```
gg :=proc(x, y)
return abs(x − y) − 1;
 end proc
```

$$gg := \mathbf{proc}(x,\ y)\ \mathbf{return}\ \mathrm{abs}(x\ -\ y)\ -\ 1\ \mathbf{end\ proc} \tag{1}$$

```
CopyArrayElem :=proc(x, n)
  local y, i;
  y := Array(1..n) ;
  for i from 1 to n do
  y(i) := x(i);
  od;
  return y;
 end proc
```

$$CopyArrayElem := \mathbf{proc}(x,\ n) \tag{2}$$
```
    local y, i;
    y := Array(1..n); for i to n do y(i) := x(i) end do; return y
end proc
```

```
PrintArray :=proc(x, n)
local i
  for i from 1 to n do
  printf("%d, ", x(i));
   if i = n/2 then printf("|,");fi;
  od;
  printf("\n");
 end proc
```

$$PrintArray := \mathbf{proc}(x,\ n) \tag{3}$$
```
    local i;
    for i to n do
        printf("%d, ", x(i)); if i = 1/2*n then printf("|,") end if
    end do;
    printf("\n")
end proc
```

```
GapArray :=proc(x, n)
 local i, y;
 y := Array(1..n−1);
 for i from 1 to n−1 do
  y(i) := gg(x(i), x(i + 1));
  od;
  return y;
 # The length of the return is n-1.
 end proc
```

$$GapArray := \mathbf{proc}(x,\ n) \tag{4}$$
```
    local i, y;
    y := Array(1..n − 1);
```

```
      for i to n − 1 do y(i) := gg(x(i), x(i + 1)) end do;
      return y
end proc
ShiftArray :=proc(x, n, d)
 local i, y;
 y := Array(1..n);
 for i from 1 to n do
 y(i) := x(i) + d;
 od;
 return y;
 end proc
ShiftArray := proc(x, n, d)                                                    (5)
    local i, y;
    y := Array(1..n); for i to n do y(i) := x(i) + d end do; return y
end proc
chost :=proc(p, q)
local i, j, m;
local Hpq, hh;
 Hpq := Array(1..p + q − 2);
 hh := Array(1..p + q − 2);
  for i from 1 to q − 1 do
  Hpq(i) := i·p;
  od;

  for j from 1 to p − 1 do;
  Hpq(i + +) := j·q;
  od;
  hh := sort(Hpq);

  return hh;
 end proc
chost := proc(p, q)                                                           (6)
    local i, j, m, Hpq, hh;
    Hpq := Array(1..p + q − 2);
    hh := Array(1..p + q − 2);
    for i to q − 1 do Hpq(i) := i*p end do;
    for j to p − 1 do Hpq(++(i)) := j*q end do;
    hh := sort(Hpq);
    return hh
end proc
Theorem2 :=proc(p, q)
 # 2<p<q<2p and q=p+r
local r, hh, g, n, xx, m, w0, k;
```

```
  n := p + q - 2;
  g := Array(1 .. n - 1);

 hh := chost(p, q); #Calculate the hosts of p and q
 printf("All the hosts are list by:\n");
 PrintArray(hh, n);

 r := q - p ;
 printf("r=%d\n", r);
 w0 := ceil( p/r );
 printf("w0:=%d\n", w0);

 m := floor( (p + 1)/(2·w0) ) - 1;
 printf("m=%d\n", m);

 printf("The 1st one is in [%d, %d]\n", (w0 - 1)·q, w0·q);


 if m ≥ 1 then
 for k from 1 to m do
 printf("The %d-th one is in [%d, %d]\n", k + 1, ((k + 1)·w0 - 1)·q, (k + 1)·w0·q);
 od;
 printf( "The half one is in [%d, %d]\n", ( (p - 1)/2 )·q, ( (p + 1)/2 )·q );
 fi;
 printf("\n");

 g := GapArray(hh, n); #Calculate gaps and store them in g.
 xx := CopyArrayElem(hh, n - 1);
 dataplot(xx, g);   #Draw the hosts and gaps.

end proc
Theorem2 := proc(p, q)                                                    (7)
   local r, hh, g, n, xx, m, w0, k;
   n := p + q - 2;
   g := Array(1 .. n - 1);
   hh := chost(p, q);
   printf("All the hosts are list by:\n");
   PrintArray(hh, n);
   r := q - p;
   printf("r=%d\n", r);
   w0 := ceil(p/r);
   printf("w0:=%d\n", w0);
```

```
m := floor(1/2*(p+1)/w0) - 1;
printf("m=%d\n", m);
printf("The 1st one is in [%d, %d]\n", (w0 - 1)*q, w0*q);
if 1 <= m then
    for k to m do
        printf("The %d-th one is in [%d, %d]\n", k+1, ((k+1)*w0 - 1)*q, (k
        + 1)*w0*q)
    end do;
    printf("The half one is in [%d, %d]\n", (1/2*p - 1/2)*q, (1/2*p+1
    /2)*q)
end if;
printf("\n");
g := GapArray(hh, n);
xx := CopyArrayElem(hh, n - 1);
dataplot(xx, g)
end proc
```

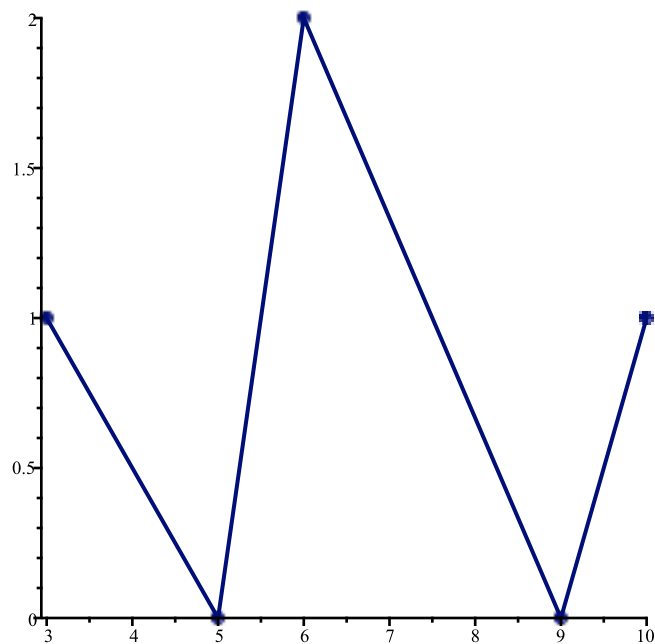*Theorem2*(3, 5)
All the hosts are list by:
3, 5, 6, |, 9, 10, 12,
r=2
w0=2
m=0
The 1st one is in [5, 10]

Theorem2(5, 7)
All the hosts are list by:
5, 7, 10, 14, 15, |, 20, 21, 25, 28, 30,
r=2
w0=3
m=0
The 1st one is in [14, 21]
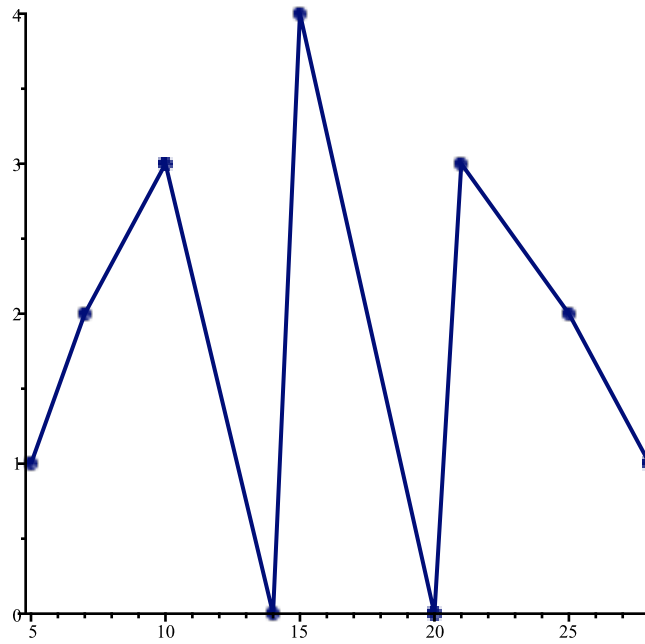
Theorem2(7, 9)
All the hosts are list by:
7, 9, 14, 18, 21, 27, 28, |, 35, 36, 42, 45, 49, 54, 56,
r=2
w0=4
m=0
The 1st one is in [27, 36]
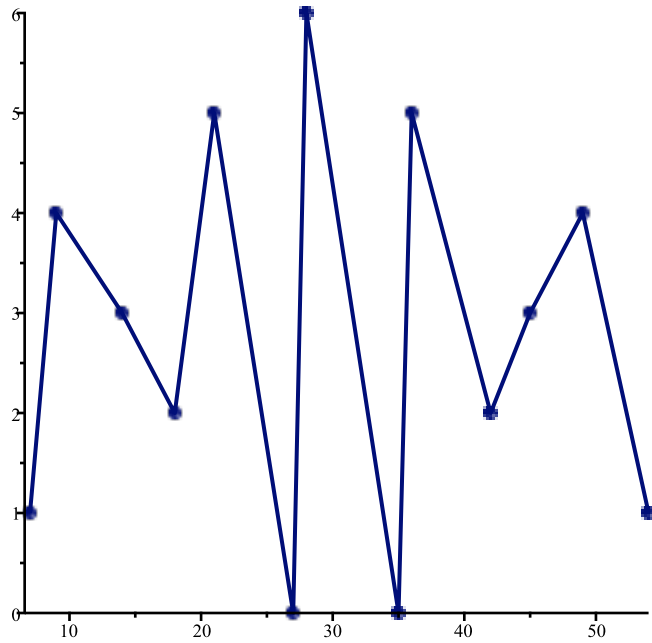
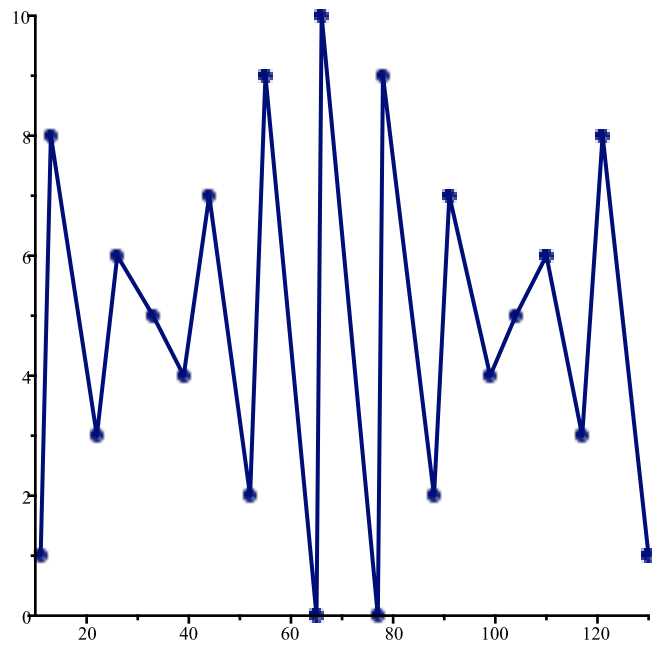*Theorem2*(11, 13)
All the hosts are list by:
11, 13, 22, 26, 33, 39, 44, 52, 55, 65, 66, |, 77, 78, 88, 91, 99, 104, 110, 117,
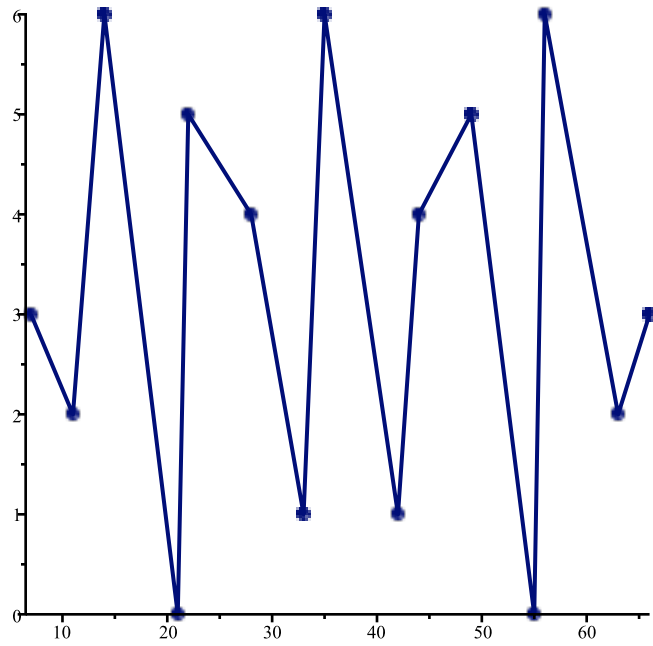**121, 130, 132,**
r=2
w0=6
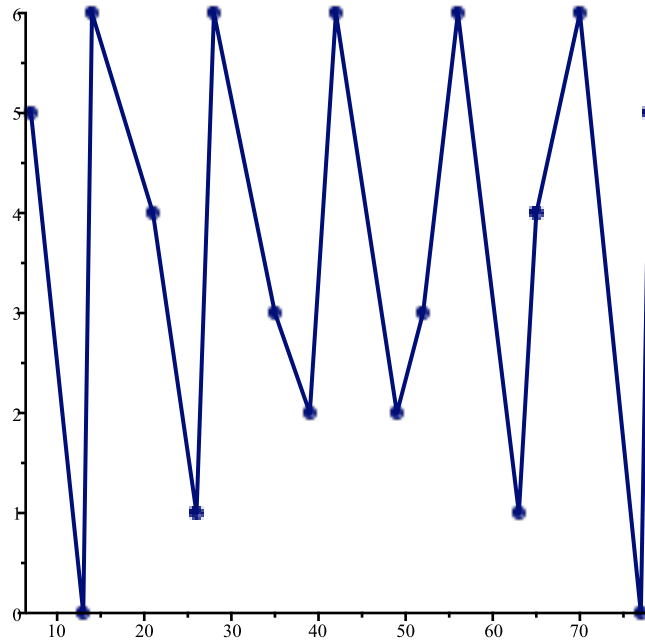**m=0**
The 1st one is in [65, 78]

Theorem2(7, 11)
All the hosts are list by:

7, 11, 14, 21, 22, 28, 33, 35, |, 42, 44, 49, 55, 56, 63, 66, 70,
r=4
w0=2
m=1
The 1st one is in [11, 22]
The 2-th one is in [33, 44]
The half one is in [33, 44]

Theorem2(7, 13)

All the hosts are list by:
7, 13, 14, 21, 26, 28, 35, 39, 42, |, 49, 52, 56, 63, 65, 70, 77, 78, 84,
r=6
w0=2
m=1
The 1st one is in [13, 26]
The 2-th one is in [39, 52]
The half one is in [39, 52]

Theorem2(11, 17)

All the hosts are list by:
11, 17, 22, 33, 34, 44, 51, 55, 66, 68, 77, 85, 88, |, 99, 102, 110, 119, 121, 132, 136, 143, 153, 154, 165, 170, 176,
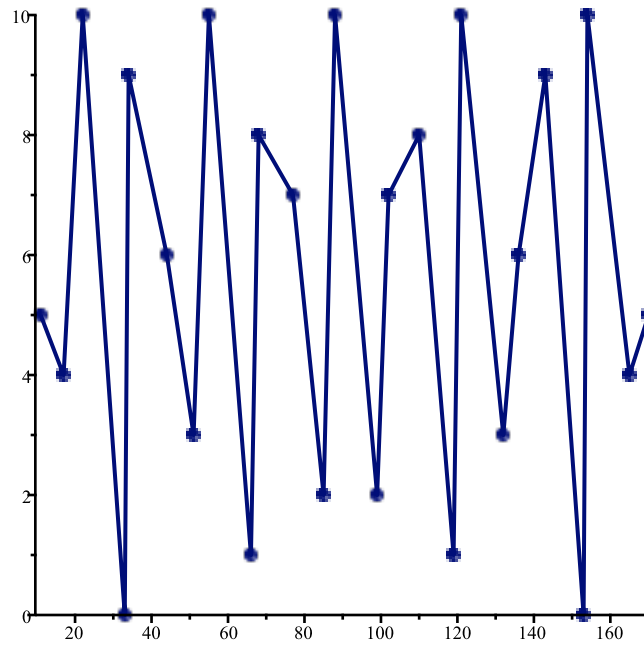r=6
w0=2
m=2
The 1st one is in [17, 34]
The 2-th one is in [51, 68]
The 3-th one is in [85, 102]
The half one is in [85, 102]

Theorem2(13, 23)

All the hosts are list by:
13, 23, 26, 39, 46, 52, 65, 69, 78, 91, 92, 104, 115, 117, 130, 138, 143, |, 156, 161, 169, 182, 184, 195, 207, 208, 221, 230, 234, 247, 253, 260, 273, 276, 286,
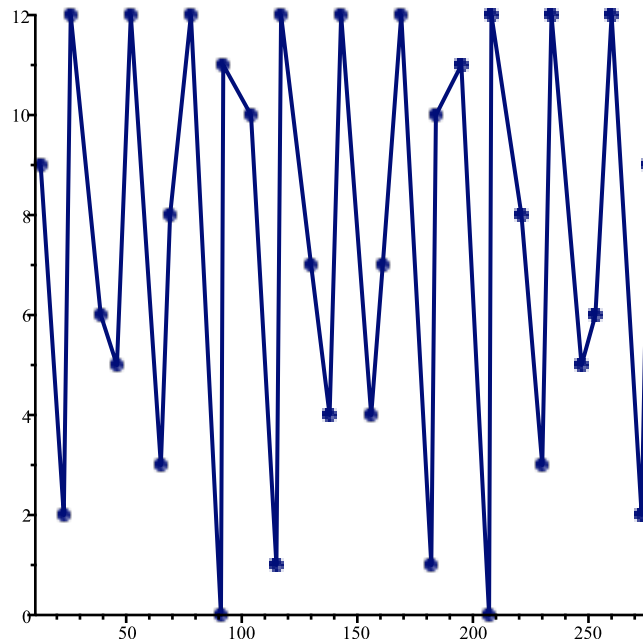r=10
w0=2
m=2
The 1st one is in [23, 46]
The 2-th one is in [69, 92]
The 3-th one is in [115, 138]
The half one is in [138, 161]

Theorem2(17, 29)

All the hosts are list by:

17, 29, 34, 51, 58, 68, 85, 87, 102, 116, 119, 136, 145, 153, 170, 174, 187, 203, 204, 221, 232, 238, |, 255, 261, 272, 289, 290, 306, 319, 323, 340, 348, 357, 374, 377, 391, 406, 408, 425, 435, 442, 459, 464, 476,
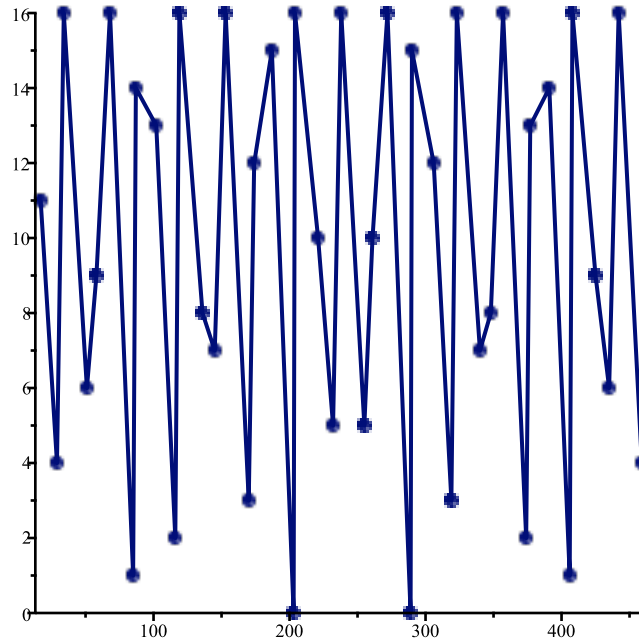
r=12

w0=2

m=3

The 1st one is in [29, 58]

The 2-th one is in [87, 116]

The 3-th one is in [145, 174]

The 4-th one is in [203, 232]

The half one is in [232, 261]

Theorem2(23, 31)

All the hosts are list by:

23, 31, 46, 62, 69, 92, 93, 115, 124, 138, 155, 161, 184, 186, 207, 217, 230, 248, 253, 276, 279, 299, 310, 322, 341, 345, |, 368, 372, 391, 403, 414, 434, 437, 460, 465, 483, 496, 506, 527, 529, 552, 558, 575, 589, 598, 620, 621, 644, 651, 667, 682, 690,
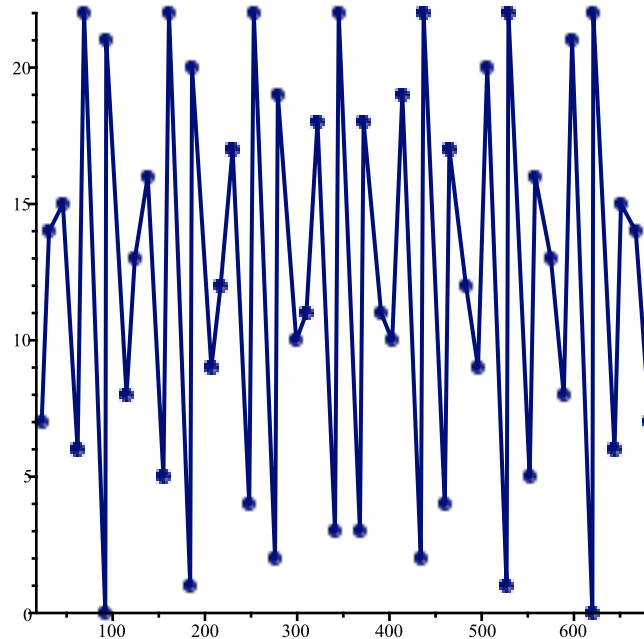
r=8

w0=3

m=3

The 1st one is in [62, 93]

The 2-th one is in [155, 186]

The 3-th one is in [248, 279]

The 4-th one is in [341, 372]

The half one is in [341, 372]

Theorem2(31, 61)

All the hosts are list by:
31, 61, 62, 93, 122, 124, 155, 183, 186, 217, 244, 248, 279, 305, 310, 341, 366,
372, 403, 427, 434, 465, 488, 496, 527, 549, 558, 589, 610, 620, 651, 671, 682, 713,
732, 744, 775, 793, 806, 837, 854, 868, 899, 915, 930, |, 961, 976, 992, 1023, 1037,
1054, 1085, 1098, 1116, 1147, 1159, 1178, 1209, 1220, 1240, 1271, 1281, 1302, 1333,
1342, 1364, 1395, 1403, 1426, 1457, 1464, 1488, 1519, 1525, 1550, 1581, 1586, 1612,
1643, 1647, 1674, 1705, 1708, 1736, 1767, 1769, 1798, 1829, 1830, 1860,
r=30
w0=2
m=7
The 1st one is in [61, 122]
The 2-th one is in [183, 244]
The 3-th one is in [305, 366]
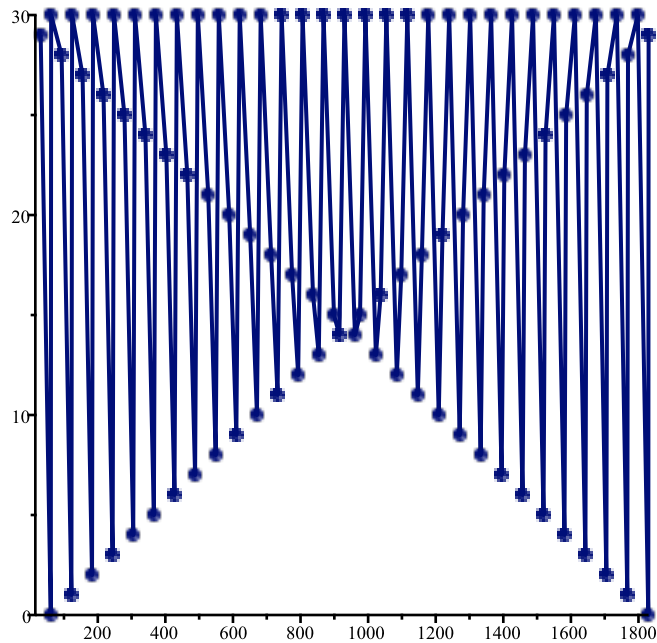The 4-th one is in [427, 488]
The 5-th one is in [549, 610]
The 6-th one is in [671, 732]
The 7-th one is in [793, 854]
The 8-th one is in [915, 976]
The half one is in [915, 976]

Theorem2(37, 73)

All the hosts are list by:
37, 73, 74, 111, 146, 148, 185, 219, 222, 259, 292, 296, 333, 365, 370, 407, 438,
444, 481, 511, 518, 555, 584, 592, 629, 657, 666, 703, 730, 740, 777, 803, 814, 851,
876, 888, 925, 949, 962, 999, 1022, 1036, 1073, 1095, 1110, 1147, 1168, 1184, 1221,
1241, 1258, 1295, 1314, 1332, |, 1369, 1387, 1406, 1443, 1460, 1480, 1517, 1533,
1554, 1591, 1606, 1628, 1665, 1679, 1702, 1739, 1752, 1776, 1813, 1825, 1850, 1887,
1898, 1924, 1961, 1971, 1998, 2035, 2044, 2072, 2109, 2117, 2146, 2183, 2190, 2220,
2257, 2263, 2294, 2331, 2336, 2368, 2405, 2409, 2442, 2479, 2482, 2516, 2553, 2555,
2590, 2627, 2628, 2664,
r=36
w0=2
m=8
The 1st one is in [73, 146]
The 2-th one is in [219, 292]
The 3-th one is in [365, 438]
The 4-th one is in [511, 584]
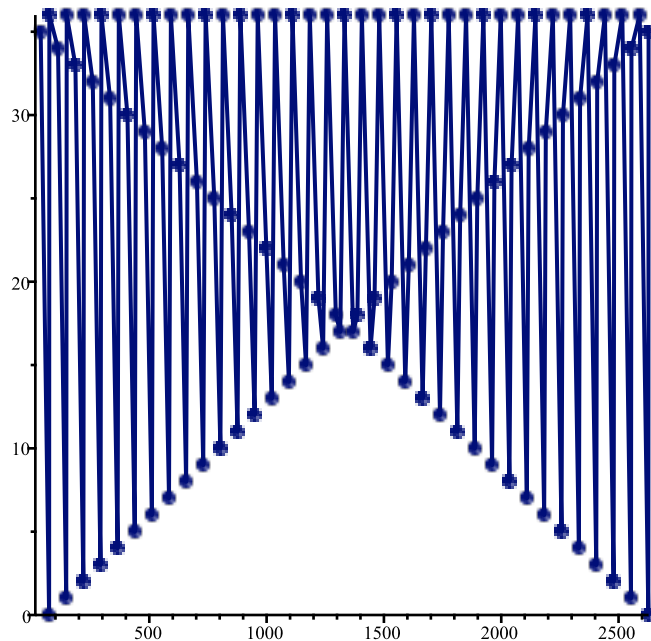The 5-th one is in [657, 730]
The 6-th one is in [803, 876]
The 7-th one is in [949, 1022]
The 8-th one is in [1095, 1168]
The 9-th one is in [1241, 1314]
The half one is in [1314, 1387]

Theorem2(47, 97)

All the hosts are list by:

47, 94, 97, 141, 188, 194, 235, 282, 291, 329, 376, 388, 423, 470, 485, 517, 564, 582, 611, 658, 679, 705, 752, 776, 799, 846, 873, 893, 940, 970, 987, 1034, 1067, 1081, 1128, 1164, 1175, 1222, 1261, 1269, 1316, 1358, 1363, 1410, 1455, 1457, 1504, 1551, 1552, 1598, 1645, 1649, 1692, 1739, 1746, 1786, 1833, 1843, 1880, 1927, 1940, 1974, 2021, 2037, 2068, 2115, 2134, 2162, 2209, 2231, 2256, |, 2303, 2328, 2350, 2397, 2425, 2444, 2491, 2522, 2538, 2585, 2619, 2632, 2679, 2716, 2726, 2773, 2813, 2820, 2867, 2910, 2914, 2961, 3007, 3008, 3055, 3102, 3104, 3149, 3196, 3201, 3243, 3290, 3298, 3337, 3384, 3395, 3431, 3478, 3492, 3525, 3572, 3589, 3619, 3666, 3686, 3713, 3760, 3783, 3807, 3854, 3880, 3901, 3948, 3977, 3995, 4042, 4074, 4089, 4136, 4171, 4183, 4230, 4268, 4277, 4324, 4365, 4371, 4418, 4462, 4465, 4512,

r=50

w0=1

m=23

The 1st one is in [0, 97]

The 2-th one is in [97, 194]

The 3-th one is in [194, 291]

The 4-th one is in [291, 388]

The 5-th one is in [388, 485]

The 6-th one is in [485, 582]

The 7-th one is in [582, 679]

The 8-th one is in [679, 776]

The 9-th one is in [776, 873]

The 10-th one is in [873, 970]

The 11-th one is in [970, 1067]

The 12-th one is in [1067, 1164]

The 13-th one is in [1164, 1261]

The 14-th one is in [1261, 1358]

The 15-th one is in [1358, 1455]

The 16-th one is in [1455, 1552]
The 17-th one is in [1552, 1649]
The 18-th one is in [1649, 1746]
The 19-th one is in [1746, 1843]
The 20-th one is in [1843, 1940]
The 21-th one is in [1940, 2037]
The 22-th one is in [2037, 2134]
The 23-th one is in [2134, 2231]
The 24-th one is in [2231, 2328]
The half one is in [2231, 2328]
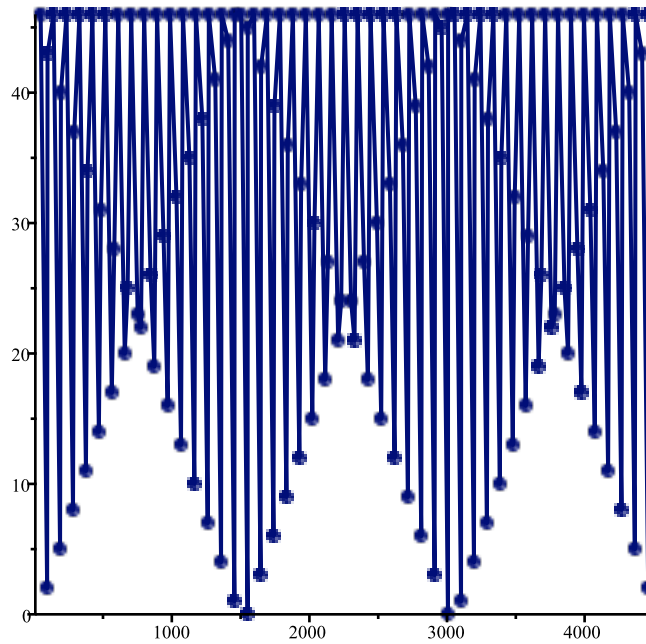
Theorem2(53, 103)

All the hosts are list by:
53, 103, 106, 159, 206, 212, 265, 309, 318, 371, 412, 424, 477, 515, 530, 583, 618,
636, 689, 721, 742, 795, 824, 848, 901, 927, 954, 1007, 1030, 1060, 1113, 1133,
1166, 1219, 1236, 1272, 1325, 1339, 1378, 1431, 1442, 1484, 1537, 1545, 1590, 1643,
1648, 1696, 1749, 1751, 1802, 1854, 1855, 1908, 1957, 1961, 2014, 2060, 2067, 2120,
2163, 2173, 2226, 2266, 2279, 2332, 2369, 2385, 2438, 2472, 2491, 2544, 2575, 2597,
2650, 2678, 2703, |, 2756, 2781, 2809, 2862, 2884, 2915, 2968, 2987, 3021, 3074,
3090, 3127, 3180, 3193, 3233, 3286, 3296, 3339, 3392, 3399, 3445, 3498, 3502, 3551,
3604, 3605, 3657, 3708, 3710, 3763, 3811, 3816, 3869, 3914, 3922, 3975, 4017, 4028,
4081, 4120, 4134, 4187, 4223, 4240, 4293, 4326, 4346, 4399, 4429, 4452, 4505, 4532,
4558, 4611, 4635, 4664, 4717, 4738, 4770, 4823, 4841, 4876, 4929, 4944, 4982, 5035,
5047, 5088, 5141, 5150, 5194, 5247, 5253, 5300, 5353, 5356, 5406,
r=50
w0=2
m=12
The 1st one is in [103, 206]
The 2-th one is in [309, 412]

The 3-th one is in [515, 618]
The 4-th one is in [721, 824]
The 5-th one is in [927, 1030]
The 6-th one is in [1133, 1236]
The 7-th one is in [1339, 1442]
The 8-th one is in [1545, 1648]
The 9-th one is in [1751, 1854]
The 10-th one is in [1957, 2060]
The 11-th one is in [2163, 2266]
The 12-th one is in [2369, 2472]
The 13-th one is in [2575, 2678]
The half one is in [2678, 2781]
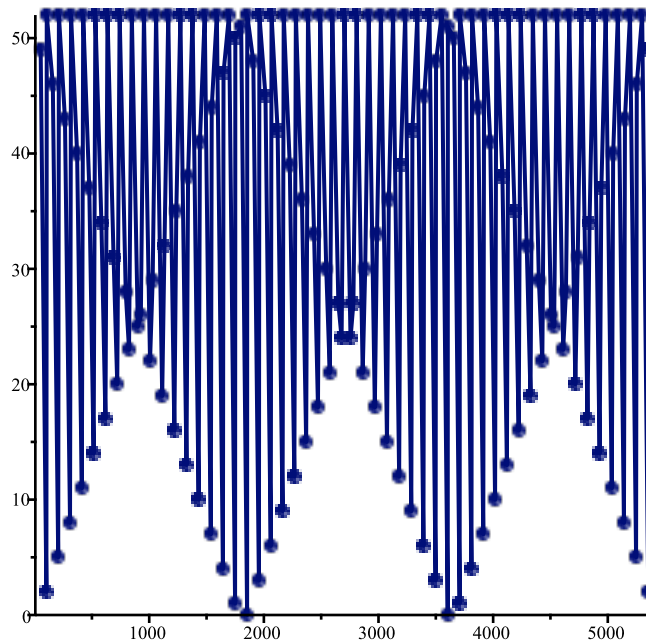

Theorem2(47, 101)

All the hosts are list by:
47, 94, 101, 141, 188, 202, 235, 282, 303, 329, 376, 404, 423, 470, 505, 517, 564,
606, 611, 658, 705, 707, 752, 799, 808, 846, 893, 909, 940, 987, 1010, 1034, 1081,
1111, 1128, 1175, 1212, 1222, 1269, 1313, 1316, 1363, 1410, 1414, 1457, 1504, 1515,
1551, 1598, 1616, 1645, 1692, 1717, 1739, 1786, 1818, 1833, 1880, 1919, 1927, 1974,
2020, 2021, 2068, 2115, 2121, 2162, 2209, 2222, 2256, 2303, 2323, 2350, |, 2397,
2424, 2444, 2491, 2525, 2538, 2585, 2626, 2632, 2679, 2726, 2727, 2773, 2820, 2828,
2867, 2914, 2929, 2961, 3008, 3030, 3055, 3102, 3131, 3149, 3196, 3232, 3243, 3290,
3333, 3337, 3384, 3431, 3434, 3478, 3525, 3535, 3572, 3619, 3636, 3666, 3713, 3737,
3760, 3807, 3838, 3854, 3901, 3939, 3948, 3995, 4040, 4042, 4089, 4136, 4141, 4183,
4230, 4242, 4277, 4324, 4343, 4371, 4418, 4444, 4465, 4512, 4545, 4559, 4606, 4646,
4653, 4700,
r= 54
w0= 1
m=23

The 1st one is in [0, 101]
The 2-th one is in [101, 202]
The 3-th one is in [202, 303]
The 4-th one is in [303, 404]
The 5-th one is in [404, 505]
The 6-th one is in [505, 606]
The 7-th one is in [606, 707]
The 8-th one is in [707, 808]
The 9-th one is in [808, 909]
The 10-th one is in [909, 1010]
The 11-th one is in [1010, 1111]
The 12-th one is in [1111, 1212]
The 13-th one is in [1212, 1313]
The 14-th one is in [1313, 1414]
The 15-th one is in [1414, 1515]
The 16-th one is in [1515, 1616]
The 17-th one is in [1616, 1717]
The 18-th one is in [1717, 1818]
The 19-th one is in [1818, 1919]
The 20-th one is in [1919, 2020]
The 21-th one is in [2020, 2121]
The 22-th one is in [2121, 2222]
The 23-th one is in [2222, 2323]
The 24-th one is in [2323, 2424]
The half one is in [2323, 2424]
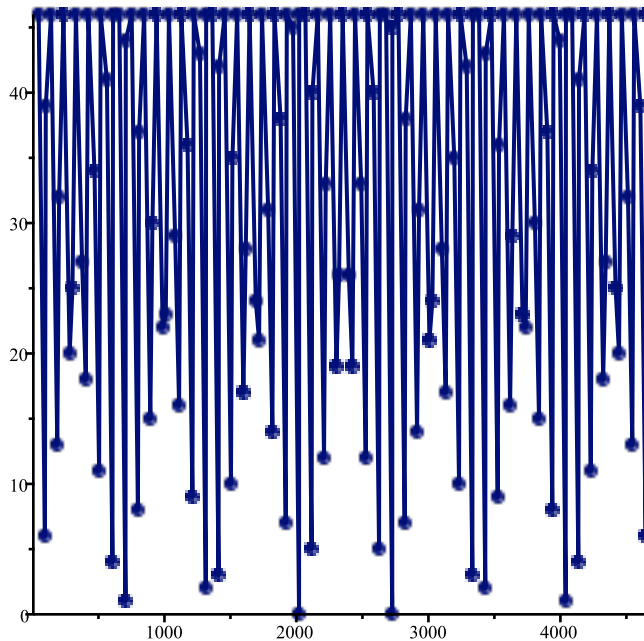
$Theorem2(37, 67)$

All the hosts are list by:
37, 67, 74, 111, 134, 148, 185, 201, 222, 259, 268, 296, 333, 335, 370, 402, 407,

444, 469, 481, 518, 536, 555, 592, 603, 629, 666, 670, 703, 737, 740, 777, 804, 814,
851, 871, 888, 925, 938, 962, 999, 1005, 1036, 1072, 1073, 1110, 1139, 1147, 1184,
1206, 1221, |,1258, 1273, 1295, 1332, 1340, 1369, 1406, 1407, 1443, 1474, 1480,
1517, 1541, 1554, 1591, 1608, 1628, 1665, 1675, 1702, 1739, 1742, 1776, 1809, 1813,
1850, 1876, 1887, 1924, 1943, 1961, 1998, 2010, 2035, 2072, 2077, 2109, 2144, 2146,
2183, 2211, 2220, 2257, 2278, 2294, 2331, 2345, 2368, 2405, 2412, 2442,

r=30

w0=2

m=8

The 1st one is in [67, 134]

The 2-th one is in [201, 268]

The 3-th one is in [335, 402]

The 4-th one is in [469, 536]
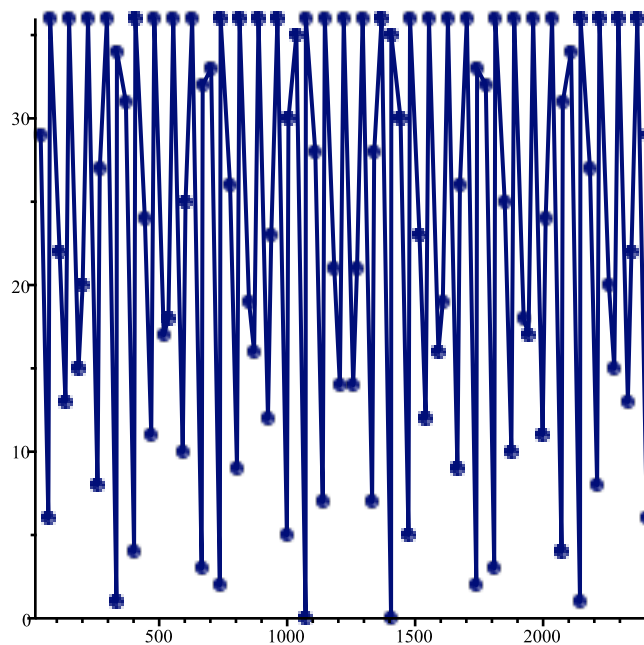
The 5-th one is in [603, 670]

The 6-th one is in [737, 804]

The 7-th one is in [871, 938]

The 8-th one is in [1005, 1072]

The 9-th one is in [1139, 1206]

The half one is in [1206, 1273]

Theorem2(41, 71)

All the hosts are list by:
41, 71, 82, 123, 142, 164, 205, 213, 246, 284, 287, 328, 355, 369, 410, 426, 451,
492, 497, 533, 568, 574, 615, 639, 656, 697, 710, 738, 779, 781, 820, 852, 861, 902,
923, 943, 984, 994, 1025, 1065, 1066, 1107, 1136, 1148, 1189, 1207, 1230, 1271,
1278, 1312, 1349, 1353, 1394, 1420, 1435, |,1476, 1491, 1517, 1558, 1562, 1599,
1633, 1640, 1681, 1704, 1722, 1763, 1775, 1804, 1845, 1846, 1886, 1917, 1927, 1968,
1988, 2009, 2050, 2059, 2091, 2130, 2132, 2173, 2201, 2214, 2255, 2272, 2296, 2337,
2343, 2378, 2414, 2419, 2460, 2485, 2501, 2542, 2556, 2583, 2624, 2627, 2665, 2698,

2706, 2747, 2769, 2788, 2829, 2840, 2870,
r=30
w0=2
m=9
The 1st one is in [71, 142]
The 2-th one is in [213, 284]
The 3-th one is in [355, 426]
The 4-th one is in [497, 568]
The 5-th one is in [639, 710]
The 6-th one is in [781, 852]
The 7-th one is in [923, 994]
The 8-th one is in [1065, 1136]
The 9-th one is in [1207, 1278]
The 10-th one is in [1349, 1420]
The half one is in [1420, 1491]