'clock in the morning.

Teachers may wish to try this problem on their senior elementary or secondary students. Some of the younger children who have not yet had algebra may have more success with it, because they have little choice but to look at it from first principles. The older students may find algebra a distraction, miss the underlying structure and make the problem much more complicated than it is.