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Multiple Descriptions Video Coding Using Coinciding Lattice Vector Quantizer for H.264/AVC and Motion JPEG2000

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Abstract: Widespread use of networked multimedia applications emphasizes the need for robust communication protocols that can provide fault tolerance and error resiliency. Multiple-description (MD) coding is a technique used to mitigate channel failure when retransmission is expensive or impossible. Multiple descriptions lattice vector quantization has become a popular choice for robust data transmission over unreliable network channels. In this paper, a new generic MD video coding scheme is proposed that is based on coinciding similar sublattices of the hexagonal lattice. The proposed video coding scheme has been exploited to form new MD video coding schemes: MDCLVQ-H.264/AVC and MDCLVQ-Motion JPEG2000. The experimental results for several reference videos show that the encoding performance of the scheme improves, while the quality of the videos in terms of PSNR of the central decoder remains acceptable. Thus, the error resiliency of the scheme improves without a significant drop in the quality of the reconstruction.

Key words: H.264/AVC · Multiple Descriptions Lattice Vector Quantization · MDLVQ · Multiple Description Coding and Wavelet transform

INTRODUCTION

Recent advances in high-performance portable processing equipment, such as mobile processors, have enabled users to experience new-generation devices, including networked gaming consuls, smart televisions and smart phones. Video coding, video compression and video communication are essential parts of the aforementioned applications. However, networking infrastructures do not offer unlimited bandwidth and storage devices do not offer unlimited capacities. Therefore, there is significant demand for reliable high-performance video communication/compression protocols. Video compression refers to the process of reducing the amount of video data used to represent digital videos; it is a combination of spatial image compression and temporal motion compensation [1].

Multiple description (MD) coding has appeared to be an attractive channel coding scheme to decrease the impact of network failures and increase the robustness of multimedia communications [2]. The MD coding is especially useful for those applications in which retransmission is not possible or is too expensive. Lattice vector quantization (LVQ) is a well-known lossy compression technique for data compression. LVQ is used for spatial compression and is less computationally complex due to the regular structure of the lattice [3]. MD image coding has been presented in several studied [4-6].

In [7], an MD video coder is presented that uses motion-compensated predictions. This MD video coder utilizes MD transform coding and three separate prediction paths at the encoder. Another MD video coding technique is introduced in [8]. In this scheme, the 3D Set Partitioning in a hierarchical tree (3D-SPIHT) algorithm is used to modify the traditional tree structure. Multiple description video coding based on LVQ was presented in [9]. In that study, MDLVQ is combined with the wavelet transform to produce a robust video coder. An error-resilient video coding scheme using the MD technique is proposed in [10] that employs MDLVQ with

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channel optimization (MDLVQ-CO). In that study, the central codebook is locally trained according to known channel erasure statistics and two translated lattices are used for side codebooks to reduce distortion in the case of erasure.

In [11], MD video coding has been used to form an authentication scheme for Motion JPEG2000 [12] streaming in a lossy network. In this study, the video frames are first transcoded into media fragments that are protected with integrity tokens and a digital signature. Then, the integrity tokens and signature are encoded into codewords using the forward error correction (FEC) for data loss resilience. Because of the use of MD video coding and sequence numbers, the scheme provides content integrity and defeats collage attacks.

In [13], two MD video coding schemes are proposed, i.e., drift-compensation multiple description video coder (DC-MDVC) and an independent flow multiple description video coder (IF-MDVC). An interesting feature of DC-MDVC is the ability to use the actual reconstructed frame as the new reference frame for the side prediction loops instead of the original frame.

In [14], a second-order predictor is inserted in the encoder prediction loop. In [15], a new MD video coding scheme for the H.264/AVC standard [16] is proposed based on multiple description scalar quantization. In that study, a splitting block is inserted in the standard H.264/AVC scheme after quantization. This block generates two descriptions by duplicating the control structures, picture parameter set, slice header and information about the motion vectors. The MD coding video scheme presented in [17] splits the video information into several encoding threads and redundant pictures are inserted to reduce the error drift packet loss that occurs. In [14], a novel RD model for H.264 video encoding in a packet loss environment is proposed. In that study, the end-to-end distortion is estimated by inserting a block-based distortion map to store the potential errors of the current frame that may propagate to the future frames. This scheme is intended for low-delay applications over lossy networks.

To the best of our knowledge, no studies have been presented with generic MD video coding using LVQ that do not rely on specific properties of the video coding scheme. The IF-MDVC is designated for transmission in environments in which the packet loss rate is directly proportional to the packet size and thus having more descriptions with a smaller packet size at the same bit-rate is advantageous [13]. The MD coding scheme in [17] requires the channel conditions to allocate the coding rate to primary and redundant pictures, to minimize the total distortion experienced at the receiver. The study described in [18] requires that the potential channel distortions of its reference frames be known a priori. In addition, the descriptions generated by the MD scheme presented in [19] based on redundant H.264 pictures are not independent; thus the decoder cannot take advantage of all of the available information.

In this paper, a new generic MD video coding based on the coinciding similar sublattices of the hexagonal lattice is presented. The coinciding similar sublattices are special sublattices because they have the same index; even though they are generated by different generator matrices [5].The proposed multiple descriptions coinciding lattice vector quantization (MDCLVQ) video coding scheme forms a diversity system for MD video coding that can be exploited for any video coding standard such as H.264/AVC or Motion JPEG2000.

The main objective of this research is to design and implement a generic MD video coding scheme that can be adopted by different kinds of video coding standards. In other words the proposed scheme does not rely on specific characteristics of the video coding standard being used and does not make any assumption about the state of the channel or probability distribution of error. In this way, the proposed scheme is different from the MD video coding schemes presented in [11, 13, 14] because it is not targeted for any specific video coding scheme and does not rely on the properties of the video encoding being used. Thus, it can effectively increase the performance of the encoding system in terms of compression efficiency and reliability, regardless of the encoding scheme being used. In addition, the proposed scheme does not require significant hardware or software changes; therefore, it can be adopted for previous hardware and software designs. Although running parallel H.264/AVC or Motion JPEG2000 encoders requires more computational resources than the original encoder, the total computation time of the MD encoders is less than that of the original because the entropies of the side videos are significantly decreased by the MDCLVQ and thus, the time required by of the arithmetic coding is decreased.

Extensive simulations and the experimental results of applying the proposed MD coding scheme to several reference QCIF video sequences demonstrate that our MD coding algorithm outperforms state-of-the-art single- and two-description video coding schemes in terms of the compression ratio and transmission robustness. In addition, small differences between the peak signal to noise ratio (PSNR) of the side video sequences compared to the central decoder show significant capability to resist channel failures. Finally, the proposed MD video coding scheme provides acceptable fidelity criteria in terms of PSNR, which means that there is a negligible drop in the quality of the video and a significant increase in error resiliency and compression efficiency. The rest of this paper is organized as follows. In Section 2, the definitions of the coinciding similar sublattices are presented and the labeling function for the MDCLVQ scheme is described. The proposed multiple description schemes of H.264/AVC (MDCLVQ-H.264/AVC) and Motion JPEG2000 (MDCLVQ-Motion JPEG2000) are presented in Section 3. The experimental results are included in Section 4 and Section 5 concludes the paper.

MDCLVQ Video Coding Scheme: In MD coding, the input data are transformed into several different descriptions. These descriptions are sent over different channels. At the receiver, if all of the descriptions are received correctly, the original data can be reconstructed accurately. However, if some of the descriptions fail to reach the destination, the rest of the descriptions are used in the side decoders to estimate the original data. In this section, the proposed MDCLVQ video coding scheme is presented. The MDCLVQ video coding scheme is based on coinciding similar sublattices of the hexagonal lattice (A_2). The coinciding similar sublattices of A_2 are defined as geometrically similar sublattices of the hexagonal lattice with the same index but are generated by different generator matrices [5]. Because MDCLVQ is a generic scheme, it applies to almost every video coding standard; however, in this research, it has only been applied to H.264/AVC and Motion JPEG2000. Thus, two MD video coding schemes, MDCLVQ-H.264/AVC and MDCLVQ-Motion JPEG2000, are defined. These MD video coding scheme are very similar and the only difference between them is the video encoding/decoding standard used. In other words, MDCLVQ-H.264/AVC uses the H.264/AVC video encoder, whereas MDCLVQ-Motion JPEG2000 uses the Motion JPEG2000 video encoder. Therefore, both schemes are shown in a single schematic diagram in Figure 1. The MDCLVQ video coding scheme can be described in six steps.

Step 1) Wavelet Decomposition: The input video file is imported as a structure that keeps an array with dimensions of height-by-width-by-three-by-frame for every movie. Therefore, it is possible to consider a video as a four dimensional array in which the first three dimensions serve as spatial directions of the moving pictures and the fourth dimension represents the time domain. In this way, a frame is defined as a set of all pixels that correspond to a single moment. In the wavelet module every plane is decomposed into several wavelet subband coefficients. In this module, the biorthogonal Cohen-Daubechies-Feauveau (CDF) 5/3 wavelet transform with a lifting implementation and one level of decomposition is used. Finally, the wavelet coefficients in the sub-bands are formed into two-dimensional vectors and streamed to the LVQ module.

Step 2) Lattice Vector Quantizer (LVQ): An L-dimensional lattice Λ in $\mathbb{R}^L$ consists of all integer linear combinations of the basis {$b_1, b_2, ..., b_L$ of R^L [20]. Thus it is denoted by $\Lambda = \langle b_1, b_2, ..., b_L$. The lattice points are usually generated using a generator matrix. The generator matrix is composed of the basis vectors of the lattice. The A_2 or the hexagonal lattice is generated by the basis vectors $\{1, \omega = -1/2 + i\sqrt{3/2}\} \subset \mathbb{C}$ [3]. Thus, it is generated by:

$$G = \begin{pmatrix} 1 & 0 \\ -1/2 & \sqrt{3/2} \end{pmatrix} \quad (1)$$

In the LVQ module, the two-dimensional vectors constructed in *Step 1* are quantized to the nearest neighboring A_2 lattice points using the fast quantizing algorithm. In this algorithm, the two-dimensional input vectors are mapped into 3D vectors by a transformation matrix. Then, the 3D vectors are mapped to the lattice

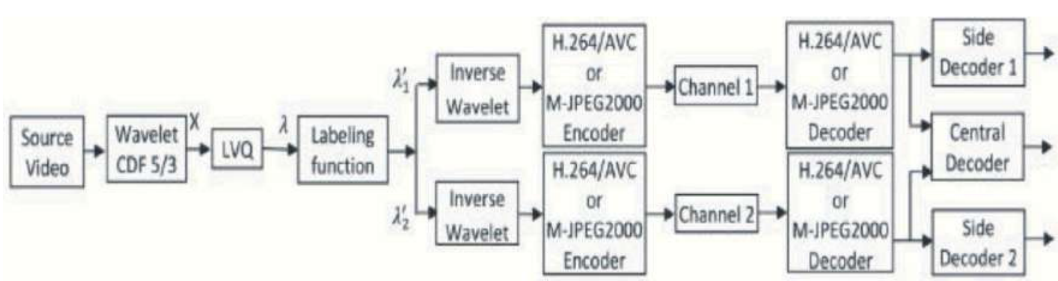

Fig. 1: MDCLVQ video coding scheme applied to H.264/AVC and Motion JPEG2000

Table 1: Different values of α and β and the generator matrices of the coinciding similar sublattices of the A_2 with index N=7

α	β		α	β		α	β	
-3	-2	$G_1 = \begin{pmatrix} -2.0 & -\sqrt{3} \\ 2.5 & -\sqrt{3}/2 \end{pmatrix}$	-1	-3	$G_5 = \begin{pmatrix} 0.5 & -3\sqrt{3}/2 \\ 2.0 & \sqrt{3} \end{pmatrix}$	2	-1	$G_9 = \begin{pmatrix} 2.5 & -3\sqrt{3}/2 \\ -0.5 & 3\sqrt{3}/2 \end{pmatrix}$
-3	-1	$G_2 = \begin{pmatrix} -2.5 & -\sqrt{3}/2 \\ 2.0 & -\sqrt{3} \end{pmatrix}$	-1	2	$G_6 = \begin{pmatrix} -2.0 & \sqrt{3} \\ -0.5 & -3\sqrt{3}/2 \end{pmatrix}$	2	3	$G_{10} = \begin{pmatrix} -0.5 & 3\sqrt{3}/2 \\ -2.5 & -\sqrt{3}/2 \end{pmatrix}$
-2	-3	$G_3 = \begin{pmatrix} -0.5 & -3\sqrt{3}/2 \\ 2.5 & \sqrt{3}/2 \end{pmatrix}$	1	-2	$G_7 = \begin{pmatrix} -2.0 & -\sqrt{3} \\ 0.5 & 3\sqrt{3}/2 \end{pmatrix}$	3	1	$G_{11} = \begin{pmatrix} -2.5 & \sqrt{3}/2 \\ -2.0 & +\sqrt{3} \end{pmatrix}$
-2	1	$G_4 = \begin{pmatrix} -2.5 & \sqrt{3}/2 \\ 0.5 & -3\sqrt{3}/2 \end{pmatrix}$	1	3	$G_8 = \begin{pmatrix} -0.5 & 3\sqrt{3}/2 \\ -2.0 & -\sqrt{3} \end{pmatrix}$	3	2	$G_{12} = \begin{pmatrix} 2.0 & \sqrt{3} \\ -2.5 & \sqrt{3}/2 \end{pmatrix}$

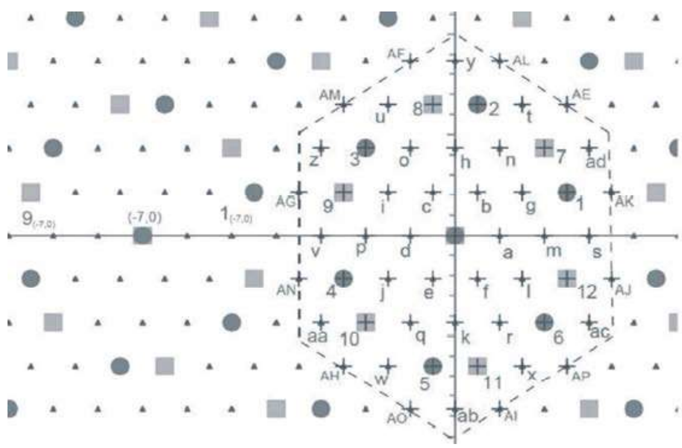

Fig. 2: The labeling scheme for N=7. The lattice, the first sublattice and the second sublattice are shown with triangles, circles and squares respectively. The SuperVoronoi set of the origin is shown with a dashed hexagon [5]

points by simple arithmetic manipulations [21]. The fundamental area of the lattice controls the granularity of the quantization. The fundamental area of the lattice is usually changed by multiplying a constant to the transformation matrix, while the shape of the lattice remains unchanged.

Step 3) Coinciding Labeling Function: In the labeling function, two independent descriptions of the video stream are produced. The labeling function can be considered as a requantization process in which the quantized input is mapped to a label. The label is composed of two sublattice points that are selected from the two coinciding similar sublattices of A_2; thus, it is called the coinciding labeling function. Assume that Λ is a hexagonal lattice with generator matrix G and that Λ' is a sublattice of Λ with generator matrix G'. For the hexagonal lattice, a sublattice Λ' is similar to Λ if $N = \alpha^2 - \alpha\beta + \beta^2$ for $\alpha, \beta \in Z$. In addition, *N* must be the sum of the number of points at the squared distance from the origin, i.e., *N* = 7, 13, 19, 37… [22]. The index $N = \sqrt{det \Lambda' / det \Lambda} = det\, G' / det\, G$ determines the coarse degree of the sublattice, which can control the amount of redundancy in the MD coder [23]. If these conditions are met then the generator matrix of the sublattice is given as follows:

$$G' = \begin{pmatrix} \text{Real}(\alpha+\beta\omega) & \text{Image}(\alpha+\beta\omega) \\ \text{Real}((\alpha+\beta\omega)\omega) & \text{Image}((\alpha+\beta\omega)\omega) \end{pmatrix} \quad (2)$$

There are 12 combinations of integer values that satisfy these conditions for *N =7*. Thus, 12 generator matrices are calculated using different values of α and β. The values of α and β and the generator matrices corresponding to *N=7* are provided in Table 1. However, there are only two distinct similar sublattices with the same index because the sublattices form two groups. In each group, there are 6 sublattices that overlap with each other. The same scenario occurs for *N=13, 19* and *37.* The two coinciding similar sublattices also overlap with each other every *N* lattice point in every direction.

The coinciding similar sublattices partition the space into several SuperVoronoi regions. The SuperVoronoi region of an overlapping sublattice point is the set of all lattice points that are closer to it than any other overlapping point [5]. The coinciding similar sublattices of the hexagonal lattice with index *N=7* and the SuperVoronoi region of the origin are plotted in Figure 2. The lattice, the first sublattice and the second sublattice are shown with triangles, circles and squares respectively. The SuperVoronoi region of the origin is shown with a dashed hexagon.

In the MDCLVQ scheme, the labeling function explained is used to map every hexagonal lattice point λ to a label that is defined as a directed edge $\vec{e}=\langle\lambda'_1,\lambda'_2\rangle$, which consists of two sublattice points that are within the same Super Voronoi region as λ. The parallelogram law is used as the rule for assigning the labels to points. According to this law, the algorithm chooses the shortest label that is as close as possible to the lattice point [23]. In other words, the sublattice points within each SuperVoronoi region compose the labels for the lattice points within the same SuperVoronoi region. Therefore, both vertices of the label will be in the vicinity of the original lattice point. Thus, in case one of the descriptions is lost, the side decoder can reconstruct an acceptable approximate point, while the side distortions are kept as low as possible. This is an important feature of the coinciding similar sublattices that results in a better prediction of the lost description and improved performance of the reconstruction in the side decoders [5]. The summary of the labeling scheme for some of the lattice points within the SuperVoronoi set of the origin is provided in Table 2. For more information, see [5].

Step 4) Inverse Wavelet Transforms: In this step two different video streams are produced. As shown in Figure 1 the side video streams are generated by performing the inverse wavelet transform on the streams generated by the labeling function. The inverse wavelet module uses the biorthogonal inverse lifting CDF 5/3 wavelet transform. Finally, the side videos are sent to the corresponding H.264/AVC or Motion JPEG2000 codecs.

Step 5) Video Encoders/Decoders: The videos are encoded in the encoder modules using the video coding standards and the user can choose either the H.264/AVC or Motion JPEG2000. In this work, the JVT reference software encoder is used for H.264/AVC, while the Open JPEG library is used for Motion JPEG2000. The bit streams are sent over the channels and are decoded using the corresponding decoders.

Step 6) The Decoders: The received video data streams are decoded by their respective video decoders. The video decoders are the H.264/AVC and the Motion JPEG 2000 decoders. These are the reverse of the encoding process that is performed in H.264/AVC and Motion JPEG 2000, respectively. As shown in Figure 1, there are two different types of decoding processes in this scheme: the MD coding decoders and video decoding process. The MD coding decoders have two side decoders and one central decoder. At the receiver, if both descriptions are received correctly, then the central decoder is used to map the labels back to the data points. The side decoders are used to find an approximation of the original lattice point if only one description is received.

Experimental Results: In this subsection, the experimental results for the proposed schemes, *MDCLVQ-H.264/AVC* and *MDCLVQ-Motion JPEG2000,* are provided. As stated before, to the best of our knowledge current MD video coding schemes focus on implementing the scheme based on a certain video coding standard and there is no generic MD video scheme presented in the literature that can be used for arbitrary video coding schemes. However, in this study, the main target was to implement an MD video coding scheme that could be exploited as a preprocessing module for any video coding standard to avoid significant hardware/software changes required by the previous MD schemes.

Therefore, the experimental simulations are divided into several modes. In the first simulation mode, the compression efficiencies of the proposed schemes are assessed. In these experiments, the video descriptions generated by the MDCLVQ scheme are encoded by

Table 2: Some of the lattice points within the SuperVoronoi set of the origin with their corresponding partitions and labels for.

Λ	(λ_x, λ_y)	$[m_x, m_y]$	e	λ	(λ_x, λ_y)	$[m_x, m_y$	e
a	(1,0)	[1,0]	<2,11>	AO-AE	$(2.5,1.5\sqrt{3})$	$[2.5,1.5\sqrt{3}]$	<5,7>
b	$(0.5,0.5\sqrt{3})$	$[0.5,0.5\sqrt{3}]$	<3,12>	AP-AF	$(-1,2\sqrt{3})$	$[2.5,2\sqrt{3}]$	<6,8>
c	$(-0.5,0.5\sqrt{3})$	$[3.0,0.5\sqrt{3}]$	<4,7>	AK-AG	$(-3.5,0.5\sqrt{3})$	$[0,0.5\sqrt{3}]$	<1,9>
d	(-1,0)	[2.5,0]	<5,8>	AL-AH	$(-2.5,-1.5\sqrt{3})$	$[1,2\sqrt{3}]$	<2,10>
e	$(-0.5,-0.5\sqrt{3})$	$[3.0,3\sqrt{3}]$	<6,9>	AM-AI	$(1,-2\sqrt{3})$	$[1,1.5\sqrt{3}]$	<3,11>

H.264/AVC and Motion JPEG2000 video encoders. Then, the average bit-rates used by the *MDCLVQ-H.264/AVC* are compared with the original single description videos encoded by the same video encoders. In addition the volume of the motion jpeg files of the side videos generated by the *MDCLVQ-Motion JPEG2000* are compared with the volume of the motion jpeg of the original videos. The performance of the MDCLVQ schemes are investigated based on its bit-rate efficiency of the *MDCLVQ-H.264/AVC* and compression efficiency of the *MDCLVQ-Motion JPEG2000*. The video encoders JM9.4 H.264/AVC and Open JPEG V1.3 have been used. In these experiments, all of the parameters are set to the standard default settings.

Bit-Rate Efficiency Analysis for MDCLVQ-H.264/AVC: The standard video sequences of “Akiyo”, “Carphone”, “Coastguard” , “Foreman” and “Miss America” in the QCIF format (144×176 pixels) with 7.5 fps are chosen to evaluate the benefits of the proposed MDCLVQ-H.264/AVC scheme over the original single description scheme in terms of bit-rate efficiency and error resiliency. The default settings of the JVT reference JM17.2 H.264/AVC are used to encode the original videos and the side videos [8]. One of the most important factors of any MD video coding system is the average required bit-rate, which is controlled by several factors, such as the fundamental area of the hexagonal lattice and the index of the coinciding similar sublattices. In other words, the required bit-rate depends on the two other factors because increasing or decreasing these factors significantly affects of the required bit-rate and the reconstruction quality. The fundamental area of the lattice affects the quantization error because it changes the density of the lattice points in the quantization plane and thus the size of the codebook used by the quantizer. The size of the codebook controls the required bit-rate for encoding the video. In this study the fundamental area of the hexagonal lattice is changed by *sigma*. The variable *sigma* is used by the fast quantizing algorithm to affect the generator matrix of the lattice being used. The fundamental area of the hexagonal lattice is changed by *sigma* = 0.1, 0.2…, 1, while maintaining the shape of the hexagonal lattice. Thus, the granularity of the quantization is adapted. Therefore central distortion, side distortions and their associated bit-rates are affected.

The other variable that influences the MD coding performance is the index of the hexagonal lattice being used. The index affects the number of lattice points within the SuperVoronoi region of the lattice and thus changes the redundancy of the MD coding scheme. According to [5], the labeling algorithm with index $N=7$ offers the best reconstruction quality; thus in this study, only index $N=7$ is used. Then, the bit-rate is calculated at the end of every encoding run. Therefore, the analyses provided below are based on different values of *sigma*.

The main target of all video encoding schemes is to decrease the required bit-rate for transmission of the video data. Therefore, it is important to check the bit-rate efficiency of the proposed MDCLVQ-H.264/AVC scheme. The required bit-rates for encoding the side videos are provided in Table 3 as well as the bit-rate required to encode the standard video. The video encoder uses an average of R (b/s) to encode a single description video. The MD video scheme uses R_1 and R_2 (b/s) to encode side video 1 and side video 2, respectively. Thus, the bit-rate efficiency of the proposed MD video scheme over the standard single description scheme is calculated as follows:

$$R_{eff} = \frac{R-(R_1+R_2)}{R} \tag{6}$$

The bit-rate efficiency R_{eff} is defined so that the performance of the proposed MD coding scheme is compared with the original video encoder in terms of the required bit-rate. Thus, if the MD video coding scheme can encode both side videos with total bit-rate $(R_1+R_2) \leq R$ then the bit-rate efficiency R_{eff} will be positive and the MD coding is outperforming the original encoder. However, if $(R_1+R_2) > R$ the bit-rate efficiency R_{eff} will be negative and it indicates that compression efficiency has been sacrificed for gaining error resiliency.

The bit-rate efficiencies are provided in Table 3 as well. In all experiments, the videos are encoded at 7.5 (f/s). It is observed from Table 3 that the maximum bit-rate efficiencies are obtained when the *sigma* is 0.1 and the minimum bit-rate efficiencies are obtained when *sigma* is 1. Thus, for small *sigma* values, the algorithm shows better performance in terms of the compression efficiency. However, the compression efficiency decreases as the value of *sigma* increases. As shown in Table 3, the bit rate efficiencies of the proposed MDCLVQ-H.264/AVC scheme corresponding to *sigma* 0.1 to 0.7 for all the videos but the “Miss America” are positive; therefore, error resiliency and compression efficiency are both obtained for these videos and with the *sigma* lower than 0.7. However, for *sigma* 0.8 to 1 the compression efficiency is not achieved.

Table 3: Average bit-rate (kb/s) required by the reference JM encoder to encode several descriptions produced by the MDCLVQ-H.264/AVC

Video	Sigma	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Akiyo	R_1	0.55	0.57	0.63	0.88	1.77	2.13	2.34	3.08	4.00	5.17
	R_2	0.55	0.57	0.64	0.89	1.28	1.77	2.42	3.17	4.16	5.45
	Total	1.10	1.14	1.27	1.77	3.05	3.90	4.76	6.25	8.16	10.62
	R	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
	R_{eff}	0.78	0.77	0.75	0.65	0.39	0.22	0.05	-0.25	-0.63	-1.12
Coastguard	R_1	0.56	0.67	1.40	2.92	5.02	10.48	13.54	20.51	29.41	40.01
	R_2	0.55	0.68	1.40	2.90	5.14	8.73	14.33	21.86	31.42	42.49
	Total	1.11	1.35	2.80	5.82	10.16	19.21	27.87	42.37	60.83	82.50
	R	38.55	38.55	38.55	38.55	38.55	38.55	38.55	38.55	38.55	38.55
	R_{eff}	0.97	0.96	0.93	0.85	0.74	0.50	0.28	-0.10	-0.58	-1.14
Carphone	R_1	0.56	0.75	1.62	3.25	5.41	9.31	12.74	17.94	24.21	31.39
	R_2	0.55	0.76	1.61	3.30	5.49	8.65	13.08	18.46	25.00	32.27
	Total	1.11	1.51	3.23	6.55	10.90	17.96	25.82	36.40	49.21	63.66
	R	30.66	30.66	30.66	30.66	30.66	30.66	30.66	30.66	30.66	30.66
	R_{eff}	0.96	0.95	0.89	0.79	0.64	0.41	0.16	-0.19	-0.61	-1.08
Foreman	R_1	0.56	1.03	2.22	3.87	6.04	10.65	12.04	15.90	20.67	26.09
	R_2	0.56	1.05	2.24	3.95	6.12	8.88	12.29	16.50	21.37	27.19
	Total	1.12	2.08	4.46	7.82	12.16	19.53	24.33	32.40	42.04	53.28
	R	25.39	25.39	25.39	25.39	25.39	25.39	25.39	25.39	25.39	25.39
	R_{eff}	0.96	0.92	0.82	0.69	0.52	0.23	0.04	-0.28	-0.66	-1.10
Miss America	R_1	0.57	0.62	0.81	1.19	1.59	2.39	2.91	3.80	5.04	6.52
	R_2	0.52	0.62	0.84	1.18	1.63	2.17	2.98	3.91	5.20	6.70
	Total	1.09	1.24	1.65	2.37	3.22	4.56	5.89	7.71	10.24	13.22
	R	5.27	5.27	5.27	5.27	5.27	5.27	5.27	5.27	5.27	5.27
	R_{eff}	0.79	0.76	0.69	0.55	0.39	0.13	-0.12	-0.46	-0.94	-1.51

Table 4: Volume of the motion jpeg files (KB) and the encoding efficiency of the MDCLVQ-Motion JPEG2000

Video	Sigma	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Akiyo	V_1	932	1855	2544	3024	3594	4080	4340	4738	5061	5378
	V_2	895	1876	2546	3134	3717	4114	4463	4840	5136	5446
	Total	1827	3731	5090	6158	7311	8194	8803	9578	10197	10824
	V	4536	4536	4536	4536	4536	4536	4536	4536	4536	4536
	V_{eff}	0.60	0.18	-0.12	-0.36	-0.61	-0.81	-0.94	-1.11	-1.25	-1.39
Coastguard	V_1	1367	2352	3161	3667	4234	4851	5138	5541	5912	6230
	V_2	1245	2364	3159	3786	4368	4838	5298	5688	6049	6358
	Total	2612	4716	6320	7453	8602	9689	10436	11229	11961	12588
	V	5797	5797	5797	5797	5797	5797	5797	5797	5797	5797
	V_{eff}	0.55	0.19	-0.09	-0.29	-0.48	-0.67	-0.80	-0.94	-1.06	-1.17
Carphone	V_1	1279	2541	3637	4269	4999	5660	6079	6613	7055	7460
	V_2	1193	2538	3630	4381	5136	5695	6206	6707	7143	7518
	Total	2472	5079	7267	8650	10135	11355	12285	13320	14198	14978
	V	6699	6699	6699	6699	6699	6699	6699	6699	6699	6699
	V_{eff}	0.63	0.24	-0.08	-0.29	-0.51	-0.70	-0.83	-0.99	-1.12	-1.24
Foreman	V_1	1080	2240	3044	3513	4067	4645	4922	5346	5699	6012
	V_2	1030	2243	3017	3630	4193	4639	5055	5441	5795	6094
	Total	2110	4483	6061	7143	8260	9284	9977	10787	11494	12106
	V	5475	5475	5475	5475	5475	5475	5475	5475	5475	5475
	V_{eff}	0.61	0.18	-0.11	-0.30	-0.51	-0.70	-0.82	-0.97	-1.10	-1.21
Miss America	V_1	368	851	1191	1403	1683	1910	2058	2261	2422	2570
	V_2	360	852	1179	1463	1755	1939	2106	2288	2456	2594
	Total	728	1703	2370	2866	3438	3849	4164	4549	4878	5164
	V	1308	1308	1308	1308	1308	1308	1308	1308	1308	1308
	V_{eff}	0.44	-0.30	-0.81	-1.19	-1.63	-1.94	-2.18	-2.48	-2.73	-2.95

As an example, the MDCLVQ-A_2 with *sigma* = 0.1, in average shows 0.89 bit rate efficiencies for all the videos, which is an improvement as compared with the original encoder. However, for the values of *sigma* from 0.8 to 1, the proposed scheme requires more bit rates than the original H.264/AVC encoder, therefore the compression efficiency has been traded off for error resiliency. The error resiliency of the proposed MCLVQ-H.264/AVC scheme is due to its MD coding nature, that is, it can provide a degraded version of the source if only one of the channels works, while a high quality reconstruction is achieved when both channels work.

Compression Efficiency Analysis for MDCLVQ-Motion JPEG2000: As stated in the first part of this section, the proposed MD video coding is a generic scheme that is applicable to almost any type of video coding standard. In this subsection the experimental results for the proposed scheme using the Motion JPEG 2000 video coding standard are presented. The proposed MDCLVQ-Motion JPEG2000 scheme generates two descriptions of the input video that are encoded by the OpenJPEG software to motion jpeg files. In this subsection, the volume of the motion jpeg files generated by encoding the MD sequences are compared with size of the motion jpeg file of the original single description scheme and the encoding efficiencies are calculated based on Equation (6). The same standard video sequences are used to evaluate the performance of the proposed MDCLVQ-Motion JPEG2000 encoding scheme. The results are presented in Table 4.

As shown in Table 4, the compression efficiency of the MDCLVQ-Motion JPEG2000 scheme decreases as the value of *sigma* is increases. This property is similar to the results for the MDCLVQ-H.264/AVC because the compression efficiency is due to the nature of the MD coding and the coarse degree of the quantization, which is determined by *sigma*. In addition, it is observed in Table 4 that the MDCLVQ-Motion JPEG2000 shows the best compression efficiency for the "Carphone" video with *sigma = 0.1* and the worst compression efficiency is for "Miss America" video with *sigma = 1*. In general, the compression efficiencies of the MDCLVQ-Motion JPEG2000 provided in Table 4 are lower than their corresponding values in Table 3 because the H.264/AVC standard has higher compression efficiency than Motion JPEG2000.

Rate and Distortion Analysis for MDCLVQ-H.264/AVC and MDCLVQ-Motion JPEG2000: A fundamental problem of rate-distortion theory is determining the bit-rate required to encode the source at a given fidelity level. Fidelity is measured by computing the distortion between the source video and the reconstructed video. The peak signal-to-noise ratio (PSNR) is a common fidelity criterion based on the mean squared error (MSE). In the following results, the PSNR is measured for the luminance component. The performance of the MDCLVQ-H.264/AVC scheme in terms of the required bit rate and the reconstruction fidelity is controlled by the fundamental area of the hexagonal lattice. The fundamental area of the hexagonal lattice is changed by *sigma*. Therefore, in Table 4 the average PSNR of the reconstructed videos in the central decoder as well as the side decoders for the "Akiyo", "Coastguard", "Carphone", "Foreman" and "Miss America" video sequences are provided in Table 4 for different values of the *sigma*.

As shown in Table 4, the qualities of the reconstructed videos are directly proportional to the value of *sigma*. In other words, the PSNR of the central decoder and the PSNR of the side decoders improve as the *sigma* is increased. On the other hand, according to Table 3, the bit rate efficiencies decrease as the *sigma* is increased. Then, the quality of the reconstructed video is reversely related to the bit rate efficiency. Thus, as the value of *sigma* increases the compression efficiency is sacrificed for better reconstruction quality. As for example, the MDCLVQ-H.264/AVC shows the best reconstruction quality for the Foreman video when the *sigma* is 1; *PSNR = 36.96* in the side 1 decoder, *PSNR = 36.82* in the side 2 decoder and *PSNR = 38.05* in the central decoder. However the MDCLVQ-H.264/AVC shows the best bit rate efficiency (R_{eff} = 0.96) for the Foreman video when the *sigma* is 0.1.

In addition, according to Table 4 the PSNR values for the central decoder and the side decoders are not significantly improved as *sigma* is increased from 0.7 to 1. This implies that the best performance of the MDCLVQ-H.264/AVC in terms of the bit rate efficiency and reconstruction quality for the Foreman video is seen with *sigma* = 0.7. As an example, for the Foreman video with *sigma = 0.7* and with almost the same bit rate as the original encoder (R_{eff} = 0.04) the MDCLVQ-H.264/AVC provides *PSNR = 34.07 dB* in the central decoder, *PSNR = 33.08 dB* in the side 1 decoder and *PSNR = 33.08 dB* in the side 2 decoder.

The performance of the MDCLVQ-Motion JPEG2000 scheme in terms of the required bit rate and the reconstruction fidelity is controlled by the fundamental area of the hexagonal lattice. The fundamental area of the hexagonal lattice is changed by *sigma*. Therefore, the average PSNR of the reconstructed videos in the central

Table 5: The average PSNR of the central and the side decoders for the selected video sequences encoded by the MDCLVQ-H.264/AVC

Video*sigma*		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Akiyo	Side 1	16.32	21.92	25.12	28.12	30.69	32.84	34.79	36.37	37.73	37.48
	Side 2	13.62	22.42	25.24	28.12	30.70	32.88	34.89	36.35	37.61	38.28
	Central	17.42	22.62	25.65	28.60	31.38	33.81	35.69	37.27	38.70	38.19
Coastguard	Side 1	15.03	19.06	22.32	24.86	27.36	29.40	31.53	33.18	34.47	35.22
	Side 2	13.64	19.23	22.32	24.92	27.40	29.63	31.59	33.17	34.34	35.00
	Central	16.29	19.47	22.22	25.33	27.97	30.40	32.53	34.32	35.65	36.26
Carphone	Side 1	15.39	19.40	23.47	26.36	29.09	31.48	33.68	35.35	36.76	37.68
	Side 2	13.51	19.83	23.46	26.42	29.19	31.58	33.72	35.35	36.68	37.50
	Central	15.47	20.11	23.98	26.90	29.80	32.41	34.70	36.50	37.98	38.78
Foreman	Side 1	14.46	19.09	23.41	25.97	28.70	30.94	33.08	34.82	36.15	36.96
	Side 2	12.67	19.31	23.30	26.02	28.71	31.04	33.08	34.80	36.05	36.82
	Central	14.58	19.67	23.82	26.46	29.38	31.97	34.07	35.96	37.35	38.05
Miss America	Side 1	16.53	23.58	28.07	31.57	34.39	36.75	38.72	39.98	40.72	41.15
	Side 2	15.42	23.65	28.00	31.68	34.19	36.74	38.60	39.97	40.63	40.97
	Central	17.18	23.97	29.02	32.18	35.21	37.69	39.60	40.88	41.56	41.74

Table 6: The average PSNR of the central and the side decoders for the selected video sequences encoded by the MDCLVQ-Motion JPEG2000

Video*sigma*		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Akiyo	Side 1	15.95	25.51	29.45	32.75	34.00	33.99	35.56	36.07	36.36	39.94
	Side 2	13.68	25.38	29.22	32.34	33.25	34.44	35.08	35.68	36.10	36.29
	Central	18.89	29.33	33.13	35.55	36.36	36.55	37.11	37.32	37.45	42.38
Coastguard	Side 1	14.90	22.85	27.53	31.21	33.22	37.47	38.45	38.45	39.41	40.38
	Side 2	12.99	23.25	27.62	30.50	32.64	34.54	35.52	36.73	37.80	38.85
	Central	18.70	27.80	32.60	35.60	38.25	37.81	39.93	40.72	41.36	42.03
Carphone	Side 1	16.27	24.76	29.89	33.44	35.68	35.62	38.96	39.90	41.04	41.94
	Side 2	12.83	25.00	29.69	32.75	34.64	36.44	37.66	38.81	39.93	40.98
	Central	18.27	28.65	34.55	37.92	41.05	41.12	43.36	44.11	45.06	46.15
Foreman	Side 1	16.81	24.40	28.75	32.54	34.86	34.16	38.32	39.40	40.50	41.50
	Side 2	11.27	24.69	28.85	31.81	33.63	35.57	36.72	38.01	39.17	40.19
	Central	16.19	28.86	33.69	36.99	39.75	39.45	41.96	42.85	43.74	44.64
Miss America	Side 1	16.34	26.07	31.36	33.82	35.52	35.95	37.60	38.08	38.59	38.95
	Side 2	15.84	26.07	30.98	33.53	34.83	36.16	36.92	37.64	38.14	38.68
	Central	19.84	29.42	35.13	37.61	39.09	39.44	39.87	40.10	40.28	40.57

Table 7: Encoding time for the MDCLVQ-Motion JPEG2000 encoder

Sigma	0.3	0.4	0.5	0.7	0.9	Original
Akiyo	4.4	4.84	5.5	6.51	7.3	8.44
Carphone	4.12	4	4.75	5.62	6.03	6.32
Coastguard	4.34	4.67	5.2	6.33	7.07	7.69
Foreman	4.34	4.67	5.2	6.33	7.07	7.69
Miss America	2.15	2.4	2.65	3.18	3.56	3.23

decoder as well as the side decoders for the “Akiyo”, “Coastguard”, “Carphone”, “Foreman” and “Miss America” video sequences are provided for different values of the *sigma* in Table 5.

The PSNR values of the central decoder and the side decoders of the MDCLVQ-Motion JPEG2000 provided in Table 5 are higher than the corresponding values of the MDCLVQ-H.264/AVC provided in Table 7. This is because of better reconstruction quality of the Motion JPEG2000 as compared with the H.264/AVC. However, the general pattern is the same and all the PSNR values increase as the *sigma* is increased because reconstruction fidelity is more affected by the performance of the MDCLVQ-A_2. On the other hand, the compression efficiencies of MDCLVQ-H.264/AVC are in general better than MDCLVQ-Motion JPEG2000 because of higher compression efficiency of the H.264/AVC.

To the best of our knowledge, current MD video coding schemes focus on implementing the MD scheme based on certain properties of the video coding standard and MD video coding schemes that are based on multiple-description lattice vector quantization are not easily found in the literature. However, to compare the performance of the proposed MDCLVQ-H.264/AVC scheme with previous

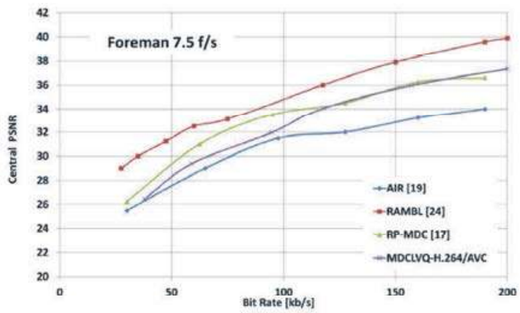

Fig. 3: PSNR values for the Foreman video encoded at 7.5 fps by the MDCLVQ-H.264/AVC and referenced algorithms

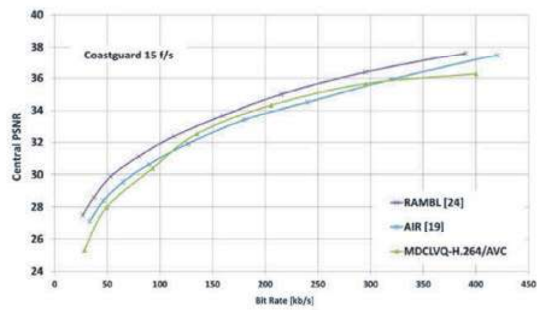

Fig. 4: PSNR values for the Coastguard video encoded at 15 f/s by the MDCLVQ-H.264/AVC and referenced algorithms

similar algorithms, the central PSNR and PSNR side values for the "Foreman" and "Coastguard" videos encoded by the MDCLVQ-H.264/AVC and the same values from AIR [19], RP-MDC [17] and RAMBL [24] are provided in Figure 3 and Figure 4.

According to Figure 3, the central PSNR of the MDCLVQ-H.264/AVC scheme is higher than the central PSNR of RP-MDC [17] and AIR [19]. However, the central PSNR of the MDCLVQ-H.264/AVC is lower than the central PSNR of the RAMBL [24]. This is because the RAMBL [24] and RP-MDC [17] are based on scalar quantization while the proposed scheme lattice vector quantization. In general the quantization error in scalar quantization is lower than the error in vector quantization. On the other hand, the computational complexity of the lattice vector quantization is lower than the scalar quantization.

According to Figure 4, the central PSNR of the proposed MDCLVQ-H.264/AVC scheme is slightly lower than RAMBL [24] and almost the same as AIR [19]. However, the disadvantage of the proposed MDCLVQ-H.264/AVC is that, it requires two parallel video encoders while the other schemes use only one video encoder. In other words, the referenced schemes try to embed the redundancy and error resiliency within a single video but the proposed schemes tries to generate two descriptions of the video that can offer error resiliency through MD coding.

According to Table 5, Figure 3 and Figure 4 the reconstruction performance of the proposed MDCLVQ-H.264/AVC scheme in terms of the central PSNR and side PSNR do not significantly increase while the *sigma* is increased from 0.7 to 1. In order to demonstrate the visual quality of the reconstructed videos, the first frames of the central videos of Foreman sequence that have been encoded by the MDCLVQ-H.264/AVC with *sigma* values that range from 0.1 to 1 are provided in Figure 5. The corresponding value of *sigma* and the PSNR of the central videos are also provided.

As shown in Figure 5, the quality of the reconstructed videos of the Foreman increase as the value of *sigma* is increased. The reconstruction quality for the Foreman videos corresponding to *sigma* lower 0.3 is very low while the videos corresponding to *sigma* higher than 0.4 is acceptable. However the difference between the PSNR corresponding to *sigma* = 0.7 and *sigma* = 1 is negligible. In other words, the extra bit rate that is required for higher values of *sigma* is not worth of the gained reconstruction fidelity, thus the best performance of the MDCLVQ-H.264/AVC for the Foreman video is achieved with *sigma* = 0.7. In other words, using *sigma = 0.7* the proposed scheme can offer acceptable side and central reconstruction qualities and error resiliency without extra bit rate.

Another important aspect of the MD video coding schemes is the time required to encode the videos and the computational complexity of the process. It is important to mention that although the MD coding schemes require two encoding procedures the time required to encode the side videos is not twice the time for encoding the original video because the most time consuming part of the encoding procedure is usually spent on the entropy coding [15]. The time required by the entropy coding is directly related to the entropy of the video and decreasing the entropy decreases the encoding time. In the MDCLVQ scheme the entropy of the video is decreased and hence the time required by the encoders is decreased. In Table 5 the time required by the two Motion JPEG2000 coders of

Fig. 5: The 1st frames of the QCIF Foreman sequence reconstructed by the central decoder corresponding to sigma = 0.1 to 1

the MDCLVQ scheme as well as the time for the original single encoder are provided. It is seen that the MDC encoders are faster than the original single encoder. The system includes an Intel® Core™2 Duo CPU @ 2.8 GHz and 4 GB memory.

The proposed MD coding schemes based on the coinciding similar sublattices A_2 require low computational capabilities due the inherent symmetries of the coinciding similar sublattices of the hexagonal lattice [5]. The labeling function and the partitioning scheme are not computation intensive processes. The partitions are generated using the simple proposed two-fold real congruency relation and the proposed shift property simplifies the problem of labeling the entire lattice space to the problem of labeling the Super Voronoi region of the origin. The Super Voronoi region of the origin with index N includes N^2+N-1lattice points and N^2 sublattice point. Thus, the labeling function is simplified to a table look up with N^2+N-1 entries. In addition the proposed labeling function, the partitioning scheme and the shift property are scalable and as a consequence using the sigma multiplier the performance of the entire scheme is adjustable [5].

CONCLUSION

Applications of multimedia communication in everyday life are found everywhere. Consequently many video compression techniques have been proposed that reduce the size of the video data. The H.264/AVC standard and the Motion JPEG2000 are the two most popular video codecs.

The multiple descriptions lattice vector quantization (MDLVQ) has become a popular choice for robust data transmission over unreliable network channels. In this paper, the multiple descriptions coinciding lattice vector quantization (MDCLVQ) is used to develop a new MD video coding scheme. The MDCLVQ scheme is based on the coinciding similar sublattices of the hexagonal lattices. The main advantage of using MD video coding is that it increases the robustness of video transmission over error-prone communication channels while conserving the bandwidth. The proposed MDCLVQ scheme is a generic MD video coding scheme that is applicable to different video coding standard. To assess the performance of the proposed MD video coding techniques, MD-H.264/AVC and MD-Motion JPEG2000, they were applied to several test videos. The experimental results show that the MD video coding schemes improves the robustness of the video transmission while preserving the required bandwidth. In addition, the compression efficiency of the proposed schemes has improved.

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