Mathematics books for young children

This essay consists of an extended review of a book on coding and brief notices of other books.

1. Here is a fine book that introduces middle school pupils to coding, a topic that surely would have immediate appeal for many of them.

Janet Beissinger & Vera Pless, *The Cryptoclub: using mathematics to make and break secret codes*

A.K. Peters, Wellesley, MA, 2006 xvi+199 pages

ISBN-13: 978-1-56881-223-6; -10: i-56881-223-X

List price: US$35.00

The authors, both at the University of Illinois in Chicago, the first at the Institute for Mathematics and Science Education and the second a professor of cryptography in the Department of Mathematics, Statistics and Computer Science, state in their preface that there is much in the mathematical theory of coding within the mathematical reach of a young adolescent. Indeed, "the Vigenère Cipher, . . ., once believed to be unbreakable, can actually be cracked by today's middle-grade students (as long as the key isn't too long) by common factors of certain numbers". A sequence of seven units takes the students from the ciphers of Julius Caesar to the RSA public key encryption made popular by Ronald Rivest, Adi Shamir and Leonard Adleman in 1977.

Despite its challenging mathematical content, there is nothing dry about this book. All the techniques employed for modern children's books are used here to good effect: a story line to connect the topics, with coloured illustrations, boxes to set off examples and exercises, as well as a number of historical anecdotes set on special pages. While the book is designed to be used by a class or club led by a teacher, a pupil or small group could work through it.

The first two units treat various sorts of substitution ciphers, including ones that use a keyword, along with techniques for deciphering them. In particular, one can analyze letter frequencies, which addresses topics in data management. A complicated form of substitution cipher is due to Vigenère, in which the shift changes with each letter according to a matching letter in a keyword which codes the letter A. The study of this occupies almost 50 pages, as the pupils must master factorization techniques required to discover the length of the key word. The remaining four units of the book provide the fundamentals of modular

arithmetic leading up to multiplicative and affine ciphers, and finally the determination of primes and raising of powers modulo a prime that will be used in the RSA public key cryptosystem.

It is worth briefly summarizing the prime number theory that the book covers. The children learn how to list primes using the sieve of Eratosthenes and to check for primality by dividing numbers by primes not exceeding their square roots. In the process, they learn about twin primes, Sophie Germaine primes (p for which $2p + 1$ is also prime), Mersenne primes and the Great Internet Mersenne Prime Search. (The largest prime determined up to February, 2005 is $2^{25,964,951} - 1$.) They find out under what conditions an integer has an inverse modulo n and invited to experiment, and finally are told how to find the RSA decryption key.

An important message conveyed in the book is that mathematics is alive and growing. While it is premature to develop much of the theory involved, the pupils have through the exercises a good opportunity to get a feel for the underlying mathematical structures.

2. The book just reviewed is the latest in a growing list of volumes suitable for presecondary students. Mitsumasa Anno has a number of charming books for the young, *Anno's Counting Book*, *Anno's Magic Seeds*, *Anno's Mysterious Multiplying Jar* and *Anno's Math Games*. My favourite Anno book *Anno's Hat Tricks*, coauthored with Akihiro Nozaki, introduces logical reasoning through puzzles in which children, seeing coloured hats on the heads of other children, have to deduce the colour of their own hats.

The Man Who Counted, by Malba Tahan, is the story of Beremiz Samir, told in *Arabian Nights* style, in which a shepherd boy develops a talent with numbers and passes through a succession of adventures involving solutions of mathematical problems to achieve both fame and fortune. The chapters can be read independently.

The Number Devil: a Mathematical Adventure by Hans Magnus Enzensberger is a deservedly popular tale of a twelve-year-old boy who hates mathematics and, in his dreams, meets the "number devil" who takes him on many mathematical adventures.

The *Math Curse*, by Jon Scieszka, affects a young girl who is constrained to see a mathematics problem in everything around her.

Theoni Pappas is a prolific mathematical expositor for children whose publications include *Fractals, Googols and other Mathematical Tales* and *Math for Kids & Other People Too!*.

In June, 2013, I learned of another book that would be suitable for chidren between the ages of 6 and 9, a brief life of Paul Erdös. The illustrations are clever and it looks to be a good read. This reminds me of the books on composers that I borrowed from the library when I was a kid.

The boy who loved math: the improbable life of Paul Erdös, by Deborah Helligman.

Barn Cat, by Carol P. Saul, is a counting book for very young children in which a cat surveys parades of increasing numbers of various creatures.