

NONLINEAR CONTRACTIONS IN METRIC SPACES UNDER LOCALLY T -TRANSITIVE BINARY RELATIONS

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Abstract. In this paper, we present a variant of Boyd-Wong fixed point theorem in a metric space equipped with a locally T -transitive binary relation, which under universal relation reduces to Boyd-Wong (Proc. Amer. Math. Soc. **20**(1969), 458-464) and Jotic (Indian J. Pure Appl. Math. **26**(1995), 947-952) fixed point theorems. Also, our results extend several other well-known fixed point theorems such as: Alam and Imdad (J. Fixed Point Theory Appl. **17**(2015), no 4, 693-702) and Karapinar and Roldán-López-de-Hierro (J. Inequal. Appl. **2014:522**(2014), 12 pages) besides some others.

Key Words and Phrases: \mathcal{R} -continuity; locally T -transitive binary relations; φ -contractions; \mathcal{R} -connected sets; fixed point.

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