Journal of Materials Science & Surface Engineering Vol. 3 (4), 2015, pp 303-307

Contents lists available at http://www.jmsse.org/

Journal of Materials Science & Surface Engineering

Relationship between J-integral and CTOD for different materials – A FE Study

Mahammad Sharif Bhudihal[1], Md. Ibrahim Kittur[2] and Krishnaraja G. Kodancha[3*],

[1]*Assistant Professor, Department of Mechanical Engineering, Anjuman Institute of Technology and Management, Bhatkal – 581 320 .*
[2]*P. G. Scholar, Department of Mechanical Engineering, B. V. Bhoomaraddi College of Engineering and Technology, Hubli – 580 031.*
[3]*Professor, Department of Mechanical Engineering, B. V. Bhoomaraddi College of Engineering and Technology, Hubli – 580 031.*

Article history

Received: 19-June-2015
Revised: 21-Aug-2015
Available online: 01-Oct-2015

Keywords:

J-integral ,
CTOD,
Steel,
Constraint Issues,
Finite Element.

Abstract

In this investigation the variation of the J considering Compact Tension (CT) specimen geometry having varying a/W and σ using 2D elastic Finite Element (FE) analysis for different state of stress has been studied. Further the investigation has been done to examine the relationship between the J and δ for varied a/W and σ. The magnitude of d_n has been computed by plastic hinge model for different materials. The plane-stress and plane-strain elastic plastic FE analyses have been conducted on the CT specimen with a/W=0.45-0.65 to extract the J and Crack – tip Opening Displacement (CTOD) values for A302 and HY80 steel. The values of J are obtained from the postprocessor of ABAQUS and CTOD are computed by converting Crack mouth opening displacement (CMOD) to CTOD using plastic hinge model as per ASTM standards. The nature of variation J, CMOD, and CTOD with normalized applied stress (σ/σ_Y) are found to be same in both the materials. The comparative study of variation of d_n with a/W of HY80 and A302 steels with earlier results of IF steel is also carried out. The study clearly infers the effect of yield stress on the variation of magnitude of d_n with reference to a/W ratio. The present analysis infers that while converting magnitude of CTOD to J one need to carefully evaluate the value of d_n depending on the material rather than considering it to be unity.

The work had been presented at an international conference ***Fatigue Durability India 2015****, 28-30th May 2015, JN TATA AUDITORIUM, Indian Institute of Science, Bangalore.*

Introduction

Elastic plastic fracture mechanics (EPFM) is a domain of fracture mechanics, which considers extensive plastic deformation ahead of a crack-tip prior to fracture. It is well known that *J*-integral (*J*) and crack-tip opening displacement, CTOD (δ) can be used as fracture parameters for analysis of fracture problems under EPFM. In EPFM it is required that J and δ should be interchangeable to each other[1]. Thus, it is essential to examine the relationship between J and δ. A well-known general relationship between J and δ given by T.L. Anderson[2] is:

$$J = m\sigma_y\delta \qquad \text{......................(1)}$$

where, σ_y is the yield stress of the material, m is constant and δ is CTOD. Earlier literature[3-5] indicates that the load intensity measured in terms of *J*-integral as a single parameter alone does not describe stress/strain field ahead of the crack-tip uniquely and accurately. Therefore, there is a necessity of introducing a second parameter with *J*, which is required to characterize the crack-tip fields. Interestingly, the constant factor m in the relationship between J and δ given in Eqn. (1) is known to be constraint dependent. Thus m can serve as a parameter to characterize constraints[5]. Shih[6] has shown that the relationship between J and δ can be obtained theoretically by HRR stress field equations[7-8] as:

$$\delta = d_n\frac{J}{\sigma_y} \qquad \text{........................(2)}$$

where, d_n is a constant, which depends on Ramberg-Osgood (R-O) constant n of the material. From Eqn. (1) and Eqn. (2) the relation between m and d_n is:

$$m = \frac{1}{d_n} \qquad \text{....................(3)}$$

Shih[6] has also shown that d_n usually varies between 0.4 to 0.8 for common structural steels and for fully plastic materials (n=∞) d_n=1, which is obtained by extrapolation. Omidvar et al.[9] using closed form solutions have confirmed that their results on relationship between J and δ fully corroborate the results of Shih[6]. The analytical solutions and 2D finite element analysis indicate, the magnitude of d_n is found to be dependent on the strain hardening component (n) of the material and specimen a/W ratio. Shih[6], Omidvar et al.[9] Panontin et al.[10] and Kulkarni et al.[11] have investigated the validity of J as a fracture parameter and the J-δ relationship for the determination of critical CTOD (δ) in predominantly plane stress fracture. As d_n can be used as a constraint parameter, it is required to examine the effect of state of stress and different material on the factor d_n, which can address constraint effects. Chiodo and Ruggieri[12] have argued that a full set of J and CTOD solutions for varying crack geometries and loading modes directly connected to the description of fracture behaviour under large scale yielding condition is still lacking. Enyang Wang et al.[13] have shown that marked variability of the predictive accuracy of Eqn. (1) implies that this equation, which was developed based on 2D plane-strain FEA, is not adequate to predict the CTOD value for 3D Single Edge Tensile (SE(T)) specimens. Work is currently being carried out to develop a more

Corresponding author. Tel: +91 98865 96953
E-mail address: krishnaraja@bvb.edu

accurate equation to predict CTOD from J based on the 3D Finite Element Analysis (FEA) results. Yifan Huang and Wenxing Zhou,[14] have studied finite element analyses with the large-displacement/large-strain formulation performed on clamped SE(T) specimens to evaluate the plastic constraint m factor that relates CTOD to J. The analysis results suggested that the value of m depends on strain hardening component n and specimen configuration (i.e. a/W, B/W and side-grooving). As per the best knowledge of authors there is a need to examine the relationship between the J and δ for varied a/W and σ, on magnitude of d_n for fracture toughness analysis of different materials using EPFM. The major emphasis in this investigation is on the study of constraints for two different materials (HY80 and A302 steels) and comparison with IF steel through different state of stress using ABAQUS software.

The objective of this investigation is to compute J and CTOD for various applied loads using 2D elastic-plastic finite element analyses considering CT specimen geometry of various a/W and different state of stress for two different materials. To examine the relationship between the J and δ for varied a/W and σ, on magnitude of d_n computed by plastic hinge model for different materials.

Finite Element Analysis

The finite element computations were performed using ABAQUS software[15]. A series of 2D elastic plastic FE analyses have been conducted on the CT specimens subjected to various applied load steps and details of the analyses are discussed in this section.

Material and Specimen Geometry

A302 steel and HY80 steel have been considered for the FE analyses. The values of mechanical properties of HY80 and A302 are obtained using pixel method from the work of Joyce and Link[16] and O'Dowd[17].

The geometry of the CT specimen used in this FE analyses is shown in Figure 1 and is in accordance with ASTM standard E1820-13[18]. The width of the specimen W=20mm is used for the analysis and other dimensions are computed accordingly.

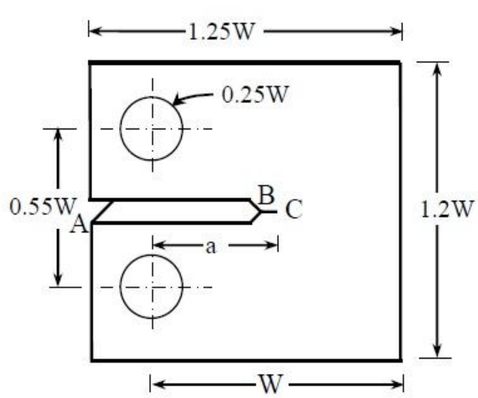

Figure 1: CT specimen geometry

The plastic deformation between two successive points in the model was assumed to be linear with a particular tangent modulus. In the Elastic-plastic analysis the Elastic properties as well as the true stress strain values for both HY80 and A302 steel were fed in to the models, developed in the initial phase. The applied stress (σ) in the analysis is computed with the analytical formulation provided in the work of Priest[19]. Different load steps were imposed during each of the finite element analysis in a manner that the magnitude of normalized applied stress (σ/σ_y) remains in the range 0-0.80. Because of mesh quality, plasticity and constraints the maximum load being limited to σ/σ_y less than or equal to 0.80. For each load step J and Crack Mouth Opening Displacement (CMOD) values were extracted using the postprocessor of ABAQUS software.

Mesh and Boundary Conditions

Finite element computations were carried out considering only one-half of the specimen geometry due to symmetry. The 2D analysis domain is descritized using 8-noded quadrilateral finite elements using reduced integration (element type CPE8R for plane strain and CPS8R for plane stress within the ABAQUS library) are used. The use of these kinds of elements were found in Kim et al.[20]. In this study, very fine mesh was used in the region around the crack tip to achieve better results. Courtin et al.[21] have compared various methods (Empirical expressions, 2D singular elements, Displacement extrapolation and J-integral) to extract stress intensity factor both in 2D and 3D crack configurations and reported that it does not require an excessive mesh refinement since the results obtained with the coarse mesh are in good agreement with fine meshing while using ABAQUS software. Typical FE mesh generated by taking origin at the crack-tip along with boundary condition is clearly depicted in Figure 2.

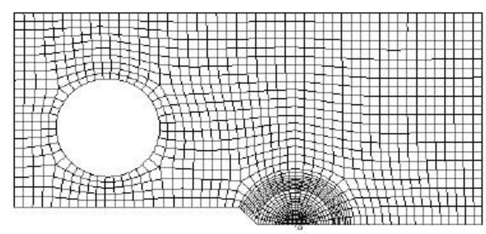

Figure 2: A typical mesh used in the FE analysis for a/W=0.55

Due to half symmetry, the symmetrical displacement boundary conditions have been imposed (u_y=0) along the ligament of the model. The load applied through pin holes is simulated by applying point loads on the circumference of the pin hole, which approximately subtends to 60^0 to keep the loading in mode-I. The nodes of loading pins holes are given displacement boundary conditions having u_x=u_z=0 and $u_y \neq 0$. A series of elastic-plastic stress analyses on compact tensile (CT) specimen (Figure 1.) of thickness 3 mm (plane stress and plane strain with thickness) and a/W=0.45 to 0.65 in steps of 0.05 are carried out for different applied load levels. In these analyses for every load steps, elastic plastic fracture parameters J-integral and CMOD are extracted.

Extraction of J values

In this study, plane-stress and plane-strain elastic plastic FE analyses have been conducted on the CT specimen with a/W=0.45 to 0.65 to extract the J and CTOD values for A302 and HY80 steel. In the present 2D analysis, using domain integral method the ABAQUS software automatically finds user defined five contours in order to carry out J-integrals. It is widely accepted that the first few contour do not provide consistent results because of numerical singularities [21]. Therefore, the magnitudes of first two contours have been neglected in the analyses, in order to get a convergent value of J. The mean value of rest of three contours is computed. ABAQUS provides a procedure for numerical evaluation of the J, based on the virtual crack extension/domain integral methods [22-23]. The method is particularly attractive because it is simple to use and provides excellent accuracy, even with rather coarse mesh.

Crack-tip opening displacement CTOD (δ)

The magnitude of CTOD for various load steps have been estimated by conversion of crack mouth opening displacement (CMOD) to CTOD using rotation factor, which is popularly referred as plastic hinge model and used in experimental fracture analysis [24].

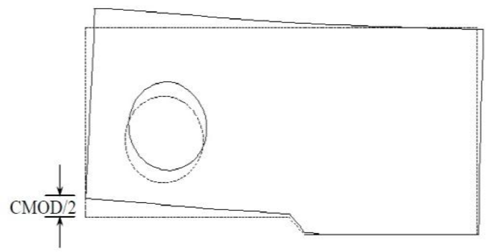

Figure 3: A typical deformed specimen compared to un-deformed specimen

In the plastic hinge model CMOD is converted to CTOD using rotation factor. At each applied load the magnitude of half CMOD is noted from the y displacement of the node at point A (Figure. 3). A typical deformed specimen compared to un-deformed specimen is shown in Figure. 3, which demonstrates the method of obtaining CMOD. The CMOD data obtained from FE results is then used to compute the magnitude of CTOD using a relation given in ASTM E1290[24] as:

$$\delta = \frac{(CMOD).r.b}{a + r.b} \qquad \text{.....................(4)}$$

where r is rotation factor, the value of r according ASTM E1290 varies with specimen a/W ratio and is between 0.44-0.47 for CT specimen, b is the un-cracked ligament and a is the crack length of the specimen. The Eqn. (4) estimates only the plastic part of CTOD, as the investigation is elastic-plastic analysis; the elastic part of CTOD is found to be insignificant and is neglected in the present work. The value of α and r can be obtained by from Eqn. (5) and (6):

$$\alpha = 2\sqrt{\left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right)^2 + \left(\frac{1}{2}\right)} - 2\left[\left(\frac{a}{b}\right) + \left(\frac{1}{2}\right)\right] \qquad \text{.....................(5)}$$

$$r = 0.4(1+\alpha) \qquad \text{.....................(6)}$$

Results and Discussion

The variations of J values of the specimen with normalized applied load, $\sigma/\sigma_y \leq 0.80$, and a/W=0.45 to 0.65 in steps of 0.05 were studied for plane stress and plane strain for A302 and HY80 Steel material. A typical variation of J vs. σ/σ_y for various a/W on CT specimen for A302 and HY80 steel for plane stress condition are shown in Figure. 4 and 5.

From the figures, it is observed that the magnitude of J is independent of material and state of stress up to $\sigma/\sigma_y \approx 0.07$ for various a/W ratio, which indicates the LEFM conditions ahead of crack tip. For $\sigma/\sigma_y > 0.07$, it is observed that the variation of J is nonlinear with applied load demonstrating the regime of EPFM. It is interesting to note from figures that for the same applied load the intensity of J-integral depends on the a/W ratio of the specimen, material and state of stress. For the similar applied loads J is large for the specimen with large a/W ratio. In order to understand the variation of J *vs.* applied load for different materials, specimens with a particular a/W are studied. A typical variation for a/W=0.50 is shown in Figure. 6.

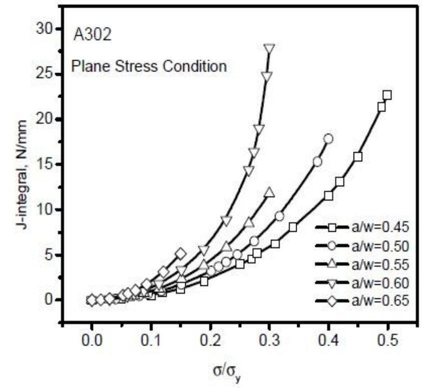

Figure 4: Effect of a/W on the variation of J vs. σ/σ_y for plane stress condition for A302 steel material.

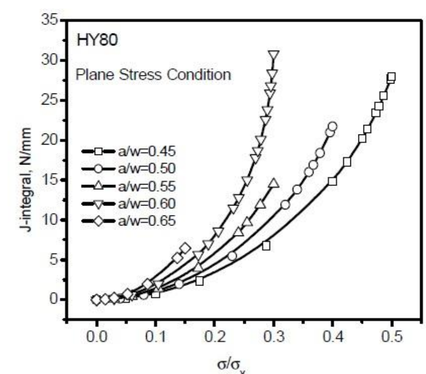

Figure 5: Effect of a/W on the variation of J vs. σ/σ_y for plane stress condition for HY80 steel material.

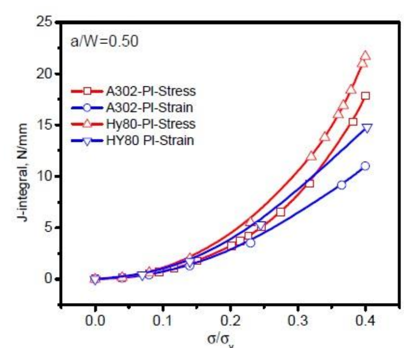

Figure 6: Variation of J vs. σ/σ_y for different state of stress condition for HY80 and A302 steel.

The figure clearly shows the magnitude of J is higher for plane stress condition than plane strain condition. The nature of variation is found to be same in both the materials.

At each applied load the CMOD is noted from the y-displacement of the node at point A (Figure. 1). A typical deformed specimen compared to un-deformed specimen is shown in Figure. 3. The variation of CMOD against the normalized applied load σ/σ_y for different state of stress for A302 and HY80 steel material are studied. A typical variation CMOD vs. σ/σ_y for plane stress condition of A302 and HY80 steel material are shown in Figure. 7 and 8. The results demonstrate that initially CMOD increases linearly and latter increases considerably and nonlinearly, which is attributed to the effect of extensive plastic deformation at crack tip.

The magnitude of linearity and nonlinearity is dependent on material, state of stress and a/W. In plane strain condition the linearity is observed more than the plane stress condition. The nature of variation is observed same in both the materials. As expected the magnitude of CMOD is more in case of plane stress condition than in plane strain condition. A typical variation of CMOD vs. σ/σ_y for different state of stress condition for HY80 and A302 steel for a/W=0.50 is shown in Figure. 9. The CMOD data are used to compute the magnitude of CTOD for different state of stress and material using Eqn. 4, 5 and 6.

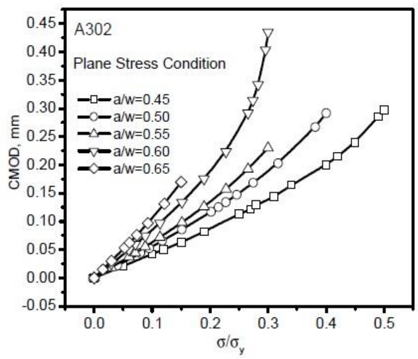

Figure 7: Effect of a/W on the variation of CMOD vs. σ/σ_y for plane stress condition for A302 steel material.

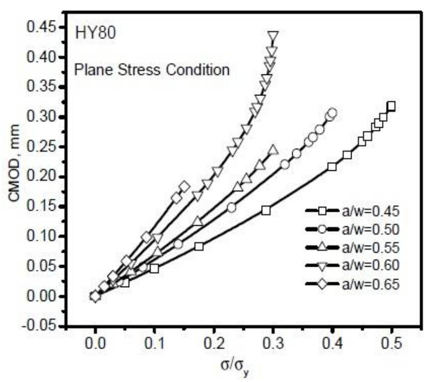

Figure 8: Effect of a/W on the variation of CMOD vs. σ/σ_y for plane stress condition for HY80 steel material

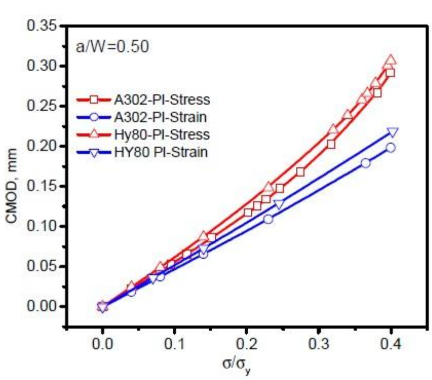

Figure 9: Variation of CMOD vs. σ/σ_y for different state of stress condition for HY80 and A302 steel

A typical variation of CTOD vs. J/σ_y. for plane stress condition of A302 and HY80 steel material is shown in Figure. 10 and 11. From these figures it is clear that the variation of CTOD against J/σ_y is non-linear; but if we neglect the elastic portion as, the variation is found to be linear. The variation of CTOD against J/σ_y is dependent on a/W. The linearity portion of the plot is considered for the estimation of d_n. The magnitude of linearity and nonlinearity is dependent on material, state of stress and a/W.

A typical variation of CTOD vs. J/σ_y for different state of stress condition for HY80 and A302 steel for a/W=0.50 is shown in Figure. 12. The nature of variation is observed to be same in both the materials. In the present investigation the constant d_n in the relationship between δ and J/σ_y (Eqn. 2) is obtained by the slopes of the linear portion of the CTOD and J/σ_y. The variation of a/W vs. d_n for different state of stress condition for HY80 and A302 steel are studied and plotted in Figure. 13. The figure infers that the value of d_n is less in plane strain than plane stress condition in both the materials. This figure clearly shows that the variation of d_n obtained by both materials and state of stress are nonlinear with respect to variation in a/W ratio. It also indicates that there is considerable difference in magnitudes of d_n obtained by both materials and state of stress in estimation of CTOD. The results in the Figure.13. indicate that for specimens with a/W < 0.50 the magnitudes of d_n are higher, and is found to be >1 in both state of stress condition.

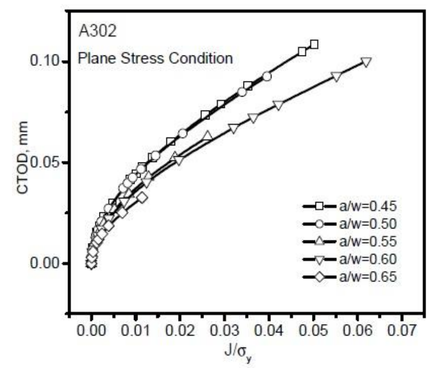

Figure 10: Effect of a/W on the variation of CTOD vs. J/σ_y for plane stress condition for A302 steel material.

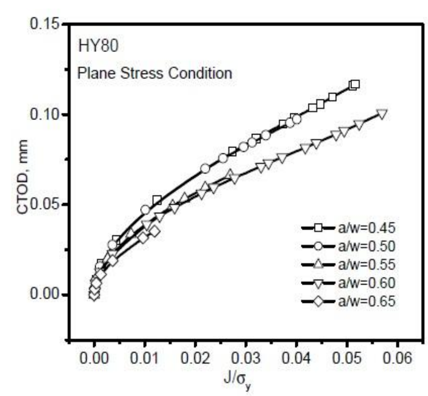

Figure 11: Effect of a/W on the variation of CTOD vs. J/σ_y for plane stress condition for HY80 steel material.

The present results demonstrate that the relation between J and CTOD strongly depends on the material, state of stress and a/W ratio of the specimens. The comparative study of variation of d_n with a/W of HY80 and A302 steels with earlier results of IF steel [1] is also made and shown in Figure. 14. The figure indicates that the value of d_n is much lower in case of IF steel than A302 and Hy80 steels. It clearly infers the effect of yield stress on the variation of magnitude of d_n with reference to a/W ratio as yield stress for IF, A302 and Hy80 are 155MPa, 450MPa and 550MPA respectively. Kulkarni et al.[11] have shown that fracture analysis of thin sheets can be done using critical CTOD, δ_c. In their analysis, critical CTOD was computed by the relation between J and δ suggested by Shih[6]. The thin sheet analysis is considered as fully plastic case and d_n in Eqn. (1) is taken as unity. But, the present analysis infers that while converting magnitude of CTOD to J one need to carefully evaluate the value of d_n depending on the material, state of stress and a/W rather than considering it to be unity.

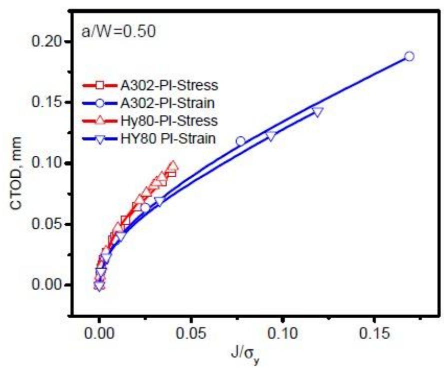

Figure 12: Variation of CTOD vs. J/σ_y for different state of stress condition for HY80 and A302 steel.

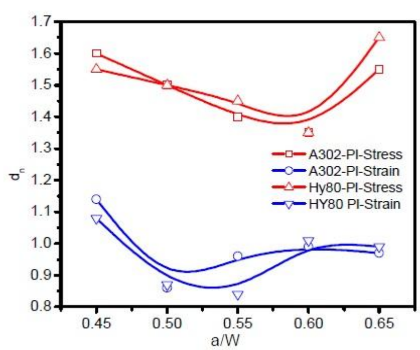

Figure 13: Variation of a/W vs. d_n for different state of stress condition for HY80 and A302 steel.

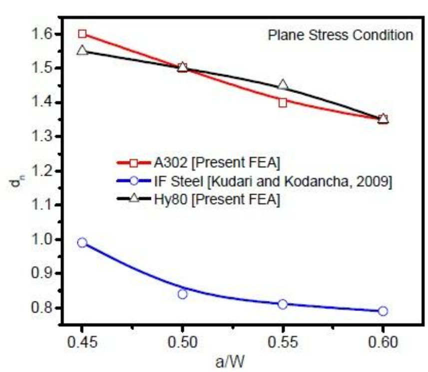

Figure 14: Variation of d_n against a/W for plane stress condition for HY80 A302 and IF steel.

Conclusions

In this investigation the relationship between J and CTOD are studied with respect to different materials, a/W and loading using 2D elastic-plastic finite element analysis. Following conclusions are drawn from the present investigation:

i. For the same applied load the intensity of J-integral depends on the a/W ratio of the specimen, material and state of stress.

ii. The variation of CTOD against J/σ_y is highly dependent on a/W material and state of stress.

iii. The variation of d_n obtained by different materials and state of stress are nonlinear with respect to variation in a/W ratio.

iv. While converting the magnitude of CTOD to J one need to carefully evaluate the value of d_n depending on the material property and a/W rather than considering it to be 1 in EPFM.

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