Bibliography on the determination of finite groups

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This article is intended to act as a reference guide to available sources on the determination of groups, with particular reference to the groups of order at most 1000. No claims are made in respect of the accuracy of the results, except where explicitly stated, or on the comprehensiveness of the references.

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The term "enumeration" is used to describe the counting of the groups of a particular type; "determination" or "listing" indicates that presentations of the groups are obtained; "classification" indicates that the groups are organised according to some criteria. The parameters p, q, r, and s are used throughout to denote distinct primes.

The determination of all isomorphism types of groups of a fixed order has interested mathematicians since the first paper on this topic by Cayley in 1854, in which he determined the cyclic groups and the groups of order 4 and 6. In an 1859 paper, he determined the groups of order 8. Netto (1882, pp. 133-137) determined the groups of order p^2 and those of order pq. Kempe (1886) listed the groups of order 8 and, inaccurately, those of order 12. The groups of order at most 12 were correctly listed by Cayley (1889).

The groups of order p^3 were independently determined by Cole & Glover (1893), Hölder (1893), and Young (1893). The groups of order p^2q and pqr were listed by Cole & Glover (1893) and Hölder (1893). The work of the former on the groups of order p^2q was corrected by Hölder (1895a). Miller (1921a and 1921b) wrongly claimed that Hölder's work was inaccurate; a new determination was carried out by Lin (1974). The groups of order p^4 were determined by Hölder (1893) and Young (1893). The groups of square-free order were determined by Hölder (1895b); the groups of order pqrs were also determined by Baudet (1918). Le Vavasseur (1896b) determined the groups of order pqrs were independently enumerated by Miller (1896a) who claimed that there are errors in the work of Le Vavasseur.

Le Vavasseur (1896a) claimed that there were more than 75 groups of order 32. A list of 51 groups was given by Miller (1896b). When Miller (1936) recalculated the groups of order 32, he obtained only 47 and put the discrepancy down to duplications in his original list. Sophie (1962) provided an explanation for the mistakes made by Miller in his 1936 paper and also verified the correctness of his original list.

Miller (1896b) provided generating sets of permutations for the groups of order less than 48. Burnside (1897, pp. 105-108) determined the groups of order 60.

The groups of order p^5 were listed by Bagnera (1898). Miller (1899b) pointed out errors for the groups of order 2^5 which were corrected by Bagnera (1899). A new list was provided by de Séguier (1904, §154-159); Schreier (1926) published a list for $p \geq 5$. Bender (1927) published a list showing errors in Bagnera's calculations for

the groups of order 3^5 but omitted a maximal class group which was included by Blackburn (1958). A correct list of the groups of order 3^5 was calculated by James (1968) and published in James (1980).

The groups of order 48 and $2p^3$ were enumerated by Miller (1899a) and those of order p^3q were determined by Western (1899). The groups of order p^2q^2 were initially determined by Le Vavasseur (1899a and 1902) and later by Lin (1974). They were also enumerated by Laue (1982, pp. 214-243). Both Lin and Laue get identical results, although Lin's summary has a counting error.

The groups of order 16p were determined by Le Vavasseur (1899b and 1903); Lunn & Senior (1934) claimed that there were errors in his work and enumerated the groups of this order. The groups of order $8p^3$ and $16p^2$ were determined by Nyhlén (1919), those of order $8p^2$ by Zhang (1983), and those of order 8pq by Wen (1984). Some groups whose orders are products of 6 primes were determined by Malmrot (1925).

The groups of order 168 were enumerated by Miller (1902). Glenn (1906) incorrectly determined the groups of order p^2qr ; the number of such groups can be found using the work of Laue (1982, pp. 244-262). The groups of order p^3q^2 were determined by Tripp (1909). Both Nyhlén (1919, p. 37) and Malmrot (1925, pp. 87-88) claim that Tripp's list is incomplete for the groups of order 72 and a list of 50 presentations is provided by Malmrot. In an independent enumeration, Miller (1929) also found 50 groups. An inaccurate enumeration of the groups of order 96 was given by Miller (1930a); an enumeration of the groups of order 32p was carried out by Lunn & Senior (1934). The groups of order 96 are listed in Laue (1982, pp. 278-296); a corrigendum to this work corrects some errors.

Potron (1904a and 1904b) gave an incomplete list of the groups of order 64. Miller (1930b) claimed that there is a total of 294 groups. A complete list was calculated by P. Hall & Senior in the 1930s and published by M. Hall & Senior (1964). Some corrections to generating sets supplied for permutation groups representing these groups were given by McKay (1969).

The first calculations of the groups of order p^6 , for p an odd prime, is also in the work of Potron. Tordella (1939) described some errors in this work but his work is incomplete. Easterfield (1940) provided presentations for the groups, for p > 3, and their classification into isoclinism families. Blackburn (1958) classified the maximal class groups of order p^6 . A list of the groups calculated by James (1968) had a number of errors and was incomplete. Leong (1974) and Miech (1975) published lists of certain subclasses of p-groups. Küpper (1979) corrected errors in the work of James. A complete list, incorporating corrections by Keane for groups of order 3^6 and incorrectly the work of Küpper, was published by James (1980). It agreed, in the relevant sections, with the works of Blackburn, Leong, and Miech. However, it contains a number of serious errors, some of which are documented by Pilyavskaya (1983). A summary of the known errors and their corrections is given in Newman & O'Brien (1986). The groups of order 3^6 were determined by Baldwin (1987) providing confirmation of the 1986 work of Newman & O'Brien.

A comprehensive listing of presentations and certain properties, in particular the subgroup lattices, of the groups of order at most 100, excluding those of order 64 and 96, was provided by Neubüser (1967).

The first work on the groups of order 128 was done by P. Hall in the 1930s. An inaccurate enumeration of the groups of order 128 was given by Rodemich (1980). An

independent determination of these groups was provided by James, Newman & O'Brien (1990). The groups of order p^7 that have exponent p were determined by Wilkinson (1988).

The groups of order 108, 120, 144, 162, 180, and 200 were enumerated by Senior & Lunn (1934 and 1935). The groups of order 180 were determined by Jabber (1941) and Taunt (1948) provided confirmation of his work. Taunt (1955) discussed the construction of soluble groups of cube-free order and Laue (1982, pp. 214-243) enumerated soluble groups of certain orders.

A complete determination of the groups of order 256 was provided by O'Brien (1988, 1991).

In addition to the published sources listed above, the provision of group descriptions in electronically accessible form has developed significantly during the 1980s. Much of the electronically accessible material has been made available for use with the computer algebra systems GAP and MAGMA.

These include a library of primitive permutation groups of degree at most 50 developed by Sims, a library of transitive groups of degree at most 12 (see Royle, 1987), and a library of simple groups of order less than 1 000 000 (see Campbell & Robertson, 1985). Holt & Plesken have calculated a list of perfect groups of order less than 1 000 000 and this material is also available as libraries for GAP and MAGMA.

Libraries are also available for the groups of order at most 100, the groups of order dividing 128, and the groups of order dividing 3⁶. These were developed by Cannon, Newman & O'Brien (1989), and Baldwin (1987), respectively. In 1989, a library for the groups of order 256 was released by O'Brien.

Besche & Eick (1998) developed algorithms to determine the groups of order n, for an arbitrary positive integer n. They used these to determine the non-nilpotent groups of order of order at most 1000 omitting 768. Combining their results with those for the groups of orders dividing 256 and 729, we now have explicit descriptions for the groups of order at most 1000 with two exceptions: those of orders 512 and 768. These descriptions are available as libraries.

Eick & O'Brien (1998) show that there are 10 494 213 groups of order 2^9 .

Appendix

Number of d-generator p-groups of order at most p^10 for p in [2..7], d in [2..9]

Computed using the algorithm of Eick & O'Brien, J. Austral. Math. Soc. Ser. A 67, 1999, 191-205.

Each entry <a, b> indicates that there b groups (having particular prime p, class c, and generator number d) of order p^a.

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- Number of 2-generator groups having 2-class 2 and orders up to 2^10 = [<3, 3>, <4, 3>, <5, 1>]
- Number of 3-generator groups having 2-class 2 and orders up to $2^10 = [<4, 4>, <5, 15>, <6, 28>, <7, 15>, <8, 4>, <9, 1>]$
- Number of 4-generator groups having 2-class 2 and orders up to 2¹⁰ = [<5, 6>, <6, 54>, <7, 604>, <8, 3566>, <9, 6709>, <10, 3566>]
- Number of 5-generator groups having 2-class 2 and orders up to $2^10 = [<6, 7>, <7, 151>, <8, 26065>, <9, 5829109>, <10, 378628831>]$
- Number of 6-generator groups having 2-class 2 and orders up to 2^10 = [<7, 9>, <8, 433>, <9, 2948829>, <10, 47698016406>]
- Number of 7-generator groups having 2-class 2 and orders up to 2^10 = [<8, 10>, <9, 1112>, <10, 726843973>]
- Number of 8-generator groups having 2-class 2 and orders up to 2^10 = [<9, 12>, <10, 2933>]
- Number of 9-generator groups having 2-class 2 and orders up to 2^10 = [<10, 13>]
- Number of 2-generator groups having 3-class 2 and orders up to 3^10 = [<3, 3>, <4, 3>, <5, 1>]
- Number of 3-generator groups having 3-class 2 and orders up to 3^10 = [<4, 4>, <5, 17>, <6, 36>, <7, 17>, <8, 4>, <9, 1>]
- Number of 4-generator groups having 3-class 2 and orders up to $3^10 = [<5, 6>, <6, 60>, <7, 1361>, <8, 23361>, <9, 66667>, <10, 23361>]$
- Number of 5-generator groups having 3-class 2 and orders up to 3^10 = [<6, 7>, <7, 178>, <8, 578478>, <9, 3676421905>, <10, 2689548451105>]
- Number of 6-generator groups having 3-class 2 and orders up to 3^10 = [<7, 9>, <8, 566>, <9, 1220110640>, <10, 5865931805742213>]
- Number of 7-generator groups having 3-class 2 and orders up to 3^10 = [<8, 10>, <9, 1713>, <10, 7967909744001>]
- Number of 8-generator groups having 3-class 2 and orders up to 3^10 = [<9, 12>, <10, 5486>]
- Number of 9-generator groups having 3-class 2 and orders up to 3^10

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= [ <10, 13> ]
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- Number of 2-generator groups having 5-class 2 and orders up to 5^{10} = [$\langle 3, 3 \rangle$, $\langle 4, 3 \rangle$, $\langle 5, 1 \rangle$]
- Number of 3-generator groups having 5-class 2 and orders up to $5^10 = [<4, 4>, <5, 19>, <6, 42>, <7, 19>, <8, 4>, <9, 1>]$
- Number of 4-generator groups having 5-class 2 and orders up to 5¹⁰ = [<5, 6>, <6, 68>, <7, 6598>, <8, 683776>, <9, 3395171>, <10, 683776>]
- Number of 5-generator groups having 5-class 2 and orders up to 5^10 = [<6, 7>, <7, 225>, <8, 84564771>, <9, 32977411084053>, <10, 515434141823071597>]
- Number of 6-generator groups having 5-class 2 and orders up to 5¹⁰ = [<7, 9>, <8, 883>, <9, 6588081120697>, <10, 40257987234383294345165>]
- Number of 7-generator groups having 5-class 2 and orders up to 5^10 = [<8, 10>, <9, 3731>, <10, 2572632501048176344>]
- Number of 8-generator groups having 5-class 2 and orders up to 5^10 = [<9, 12>, <10, 18312>]
- Number of 9-generator groups having 5-class 2 and orders up to 5^10 = [<10, 13>]
- Number of 2-generator groups having 7-class 2 and orders up to $7^10 = [<3, 3>, <4, 3>, <5, 1>]$
- Number of 3-generator groups having 7-class 2 and orders up to $7^10 = [<4, 4>, <5, 21>, <6, 48>, <7, 21>, <8, 4>, <9, 1>]$
- Number of 4-generator groups having 7-class 2 and orders up to 7^10 = [<5, 6>, <6, 76>, <7, 26156>, <8, 8254097>, <9, 57683849>, <10, 8254097>]
- Number of 5-generator groups having 7-class 2 and orders up to 7^10 = [<6, 7>, <7, 279>, <8, 2824355125>, <9, 16277580699921428>, <10, 1915154483198662394396>]
- Number of 6-generator groups having 7-class 2 and orders up to 7^10 = [<7, 9>, <8, 1379>, <9, 2324584428892253>, <10, 1577131637384923517587108603>]
- Number of 7-generator groups having 7-class 2 and orders up to 7^10 = [<8, 10>, <9, 8101>, <10, 13399965568604995907122>]

- Number of 8-generator groups having 7-class 2 and orders up to 7^10 = [<9, 12>, <10, 56620>]
- Number of 9-generator groups having 7-class 2 and orders up to $7^10 = [<10, 13>]$

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