

Graphical Representation of Conjugacy Classes in the Order – Preserving Partial One – One Transformation Semigroup

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Abstract: Let $X_n = \{1, 2, 3, \dots, n\}$ be the finite set of n objects. The set of all partial transformation of n objects forms a semigroup under the usual composition of functions. It is denoted by T_n when it is full or total, I_n when it is partial one - one and P_n when it is strictly partial. The elements of conjugacy classes of partial one – one transformation were studied. The elements in each conjugacy were represented using two – line notation. Finally the elements were arranged in their respective conjugacy class with the aids of unlabelled graph. The elements with the same graph structures will contained in the same conjugacy class.

Keywords: Semigroup, transformation semigroup, conjugacy, conjugacy classes, graph, order – preserving partial one – one transformation semigroup.

1. Introduction

The full transformation semigroup and some special subsemigroups of T_n have been studied over the last fifty years(See [1],[3],[4], Let $X_n = \{1, 2, 3, \dots, n\}$. Then a (partial) transformation $\alpha : \text{Dom} \subseteq X_n \rightarrow \text{Im} \alpha$ is said to be full or total if $\text{Dom} \alpha = X_n$, otherwise it is called strictly partial.

Howie [5], a mapping in T_n is called order- preserving if for all $i, j \in \{1, 2, 3, \dots, n\}$,

$i \leq j \Rightarrow i\alpha \leq j\alpha$. The semigroup of partial one – one of X_n will be denote by IO_n respectively. In his contribution, Howie [6] investigated that the order IO_n is $|IO_n| = \binom{2n}{n}$ respectively. Richard, F. P. [2] represent

the elements in each conjugacy of Symmetrics group (S_n) and full transformation semigroup (T_n) using one – line notations and the unlabelled graph to described the generalized cycle structure (or “type”) of each element.

2. Preliminaries

2.1 Conjugacy classes of. S_n and T_n

In any group G , elements g and k are conjugates if $g = khk^{-1}$, for some $k \in G$.

The set of all elements conjugates to a given $g \in G$ is called the conjugacy class of g .

In S_n if

$$\pi = (x_1, x_2, x_3, \dots, x_i) (x_m, x_{m+1}, x_{m+2}, \dots, x_n) \text{ in cycle notation, then for any } \sigma \in S_n$$

$$\sigma \pi \sigma^{-1} = (\sigma(x_1), \sigma(x_2), \sigma(x_3), \dots, \sigma(x_i)) (\sigma(x_m), \sigma(x_{m+1}), \sigma(x_{m+2}), \dots, \sigma(x_n))$$

Conjugacy is an equivalence relation, so the distinct conjugacy classes partition G

Consider the full transformation semigroup, T_n which consists of the mapping $X_n \rightarrow X_n$.

Let $\alpha, \beta \in T_n$ and $\sigma \in T_n$. We say $\alpha \sim \beta$ if $\sigma^i(\alpha) = \sigma^j(\beta)$, for some i, j . the relation allows us to decompose into cycles.

2.2 Theorem 1 [2]

The relation $\alpha \sim \beta$ if $\sigma^i(\alpha) = \sigma^j(\beta)$, for some i, j is an equivalence relation

For example, in O_5 , $(\begin{smallmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & - & 3 \end{smallmatrix})$ and $(\begin{smallmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & 3 & - \end{smallmatrix})$ are elements in the same conjugacy class and same graph structures as shown in figure 1.

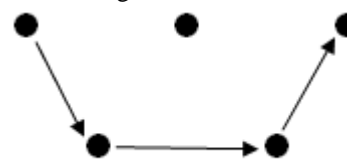


Figure 1

2.2 Directed Graph notation

Consider an element $\alpha \in T_n$ where $\alpha = (i, j, k, \dots, l)$ in one – line notation. Draw n vertices and labeled then i, j, k, \dots, l . Indicate $\pi(i) = j$ by drawing a directed line segment from i to j . For instance, if $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix} \in T_5$, the directed graph is shown in the figure 2.

For instance, if $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \in T_5$, the directed graph is shown in the figure 2

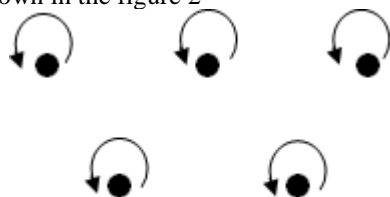


Figure 2

The conjugacy classes of IO_n

The Conjugacy Classes in IO_1 .

The conjugacy class, $C_1 = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$, consists of elements of the form;



The conjugacy class, $C_2 = \left\{ \begin{pmatrix} 1 \\ - \end{pmatrix} \right\}$, consists of elements of the form;



We see that IO_1 has two conjugacy classes.

The Conjugacy Classes in IO_2 .

The conjugacy class, $C_1 = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \right\}$, consists of elements of the form;



The conjugacy class, $C_2 = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 \\ - & 2 \end{pmatrix} \right\}$, consists of elements of the form;



The conjugacy class, $C_3 = \left\{ \begin{pmatrix} 1 & 2 \\ - & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & - \end{pmatrix} \right\}$, consists of elements of the form;



The conjugacy class, $C_4 = \left\{ \begin{pmatrix} 1 & 2 \\ - & - \end{pmatrix} \right\}$, consists of elements of the form;



We see that IO_2 has four conjugacy classes;

The Conjugacy Classes in IO_3 .

The conjugacy class, $C_1 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \right\}$, consists of elements of the form;



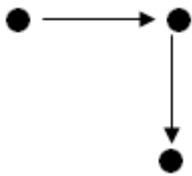
The conjugacy class, $C_2 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & 2 & 3 \end{pmatrix} \right\}$, consists of elements of the form;



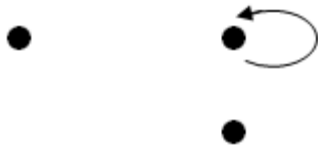
The conjugacy class, $C_3 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & - & 3 \end{pmatrix} \right\}$, consists of elements of the form;



The conjugacy class, $C_4 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ - & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & - \end{pmatrix} \right\}$, consists of elements of the form;

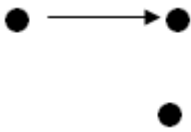


The conjugacy class, $C_5 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & - & 3 \end{pmatrix} \right\}$, consists of elements of the form;



The conjugacy class, $C_6 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ - & 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & - & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & 3 & - \end{pmatrix} \right\}$

consists of elements of the form;



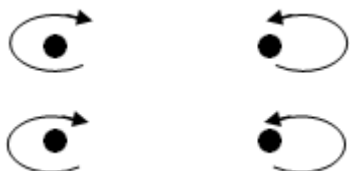
The conjugacy class, $C_7 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ - & - & - \end{pmatrix} \right\}$, consists of elements of the form;



We see that IO_3 has seven conjugacy classes;

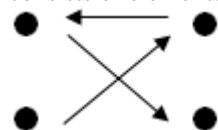
The Conjugacy Classes in IO_4 .

The conjugacy class, $C_1 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \right\}$, consists of elements of the form;



The conjugacy class, $C_2 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & - \end{pmatrix} \right\}$,

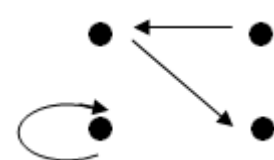
consists of elements of the form;



The conjugacy class, $C_3 =$

$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & - & 4 \end{pmatrix} \right\}$,

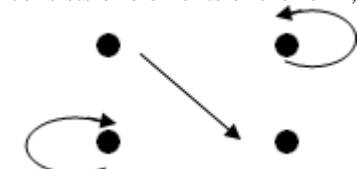
consists of elements of the form;



The conjugacy class, $C_4 =$

$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 3 & 4 \end{pmatrix} \right\}$

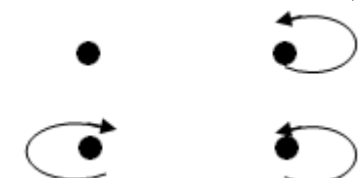
consists of elements of the form;



The conjugacy class, $C_5 =$

$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & - \end{pmatrix} \right\}$,

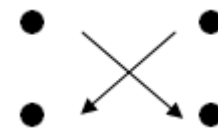
consists of elements of the form;



The conjugacy class, $C_6 =$

$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & - & - \end{pmatrix} \right\}$

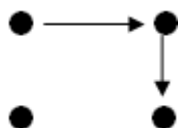
consists of elements of the form;



The conjugacy class, $C_7 =$

$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & - & - \end{pmatrix} \right\}$

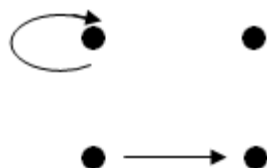
consists of elements of the form;



The conjugacy class, $C_8 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 1 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 2 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & - & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & - & 3 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & - & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 3 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 4 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & - & 2 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & - & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & - & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & - & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 2 & - \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 4 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 3 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & - & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & - & - \end{pmatrix} \right\}$$

consists of elements of the form;



The conjugacy class, $C_9 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & - & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 3 & - \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & - & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 3 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & - & - \end{pmatrix} \right\}$$

consists of elements of the form;



The conjugacy class, $C_{10} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & - & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & - & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 1 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & - & 2 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 2 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & - & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & - & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & - & - \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & - & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 4 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 4 & - & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & - & - & - \end{pmatrix} \right\}$$

consists of elements of the form;



The conjugacy class, $C_{11} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & - & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & - & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 3 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & - & 4 \end{pmatrix} \right\},$$

consists of elements of the form;



The conjugacy class, $C_{12} = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & - & - \end{pmatrix} \right\}$, consists of elements of the form;



We see that IO_4 has twelve conjugacy classes;

The Conjugacy Classes in IO_5 .

The conjugacy class, $C_1 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \right\}$, consists of elements of the form;



The conjugacy class, $C_2 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 3 & 4 & 5 \end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & 3 & 4 & 5 \end{pmatrix} \right\}$$

consists of elements of the form;

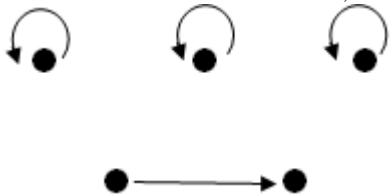


The conjugacy class, $C_3 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & - & 5 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 2 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & - & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 3 & 4 & 5 \end{pmatrix} \right\}$$

consists of elements of the form;

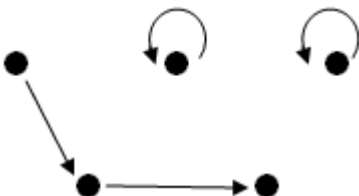


The conjugacy class, $C_4 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 2 & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 5 & - \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & - & 4 & 5 \end{pmatrix} \right\}$$

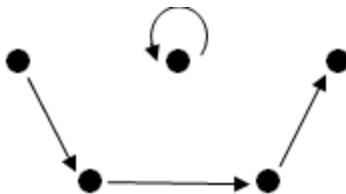
consists of elements of the form



The conjugacy class, $C_5 =$

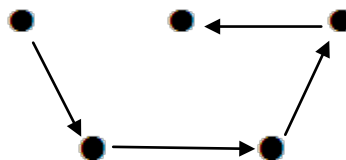
$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & - & 5 \end{pmatrix} \right\}$$

consists of elements of the form;



The conjugacy class, $C_6 =$

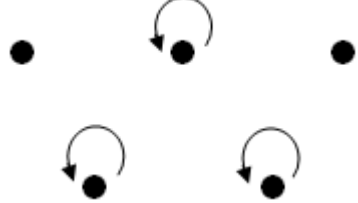
$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & - \end{pmatrix} \right\}$ consists of elements of the form;



The conjugacy class, $C_7 =$

$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 3 & 4 & - \end{pmatrix} \right.$
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 3 & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & - & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & 3 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & 3 & - & 5 \end{pmatrix} \right.$
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & - & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 3 & 4 & 5 \end{pmatrix} \right\}$

consists of elements of the form;



The conjugacy class, $C_8 =$

$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & - & 4 \end{pmatrix} \right.$
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 2 & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & - & 2 & 5 \end{pmatrix} \right.$
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 3 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 3 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & - & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & - & 3 & 5 \end{pmatrix} \right.$
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 3 & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & - & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 4 & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & 4 & 5 \end{pmatrix} \right.$
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & 3 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 3 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 3 & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & - & 3 & 5 \end{pmatrix} \right.$
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & - & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 2 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & 4 & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & - & - & 4 & 5 \end{pmatrix} \right.$
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 3 & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 2 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & - & 4 & - \end{pmatrix} \right.$
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & 3 & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 3 & - & 4 & 5 \end{pmatrix} \right\}$

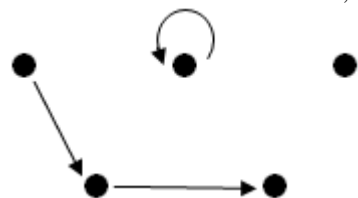
consists of elements of the form;



The conjugacy class, $C_9 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 3 & - & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 2 & 3 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & - & 2 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & 4 & - \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & - & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & - & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & - & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & - & - \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & - & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 4 & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & - & 4 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & - & 3 & 4 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 2 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & - & - & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & 4 & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & - & 4 & - & 5 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & 2 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & - & - & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 3 & 4 & - & 5 \end{pmatrix} \right\}$$

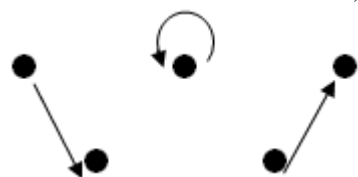
consists of elements of the form;



The conjugacy class, $C_{10} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & - & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 2 & - & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & 2 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 2 & 5 & - \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & - & - & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 3 & - & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & - & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & 3 & 5 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 3 & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 5 & - & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 4 & - & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 3 & - & 4 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & - & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 3 & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 4 & - & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & - & - & 5 \end{pmatrix} \right\}$$

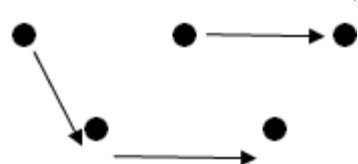
consists of elements of the form;



The conjugacy class, $C_{11} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & 2 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & - & 4 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 4 & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & - & - & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & - & 3 & 4 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & - & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 4 & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & - & - \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & - & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & - & 5 & - \end{pmatrix} \right\}$$

consists of elements of the form;



The conjugacy class, $C_{15} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & 3 & - \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 2 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 2 & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & - & - & - \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & - & 4 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 3 & 4 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 3 & 4 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & - & 5 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 3 & 5 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 3 & - & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & - & - & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 4 & 5 & - & - \end{pmatrix} \right\}$$

consists of elements of the form;



The conjugacy class, $C_{16} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & 3 & - \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 4 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & - & 4 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & - & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & - & - & 3 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 4 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & - & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 2 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 5 & - & - \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 2 & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & - & - & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 3 & - & - & 4 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 5 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & - & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & - & - & - \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & - & - & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & - & 5 & - & - \end{pmatrix} \right\}$$

consists of elements of the form;

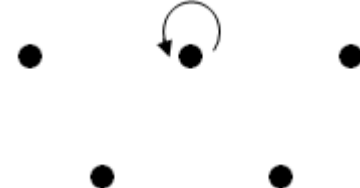


The conjugacy class, $C_{17} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 3 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 4 & - \end{pmatrix} \right\}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & - & 5 \end{pmatrix}$$

consists of elements of the form;



The conjugacy class, $C_{18} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 1 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & - & 1 \end{pmatrix} \right.$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 2 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & - & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & - & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 4 & - & - & - \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & - & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 5 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 5 & - & - \end{pmatrix}$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & - & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 3 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 4 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 5 & - \end{pmatrix} \right\}$$

consists of elements of the form;



The conjugacy class, $C_{19} = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & - & - \end{pmatrix} \right\}$,

consists of elements of the form;



Thus, we see that IO_5 has nineteen conjugacy classes.

4. Result

The summary of the sequence on the number of conjugacy classes of Order - preserving partial one - one transformation semigroup is **2,4,7, 12,19, . . .** A000070 of the Online Encyclopedia of Integer Sequences (OEIS) [7].

5. Conclusion

It was discovered that the transformation semigroup can be understood better in nature and properties with the aids of graphs and the unlabelled graph can be effectively use to determine the conjugacy class of Order - preserving partial one - one transformation semigroup.

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