1

DISSERTATION ON THE ART OF COMBINATIONS

1666

(Selections)

The Dissertatio de arte combinatoria, *which Leibniz published in 1666, was an expansion of the dissertation and theses submitted for disputation the same year to qualify for a position in the philosophical faculty at Leipzig. The work contains the germ of the plan for a universal characteristic and logical calculus, which was to occupy his thinking for the rest of his life. That project is here conceived as a problem in the arithmetical combination of simple into complex concepts, Leibniz deriving basic theorems on permutation and combination and applying them to the classification of cases in logic, law, theology, and other fields of thought. His later judgment on the work was that in spite of its immaturity and its defects, especially in mathematics, its basic purpose was sound.*

Three introductory sections which supply the metaphysical and logical foundations of work are given here. They are (I) a demonstration of the existence of God with which he prefaced the work; (II) the 'corollaries' prepared for the disputation; and (III) the definitions introducing the work itself. The solution of the first two problems and several applications are also included.

I. DEMONSTRATION OF THE EXISTENCE OF GOD

[G, IV, 32–33]

Hypotheses [*Praecognita*][1]*:*

1. Definition 1. *God* is an incorporeal substance of infinite power [*virtus*].

2. Definition 2. I call *substance* whatever moves or is moved.

3. Definition 3. *Infinite power* is an original capacity [*potentia*] to move the infinite. For power is the same as original capacity; hence we say that secondary causes operate by virtue [*virtus*] of the primary.

4. Postulate. Any number of things whatever may be taken simultaneously and yet be treated as one whole. If anyone makes bold to deny this, I will prove it. The concept of *parts* is this: given a plurality of beings all of which are understood to have something in common; then, since it is inconvenient or impossible to enumerate all of them every time, one name is thought of which takes the place of all the parts in our reasoning, to make the expression shorter. This is called the *whole*. But in any number of given things whatever, even infinite, we can understand what is true of all, since we can enumerate them all individually, at least in an infinite time. It is therefore permissible to use one name in our reasoning in place of all, and this will itself be a *whole*.[2]

5. Axiom 1. If anything is moved, there is a mover.

6. Axiom 2. Every moving body is being moved.

7. Axiom 3. If all its parts are moved, the whole is moved.

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8. Axiom 4. Every body whatsoever has an infinite number of parts; or, as is commonly said, the continuum is infinitely divisible.

9. Observation. There is a moving body.

Proof [Ἔκθεσις]:

1. Body *A* is in motion, by hypothesis No. 9.
2. Therefore there is something which moves it, by No. 5,
3. and this is either incorporeal
4. because it is of infinite power, by No. 3;
5. since *A*, which it moves, has infinite parts, by No. 8;
6. and is a substance, by No. 2.
7. It is therefore God, by No. 1 Q.E.D.
8. Or it is a body,
9. which we may call *B*.
10. This is also moved, by No. 6,
11. and what we have demonstrated about body *A* again applies, so that
12. either we must sometime arrive at an incorporeal power, as we showed in the case of *A*, in steps 1–7 of the proof, and therefore at God;
13. or in the infinite whole there exist bodies which move each other continuously.
14. All these taken together as one whole can be called *C*, by No. 4.
15. And since all the parts of *C* are moved, by step 13,
16. *C* itself is moved, by No. 7,
17. and by some other being, by No. 5,
18. namely, by an incorporeal being, since we have already included all bodies, back to infinity, in *C*, by step 14. But we need something other than *C*, by 17 and 19,
19. which must have infinite power, by step No. 3, since *C*, which is moved by it, is infinite, by steps 13 and 14;
20. and which is a substance, by No. 2,
21. and therefore God, by No. 1.

Therefore, God exists. Q. E.D.[3]

II. COROLLARIES FOR DISPUTATION[4]

[G., IV, 41–43]

An Arithmetical Disputation on Complexions, which Mr. Gottfried Wilhelm Leibniz of Leipzig will hold in the famous university of Leipzig, by permission of its distinguished philosophical faculty, on March 7, 1666.

I. *Logic*

1. There are two primary propositions. The first is the principle of all theorems or necessary propositions: *what is (so) either is or is not (so)*, or conversely. The other is the basis of all observations or contingent propositions: *something exists.*

2. Perfect demonstrations are possible in all disciplines.

3. If we regard the disciplines in themselves, they are all *theoretical;* if their application, they are all *practical.* Those, however, from which the application follows more immediately are rightly called practical par excellence.

4. Although every method can be employed in every discipline, as we follow the traces either of our own investigation or of the producing nature in our treatment, it yet

happens in the practical disciplines that the *order of nature* and *that of knowledge* coincides, because here the *nature of the thing* itself originates in our *thought* and production. For the end in view both moves us to produce the means and leads us to know them, which is not true in the matters which we can merely know but cannot also produce. Moreover, although every method is allowed, not every one is expedient.

5. The end of logic is not the *syllogism* but *simple contemplation*. The *proposition* is, in fact, the means to this end, and the syllogism is the means to the proposition.

II. *Metaphysics*

1. One infinite is greater than another. (Cardan, *Pract. Arith.*, chap. 66, nn. 165 and 260. Seth Ward is said to dissent in his *Arithmetic of Infinites.*[5])

2. God is substance; creature is accident.

3. A discipline concerning created beings in general is needed, but this is nowadays usually included in metaphysics.

4. It is very improbable that the term *cause* expresses an unequivocal concept to cover efficient, material, formal, and final causes. For what is the word *influx*, more than a mere word?[6]

III. *Physics*

1. Since we may observe that other cosmic bodies move about their own axes, it is not absurd that the same should be true of the earth; but neither is the contrary.

2. Since the most general difference between bodies is that of *density* and *rarity*[7], the four *primary qualities* may obviously be explained as follows: the *humid* is the rare, the *dry* is the dense, the *warm* is the rarefying, and the *cold* is the condensing. Everything rare is easily confined within external boundaries, but with difficulty within its own boundaries; everything dense, the contrary. In the rare, everything that rarefies facilitates the quickening of the homogeneous with respect to itself and the separation of the heterogenous; in the dense the way to this is blocked. A reason is thus supplied for the Aristotelian definitions. Nor does fire, which seems to be rare but must actually be dry, provide an exception to this, for I reply that one thing is to be said about fire per se and another of fire which inheres in other bodies, for in this case it follows the nature of these bodies. Thus it is clear that a flame, which is nothing but burning air, must be *fluid* just as is air itself. On the other hand, the fire which consists of burning iron is like iron itself.

3. It is a fiction that the force of the magnet is checked by steel.

IV. *Practical*

1. *Justice* (particular) is a virtue serving the mean in the affections of one man toward another, the affections of enjoying and of harming, or those of good will and hate. The rule of the mean is *to gratify another (or myself) as long as this does not harm a third person (or another)*. This must be noted in order to defend Aristotle against the cavil of Grotius, who speaks as follows in the Prolegomena of his *de Jure belli et pacis* (Sec. 4):

That this principle (that virtue consists in the mean) cannot correctly be assumed as universal is clear even in the case of justice. For since he (Aristotle) was unable to find the opposites of excess and defect in the affections and the actions which follow from them, he sought them both in the things themselves with which justice is concerned. But this is obviously to leap from one genus of things to another, a fault which he rightly criticizes in others.[8]

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Grotius, namely, maintains that it is inconsistent to introduce into the species of a classification something which is derived by another principle of classification; he calls this, not too philosophically, "leaping over into another genus". Certainly the mean in affections is one thing, the mean in things another, and virtues are habits, not of things but of minds. Therefore I show that justice is also found in a moderation of the affections.

2. Thrasymachus well says, in Plato's *Republic*, Book I, that *justice* is what is useful to the more powerful. For in a proper and simple sense, God is more powerful than others. In an absolute sense one man is not more powerful than another, since it is possible for a strong man to be killed by a weak one. Besides, usefulness to God is not a matter of profit but of honor. Therefore *the glory of God is obviously the measure of all law*. Anyone who consults the theologians, moralists, and writers on cases of conscience will find that most of them base their arguments on this. Once this principle is established as certain, therefore, the doctrine of justice can be worked out scientifically. Until now this has not been done.[9]

III. CUM DEO!

[G., IV, 35–75]

1. Metaphysics, to begin at the top, deals with being and with the affections of being as well. Just as the affections of a natural body are not themselves bodies, however, so the affections of a being are not themselves beings.

2. An affection (or mode) of a being, moreover, is either something absolute, which is called *quality*, or something relative, and this latter is either the affection of a thing relative to its parts if it has any, that is, *quantity*, or that of one thing relative to another, *relation*. But if we speak more accurately and assume a part to be different from the whole, the quantity of a thing is also a relation to its part.

3. Therefore, it is obvious that neither quality nor quantity nor relation is a being; it is their treatment in a signate actuality that belongs to metaphysics.

4. Furthermore, every relation is either one of *union* or one of *harmony* [*convenientia*]. In union the things between which there is this relation are called *parts*, and taken together with their union, a *whole*. This happens whenever we take many things simultaneously as *one*. By *one* we mean whatever we think of in one intellectual act, or at once. For example, we often grasp a number, however large, all at once in a kind of blind thought, namely, when we read figures on paper which not even the age of Methuselah would suffice to count explicitly.

5. The concept of *unity* is abstracted from the concept of one being, and the whole itself, abstracted from unities, or the totality, is called *number*.[10] *Quantity* is therefore the number of parts. Hence quantity and number obviously coincide in the thing itself, but quantity is sometimes interpreted extrinsically, as it were, in a relation or ratio to another quantity, to aid us, namely, when the number of parts is unknown.

6. This is the origin of the ingenious specious analysis[11] which *Descartes* was the first to work out, and which *Francis Schotten* and *Erasmus Bartholin* later organized into principles, the latter in what he calls the *Elements of Universal Mathematics*. *Analysis* is thus the science of ratios and proportions, or of unknown quantity, while *arithmetic* is the science of known quantity, or numbers. But the Scholastics falsely believed that number arises only from the division of the continuum and cannot be

applied to incorporeal beings. For number is a kind of incorporeal figure, as it were, which arises from the union of any beings whatever; for example, God, an angel, a man, and motion taken together are four.

7. Since number is therefore something of greatest universality, it rightly belongs to metaphysics, if you take metaphysics to be the science of those properties which are common to all classes of beings. For to speak accurately, mathematics (adopting this term now) is not one discipline but small parts taken out of different disciplines and dealing with the quantity of the objects belonging to each of them. These parts have rightly grown together because of their cognate nature. For as arithmetic and analysis deal with the quantity of beings, so geometry deals with the quantity of bodies, or of the space which is coextensive with bodies. Far be it from us, certainly, to destroy the social distribution of disciplines among the professions, which has followed convenience in teaching rather than the order of nature.

8. Furthermore, the whole itself (and thus number or totality) can be broken up into parts, smaller wholes as it were. This is the basis of *complexions*, provided you understand that there are common parts in the different smaller wholes themselves. For example, let the whole be *ABC*; then *AB*, *BC*, and *AC* will be smaller wholes, its parts. And the disposition of the smallest parts, or of the parts assumed to be smallest (that is, the unities) in relation to each other and to the whole can itself also be varied. Such a disposition is called *situs*.[12]

9. So there arise two kinds of *variation: complexion* and *situs*. And viewed in themselves, both complexion and situs belong to metaphysics, or to the science of whole and parts. If we look at their variability, however, that is, at the quantity of variation, we must turn to numbers and to arithmetic. I am inclined to think that the science of complexions pertains more to pure arithmetic, and that of situs to an arithmetic of figure. For so we understand unities to produce a line. I want to note here in passing, however, that unities can be arranged either in a straight line or in a circle or some other closed line or lines which outline a figure. In the former case they are in absolute situs or that of parts to the whole, or *order*; in the latter they are in relative situs or that of parts to parts, or *vicinity*. In definitions 4 and 5, below, we shall tell how these differ. Here these preliminary remarks will suffice to bring to light the discipline upon which our subject matter is based.[13]

DEFINITIONS

1. *Variation* here means change of relation. For change may be one of substance, or of quantity, or of quality; still another kind changes nothing in the thing but only its relation, its situs, its conjunction with some other thing.

2. *Variability* is the quantity of all variations. For the limits of powers taken in abstraction denote their quantity; so it is frequently said in mechanics that the power of one machine is double that of another.

3. *Situs* is the location of parts.

4. Situs is either absolute or relative; the former is that of the parts with respect to the whole, the latter that of parts to parts. In the former the number of places is considered, and the distance from the beginning and the end; in the latter neither the beginning nor the end is considered, but only the distance of one part from another part is viewed. Hence the former is expressed by a line or by lines which do not inclose

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a figure or close upon themselves, and best by a straight line; the latter is expressed by a line or lines inclosing a figure, and best by a circle. In the former much consideration is given to priority and posteriority; in the latter, none. We will therefore do well to call the former *order*,

5. And the latter *vicinity*. The former is disposition; the latter, composition. Thus by reason of order the following situses are different: *abcd, bcda, cdab, dabc.* But in vicinity there can be no variation but only situs, namely, this:

$$\begin{array}{ccc} & b & \\ a & & c \\ & d & \end{array}$$

Thus when the very witty Taubman was dean of the philosophical faculty at Wittenberg, he is said to have placed the names of Master's candidates on the public program in a circular arrangement, so that eager readers should not learn who held the position of 'swine'.[14]

6. We will usually mean the variability of order when we take variations par excellence; for example, 4 things can be arranged in 24 ways.[15]

7. The variability of a complex we call *complexions*; for example, 4 things can be put together in 15 different ways.[16]

8. The number of varying things we shall call simply *number*; for example, 4 in the case proposed.

9. A *complexion* is the union of a smaller whole within the greater, as we have said in the introduction.

10. In order to determine a certain complexion, however, the greater whole is to be divided into equal parts assumed as minima (that is, parts now not to be considered as further divisible). Of these parts it is composed, and by the variation of them the complexion or lesser whole may be varied. Because the lesser whole itself is greater or less according as more parts are included at any time, we call the number of parts or unities to be connected together at one time the *exponent*, after the example of a geometric progression. For example, let the whole be *ABCD*. If the lesser whole is to consist of two parts, for example, *AB, AC, AD, BC, BD, CD*, the exponent will be 2; if of three parts, for example, *ABC, ABD, ACD, BCD*, the exponent will be 3.

11. We shall write the complexions with a given exponent as follows: if the exponent is 2, *com2nation* (combination); if 3, *con3nation* (conternation); if 4, *con4nation*; etc.

12. *Complexions taken simply* are all the complexions computed for all exponents; for example, 15 of the number 4. These consist of 4 units, 6 com2nations, 4 con3nations, 1 con4nation.

13. A useful (useless) variation is one which can (cannot) occur because of the nature of the subject matter; for example, the four [physical] elements can be com2ned six times, but two com2nations are useless, namely, those in which the contraries fire and water and the contraries air and earth are com2ned....

PROBLEMS

Three things should be considered: problems, theorems, and applications. We have added the application to individual problems wherever it seemed worth while, and the theorems also. To some of the problems, however, we have added a demonstration. Of these, we owe the latter part of the first problem, and the second and fourth, to others; the rest we ourselves have discovered. We do not know who was the first to discover them. Schwenter (*Delic.*, Book i, Sec. 1, prop. 32) says they exist in Jerome Cardan, John Buteonis, and Nicolas Tartalea. But we have not found them in Cardan's *Arithmetica practica*, published in Milan in 1539. Christopher Clavius set forth especially clearly what has been found recently, in his *Commentarium in Sphaeram Joannis de Sacro Bosco*, published in Rome in 1585, pages 33 ff.[17]

Problem I

To Discover the Complexions for a Given Number and Exponent

1. There are two ways of solving this problem, one for all complexions, the other for com2nations only. The former is more general, but the latter requires fewer data, namely, only the number and the exponent, while the former also presupposes the discovery of antecedent complexions.[18]

2. We have developed the more general method; the special one is popularly known. The more general solution is this: *Add the complexions of the number preceding the given number, by the given exponent and by the exponent preceding it; the sum will be the desired complexions.* For example, let the given number be 4 and the exponent 3; add the 3 com2nations and the 1 con3nation of the preceding number 3; $(3+1=4)$. The sum 4 will be the answer.

3. But since the complexions of the preceding number are required for this solution, Table ℵ must be constructed. In it the top line contains the *numbers* from 0 to 12 inclusive from left to right (we believe this is far enough, since it is easily extended); the vertical line at the left contains the *exponents* from 0 to 12, reading from top to bottom; and the bottom line, from left to right, contains the *total complexions* [*complexiones simpliciter*]. The lines between contain the complexions for the number given at the head of the corresponding column and for the exponent given at the left.[19]

4. The *reason for this solution*, and the basis of the table, will be clear if we demon-

TABLE ℵ

	0	1	1	1	1	1	1	1	1	1	1	1	1	1
	1	0	1	2	3	4	5	6	7n	8u	9m	10b	11e	12r
	2	0	0	1	3	6	10	15	21	28	36	45	55	66
	3	0	0	0	1	4	10	20	35	56	84	120	165	220
	4	0	0	0	0	1	5	15	35	70	126	210	330	495
Exponents	5	0	0	0	0	0	1	6	21	56	126	252	462	792
	6	0	0	0	0	0	0	1	7	28	84	210	462	924
	7	0	0	0	0	0	0	0	1	8	36	120	330	792
	8	0	0	0	0	0	0	0	0	1	9	45	165	495
	9	0	0	0	0	0	0	0	0	0	1	10	55	220
	10	0	0	0	0	0	0	0	0	0	0	1	11	66
	11	0	0	0	0	0	0	0	0	0	0	0	1	12
	12	0	0	0	0	0	0	0	0	0	0	0	0	1
	*	0	1.	3.	7.	15.	31.	63.	127.	255.	511.	1023.	2047.	4095.
	†	1.	2.	4.	8.	16.	32.	64.	128.	256.	512.	1024.	2048.	4096.

Complexions

* The complexions taken simply (or the sum of the complexions of all given exponents), added to 1, equal the total of a geometric progression with base 2†.[20] ...

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strate that *the complexions for a given number and exponent arise from the sum of the complexions of the preceding number, for both the given and the preceding exponents.* Taking the given number as 5 and the given exponent as 3, the antecedent number will be 4; it will have 4 con3nations and 6 com2nations, by Table ℵ. Now the number 5 has all the con3nations of the preceding number (since the part is contained in the whole), namely, 4, and it has besides as many con3nations as the preceding number has com2nations, since the unit by which the number 5 exceeds 4, added to each of the individual com2nations of 4, will make the same number of con3nations. Thus $6+4=10$. Therefore *the complexions for a given number*, etc. Q.E.D.

Problem II

To Discover the Complexions Taken Simply for a Given Number

Seek the given number among the exponents of a geometric progression with base 2; then the total of complexions sought will be the number or term of the progression whose exponent is the given number, minus 1. It is difficult to understand the reason or demonstration for this, or to explain if it is understood. The fact, however, is apparent from Table ℵ. For when added together, and the sum added to unity, the particular complexions of a given number always constitute, when one is added, the term of that geometric progression with base 2, whose exponent is the given number. But if anyone is interested in seeking the reason for this, it will have to be found in the process of resolving used in the *Practica italica, vom Zerfallen.* This must be such that a given term of the geometrical progression is separated into more parts by one than there are units (i.e., numbers) in its exponent. The first of these must always be equal to the last, the second to the next to the last, the third to the third from the last, etc., until, if it is broken up into an equal number of parts, the exponent or number of things being odd, the two parts in the middle will be equal (for example, 128 or 2^7 may be broken up into eight parts according to Table ℵ: 1, 7, 21, 35, 35, 21, 7, 1); or, if the exponent is even and it must be broken into an odd number, the number left in the middle will have none corresponding to it (for example, 256 or 2^8 may be broken up into nine parts according to Table ℵ: 1, 8, 28, 56, 70, 56, 28, 8, 1). Someone may therefore think that this brings to light a new method which is absolute for solving problem 1; namely, by breaking up the complexions taken simply, or the terms of a geometric progression with base 2, by a method discovered with the aid of algebra. In fact, however, there are not sufficient data, and the same number can be broken up in several ways yet according to the same form.

Application of Problems I and II

Since everything which exists or which can be thought must be compounded of parts, either real or at least conceptual, whatever differs in kind must necessarily either differ in that it has other parts, hence the use of complexions; or by another situs, hence the use of dispositions. The former are judged by the diversity of matter; the latter, by the diversity of form. With the aid of complexions, indeed, we may discover not only the species of things but also their attributes. Thus almost the whole of the *inventive* part of logic is grounded in complexions – both that which concerns simple terms and that which concerns complex terms; in a word, both the doctrine of *divisions* and the doctrine of *propositions;* not to mention how much we hope to illumine the *analytic* part of logic, or the logic of judgment, by a diligent examination of the modes of the syllogism in Example VI.

The use of complexions in divisions is threefold: (1) given the principle of one division, to discover its species; (2) given many divisions of the same genus, to discover the species mixed from different divisions (this we will treat in Problem III, however); (3) given the species, to discover the subaltern genera. Examples are scattered throughout all of philosophy, and we will show that they are not lacking in jurisprudence. And in medicine every variety of compounded medicaments and pharmaceuticals is made by mixing various ingredients, though the greatest care is necessary in choosing useful mixtures. First, therefore, we will give examples of species to be discovered by this principle.[21]

I. Among jurisconsults the following division is proposed (*Digests*, Gaius, XVII, 1, 2). A mandate is contracted in five ways: in favor of the mandator, of the mandator and mandatory, of a third person, of the mandator and a third person, of the mandatory and a third person. We shall seek out the adequacy of the division in this way: its basis is the question, *for whom*, or the person in whose favor the contract is made; there are three of these, the mandator, the mandatory, and a third person. But there are seven complexions of three things:

Three 1nions: since contract may be in favor of only (1) the mandator; (2) the mandatory; or (3) a third person.

The same number of com2nations: (4) in favor of the mandator and mandatory; (5) of the mandator and a third person; or (6) of the mandatory and a third person.

One con3nation: (7) in favor of the mandator, the mandatory, and a third person all together.

Here the jurisconsults reject as useless that 1nion in which the contract is in favor of the mandatory alone, because this would be advice rather than a mandate. There remain thus six classes. Why they kept only five, omitting the con3nation, I do not know.

II. Aristotle (*On Generation and Corruption*, Book ii), with Ocellus Lucanus the Pythagorean, deduces the number of elements, or of the mutable species of a simple body, from the number of primary qualities, of which he assumes there are four, but according to these laws: (1) that every element is to be a compound of two qualities and neither more nor less, and it is thus obvious that 1nions, con3nations, and the con4nation are to be discarded and only com2nations retained, of which there are six; and (2) that contrary qualities can never enter into one com2nation and that therefore two of the com2nations are useless because there are two contraries among these primary qualities. Hence there remain four com2nations, the same as the number of elements. ... Moreover, as Aristotle derived the elements from these qualities, so Galen derived from them the four temperaments, the various mixtures of which later medics have studied; all of whom Claudius Campensis opposed in the past century, in his *Animadversiones naturales in Aristotelem et Galenum* (Leyden 1576)....

IV. In wind organs we call a register, in German *ein Zug*, a little shaft by whose opening the sound may be varied, not with respect to the perceived melody or pitch itself, but in its basis in the pipe, so that sometimes a shaking, sometimes a whisper, is achieved. More than thirty of such qualities have been discovered by the industry of our contemporaries. Assume that there are in some organs only twelve such simple effects; then there will be in all about 4095, as many as there are complexions taken simply of twelve things. So a great organist can vary his playing as he opens them

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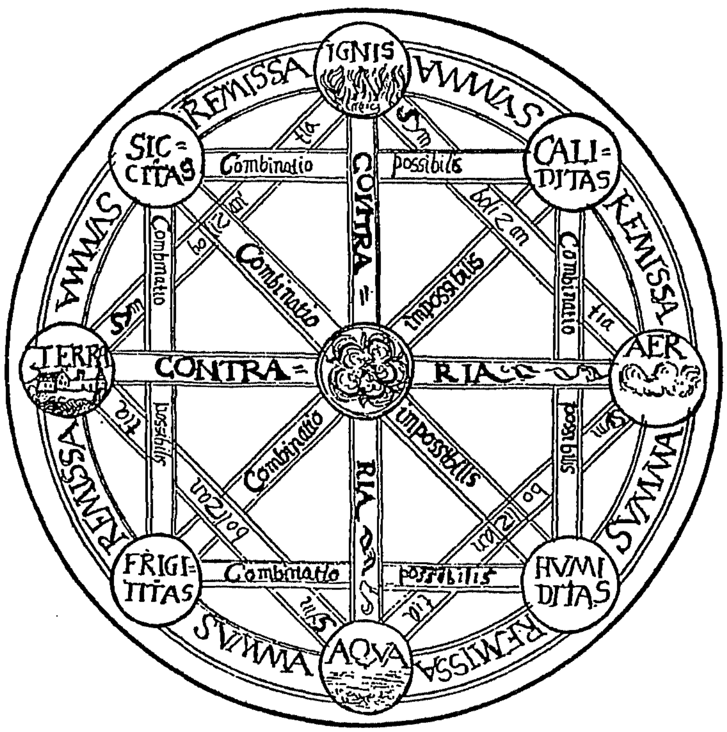
together, sometimes many, sometimes a few, sometimes these, sometimes those.

V. Thomas Hobbes, *Elementa de corpore*, Part I, chapter 5, classifies the things for which there are terms built into a proposition, or in his own terminology, the named things [*nominata*] for which there are names [*nomina*], into *bodies* (that is substances, since for him every substance is a body), *accidents, phantasms*, and *names*. Thus a name is a name either of bodies, for example man; or of accidents, for example, all abstractions, rationality, motion; or of phantasms, in which he includes space, time, all sensible qualities, etc.; or of names, in which he includes second intentions. Since these are com2ned with each other in six ways, there arise the same number of kinds of propositions, and adding to these the cases in which homogeneous terms may be com2ned (a body ascribed to body, accident to accident, phantasm to phantasm, secondary concept to secondary concept), namely, four, the total is ten. Out of these Hobbes thinks that only homogeneous terms can be usefully com2ned. If this is so, as the common philosophy certainly also acknowledges, and abstract and concrete, accident and substance, primary and secondary concepts, are wrongly predicated of each other, this will be useful for the art of discovering propositions or for observing the selection of useful com2nations out of the uncountable mixture of things....

VIII. The eighth application is in the formation of cases by the jurisconsults. For one cannot always wait for the lawmaker when a case arises, and it is more prudent to set up the best possible laws without defects, from the first, than to intrust their restriction and correction to fortune; not to mention the fact that in any state whatsoever, a judicial matter is the better treated, the less is left to the decision of the judge (Plato, *Laws*, Book ix; Aristotle, *Rhetoric*, Book i; Jacob Menochius, *De arbitrariis judicum questionibus et causis*, Book i, proem. 1).

Moreover, the art of forming cases is founded on our doctrine of complexions. For as jurisprudence is similar to geometry in other things, it is also similar in that both have elements and both have cases. The elements are simples; in geometry figures, a triangle, circle, etc; in jurisprudence an action, a promise, a sale, etc. Cases are complexions of these, which are infinitely variable in either field. Euclid composed the *Elements of Geometry*, the elements of law are contained in the *Corpus Juris*, but in both works more complicated cases are added. The simple terms in the law, however, out of the combinations of which the rest arise, and as it were, the *loci communes* and highest genera, have been collected by Bernhard Lavintheta, a Francisan monk, in his commentary on the *Ars magna* of Lully (which see). To us it seems thus: the terms from whose complexions there arises the diversity of cases in the law are persons, things, acts, and rights....

The basis of terms is the same in theology, which is, as it were, a kind of special jurisprudence, but fundamental for the same reason as the others. For theology is a sort of public law which applies in the Kingdom of God among men. Here the unfaithful are like rebels; the church is like good subjects; ecclesiastical persons, and indeed also the political magistrate, are like the subordinate magistrates; excommunication is like banishment; the teaching of Sacred Scripture and the Word of God is like that of the laws and their interpretation; that of the canon like the question of which of the laws are authentic; that of fundamental errors like that of capital crimes; that of the Final Judgment and the Last Day like that of the judiciary process and the rendered judgment; that of the remission of sins like that of the pardoning power; that of eternal punishment like that of capital punishment, etc....



REFERENCES

[1] The term 'hypothesis' is used here in its Platonic and mathematical sense, applying to the given principles upon which the demonstration is based. The term *praecognitum* appears in late Scholastic works in the context of ontology.
[2] Leibniz's facile identification of the whole with a collective name, and the part with a particular subsumed under this name, is supplemented below (No. 1, III, 4) with a definition of the whole as a numerical relation between parts. Kabitz has shown that Leibniz's epistemology in this early period was nominalistic, sensationalistic, and naively realistic. His attitude toward nominalism is corrected in the introduction to the work of Nizolius (No. 6), but the distinction between names or symbols and the real order represented is developed later, as the theory of expression or representation becomes explicit.
[3] For the next stage in Leibniz's cosmological argument for God's existence see No. 5, I; the mature formulation is in No. 51.
[4] These theses were prepared for public disputation and first printed together with the definitions and first two problems. They are found in the footnote in G., IV, 41–43.
[5] Leibniz has in mind the *Arithmetic of Infinites* of John Wallis rather than Seth Ward.
[6] The theory that causality consists of an *influxus physicus* had been proposed by Francis Suárez (1548–1617) as a solution to the problem of efficient causality and had established itself through the wide use of his *Disputationes metaphysicae* (cf. *Disp.*, XII, ii, 4).
[7] Leibniz much later reports that as a boy of 15 or younger he had walked in the Rosental near Leipzig, debating whether to accept the old philosophy of forms or the new atomism and

mechanism and that he had decided in favor of the latter (G., III, 205, 606). Kabitz has shown that his memory was in error here and that this decision could not have been made earlier than 1664 (K. Fischer, *Gottfried Wilhelm Leibniz* [5th ed.], p. 715). He is thus a mechanist at this point, convinced that a quantitative determination is possible of all qualities. But how much his thinking is still in the framework of Aristotelianism this and the following selections show.

[8] Hugo Grotius, *On the Law of War and Peace*, Oxford 1925, p. 25. The influence of Grotius was of decisive importance in the formation of Leibniz's legal thinking. Cf. p. 58, note 10.

[9] The letter to Thomas Hobbes (No. 4) contains another statement of this argument.

[10] In providing a metaphysical basis for the category of number, Leibniz is already searching, following the tradition of Nicolas of Cusa, Galileo, and Descartes, for a mathematical science more general than arithmetic and geometry but including both and thinks of it in terms of the numbering of parts and their possible relationships. It was in the Paris period that he found that numbers are incapable of maximal determination and separated mathematics from metaphysics.

[11] That is, algebra. Leibniz here overlooks the algebraic discoveries of Vieta, Cardan, and others. At this time he knew Descartes only by hearsay (cf. No. 3) and little mathematics beyond Euclid. Francis Schooten, professor at Leyden, edited the *Opera mathematica* of Francis Vieta in 1646 and also prepared an arrangement of Descartes's geometry which Erasmus Bartholinus published as the *Principia matheseos universalis seu introductio in geometriam Cartesii.*

[12] "Disposition is the arrangement of that which has parts" (Aristotle, *Metaphysics* 1027*b*).

[13] Mathematics is thus in this period of Leibniz's thought an abstract consideration of metaphysical relations, particularly of those relational properties of substance upon which quantity rests. Leibniz's general mathematics and logic were always grounded in metaphysical principles.

[14] Variations thus include, in modern terminology, both combinations and permutations, while situs is a permutation. When this permutation is in circular order (as in the case of the rearrangement in relation to each other of men seated about a table), it is vicinity; when relative position to the whole is involved, it is order.

[15] That is, the number of permutations of four things taken four at a time is 24.

[16] Complexions are in modern terminology combinations; the latter term Leibniz reserves for the special case of complexions of the second order. His example involves the total combinations of four things, with exponents from 1 to 4. Later he calls this "complexions taken simply" (Sec. 12).

[17] Leibniz's source was Daniel Schwenter, *Deliciae physico-mathematicae*, Nürnberg 1651–53.

[18] Leibniz seems to have known a formula only for the combination of n things taken two at a time, namely, $C_2^n = n(n-1)$, and to have been ignorant of the more general form. He therefore chooses a tabular method suggestive of Pascal's triangle.

[19] Note that the table involves a derivation of complexions by the additive use of 0 and 1 only and thus anticipates Leibniz's later interest in the binary number system.

[20] Thus the total combinations of n terms $= 2^n - 1$. We omit several sections dealing with special aspects of the rule for complexions.

[21] Leibniz now offers twelve applications, most of which must be omitted because of length. For his thought about traditional logic the sixth, here omitted, is perhaps most significant. It contains his deduction of the valid modes and figures of the Aristotelian syllogism.