Red and green hats

Suppose five people, Anne, Barbara, Cathy, Dorothy and Elsie are seated in a circle. Upon the head of each is placed a hat, either red or green. Each can see the hats worn by the other four, but nhot her own. However, they are given the true fact that not all the hats have the same colour. They are not allowed to communicate in any way. Every ten seconds, a bell sounds. At each sounding of the bell, everyone who has reasoned the colour of her hat is leaves the room.

Assume that all five ladies are rational and that each leaves the room at the earliest moment she determines the colour of her hat. Ecplain why everyone eventually leaves the room.

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To get a handle on the situation, you might consider the simpler problems where there are two, three or four people and not all are wearing the same coloured hat. For the five hat problem, there are essentially two situations.

First, suppose there are four red and one green hat and that Anne is wearing the green hat. Anne sees four red hats, and realizing that there is at least one green hat, immediately deduces the colour of her hat and leaves the room at the first bell. The rest now realize that Anne would not have left if she saw a green hat and deduce that they are wearing red hats and leave the room at the seond bell.

Now, suppose there are three red and two green hats, and that Anne and Barbara are wearing the green hats. In this case, each woman sees a red and a green hat and so cannot leave the room on the first bell. Each of Anne and Barbara realizes that if her own hat were red, the other would have left the room on the first bell; since this did not happen, both realize that they are wearing green hats and leave the room on the second bell. Consider the remaining three women. Each of them saw two red and two green hats, and none of them would have determined the colour of their hat by the seoond bell. After the second bell, each can reject the notion that she is wearing a green hat. Otherwise, there would have been two red hats and their wearers would have left the room on the second bell (using the analogous reasoning for the two green hat case). So they deduce that their hats are red and leave the room on the third bell.

There is a children's book that deals with this type of problem, *Anno's Hat Tricks*, by Mitsumasa Anno and Akihiro Nazaki. Anno. Anno has written a number of books directed at children that I recommend to any parents wanting to give their offspring a gentle introduction to mathematics. In fact, the children's literature in mathematics is growing all the time, and I have listed a few possibilities on my website: *www.math.utoronto.ca/barbeau/home.html*. Click on the button, *Books for elementary pupils*.