PROCESSING STRATEGIES FOR PHASE UNWRAPPING FOR INSAR APPLICATIONS

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ABSTRACT

One of the most challenging aspects in the successful application of SAR interferometry (INSAR) is unwrapping the interferometric phase. The difficulties arise in attempting to find global optimization procedures with the best possible cost criteria for data that are both noisy and incomplete. Recent progress in this problem includes introduction of network flow optimization, and the use of triangular irregular networks for sparse data. Interferograms differ greatly in the difficulty to unwrap depending on the interferogram fringe complexity and correlation. We examine the characteristics of these types and present phase unwrapping strategies for each of these.

1 INTRODUCTION

The absolute unwrapped interferometric phase derived is directly proportional to the difference in path lengths for the SAR image pair. Applications of interferometry relate the unwrapped phase to geophysical parameters such as elevation, and deformation due to ice motion, surface subsidence, earthquakes, volcanic inflation/deflation, and tectonic motion [1,2]. The complex-valued interferogram values however, are known only modulo 2π and must be unwrapped to obtain a quantitative interpretation.

Phase unwrapping is the process of restoring the correct multiple of 2π to each point of the interferometric phase image.

For a well-behaved smooth phase field all the unwrapped phase differences between adjacent interferogram samples lie between $-\pi$ and $+\pi$. When this is true, phase unwrapping is straightforward. The unwrapped phase can be evaluated by a simple path-independent integration of the phase differences of adjacent wrapped phases, starting from a reference location and using the assumption that all phase differences are in the interval $(-\pi, \pi)$.

In actual interferograms phase unwrapping is more complicated because of phase steps outside the interval $(-\pi, \pi)$. Causes for local phase gradients larger than π are:

1. **Phase Noise:** Temporal decorrelation, shadow and low SNR cause phase noise. For repeat-track interferometry temporal decorrelation is often the main cause.

2. **Phase Under-sampling**: The phase is under-sampled when the phase gradient exceeds half a fringe (phase cycle) per sample. In the presence of phase noise the under-sampling already occurs at lower gradients.

3. **Phase Discontinuities:** In interferograms layover and discontinuous surface deformation (e.g. at sliding faults or at glacier rock interfaces) cause discontinuities in the interferometric phase.

2 PHASE UNWRAPPING ALGORITHMS

One group of unwrapping algorithms poses the problem in terms of a solution of a two-dimensional partial differential equation. Among these is the least-squares algorithm [3]. Intrinsically these methods are not developed for discontinuous functions. Consequently large deviations in the unwrapped phase occur in the vicinity of true phase discontinuities relating to physical features such as layover. Another characteristic of these methods is that the output phase can differs by values different than an exact multiple of 2π.

2.1 BRANCH CUT ALGORITHMS

The principle of branch cut algorithms is to restrict the integration through the image to paths with local phase differences in the interval $(-\pi, \pi)$. Summing the phase finite differences about short circular paths permits localization of discontinuities in the wrapped phase field. If the sum is non-zero, a so-called "residue" lies in the region. The residue value or "charge" can be positive (+1) or negative (-1) depending on the sign of the sum. Line segments (branches) are drawn between positive and negative residues in a systematic way to function as barriers during the path integration that cannot be crossed. This discharging of residues results in a consistent, path independent solution. The appearance of these cuts and the residues for a typical interferogram are shown in Figure 3. One can observe that most of the residues come in pairs that lie close together. This is due to phase noise that generates a pair of residues. The basic ideas of the *branch-cut* algorithms and a first implementation were presented by Goldstein [4]. Presently used implementations differ in the methodology used to determine the branch cuts.

In our implementation the branches form tree-like networks that have zero net charge. The goal of the algorithm is to connect residues in such a way that minimizes the net length of branches. Construction of a new tree begins by finding an unvisited residue and connecting to its nearest residue neighbor, regardless of sign. A line ("branch cut") is drawn connecting this residue to the neighbor. If the neighbor has not been visited previously, its charge is added to the net sum of charges for the current tree. If the tree is neutral, then the algorithm searches for a unvisited residue and constructs a new tree, otherwise, the regions around all the current tree residues are search for new tree members. After a residue has been incorporated in a tree it is marked as visited such that its charge is not counted multiple times. The tree-building terminates when all residues in the interferogram have been visited.

Refinements of this algorithm have include modification of the search strategy in areas of low correlation, and the use of distance weighting of the data to optimize the tree structure. In the original implementation, many unnecessary cuts may be drawn that completely wall of areas. Our implementation incorporates exiting trees into the current tree directly thereby reducing redundant branches.

In many cases our implementation of the branch-cut algorithm provides a robust and efficient unwrapping solution that works well for images with high correlation. Under certain conditions multi-looking of the interferogram or filtering can facilitate the unwrapping. Especially, when limiting the unwrapping to areas of higher coherence, respectively lower phase noise, the branch-cut solution is rather conservative, with few unwrapping errors.

The spatially incomplete solution is a significant limitation of the branch-cut solution. In the case of longer-time differential interferograms higher coherence is often restricted to relatively small, spatially disconnected urban areas. In such a case the operator supported bridging of the unwrapped phase can be tedious and uncertain. And, from the conceptual point of view, it is a disadvantage that the optimization is done on a local rather than on a global basis.

The demand for global optimization and automated and optimized unwrapping of disconnected areas of high coherence and the available advanced methodology in network flow optimization lead to the development of the minimum cost flow and triangulation network based phase unwrapping solution presented below.

2.2 MINIMUM COST FLOW AND TRIANGULATION

The minimum cost flow problem defines a network consisting of *nodes* and directed *arcs* that connect the nodes. Associated with each of the arcs are a *flow*, a *cost per unit flow*, and an *arc capacity*. Flow moves from *source nodes* to *sink nodes* through the network arcs. Nodes that are neither sources nor sinks are *trans-shipment* nodes. The MCF problem solution gives the optimum flow in each of the arcs that minimize the total cost. The total cost is defined as the sum of costs for all the arc flows. In the MCF paradigm, flows are integer quantities. This problem has been extensively studied and efficient algorithms have been found and presented in the literature (e.g. Ahuja [7]). The application of minimum cost flow (MCF) techniques to phase unwrapping and thereby achieve a global optimization was first presented by Costantini [6].

In our formulation of the phase unwrapping problem in the network flow form, a node is associated with each of the local closed paths used to evaluate the residues. Source nodes are associated with the positive residues and sink nodes with the negative residues. Integer values of arc flow are equivalent to additional multiples of 2π to add to the gradient derived from the wrapped phase. Two arcs are necessary between every pair of adjacent nodes because flow is defined only to be positive flow in the MCF problem formulation. The pixels lie between the network arcs and nodes. When calculating the unwrapped phase by summing along the path, the gradient is adjusted if there is flow in the arc crossing the path of integration.

The cost per unit flow assigned to each arc is a critical parameter in the optimization process. Generally, it should be expensive to cause a phase jump in an area of low phase noise, and inexpensive in noisy regions where it is known that the phase should jump by several multiples of 2π. In our implementation the cost is proportional to a function of the coherence.

Another important element for improving the unwrapping has been to generalize the network topology to be a triangulation network as was previously proposed by [8]. In our implementation a Delaunay triangulation is used [9]. Using the triangulation has several distinct advantages. Only those points that have reliable enough phase values are considered in the unwrapping. Areas of too high phase noise are not considered.

As was the intention, the combination of triangulation with the MCF algorithm permits robust phase unwrapping in many cases of isolated areas of high coherence. This is particularly advantageous in the case of long interval differential interferograms. An example of this is shown in Figure 4 where the discontinuous regions were correctly unwrapped without operator interaction. Another advantage of the triangular network is the two times higher density of the network which permits better localization of phase discontinuities leading to more precise unwrapping. The MCF solution is rather memory intensive. A special method was implemented to still permit unwrapping of very large interferograms. Figure 5 shows the use of adaptive thinning the input interferogram to reduce the total number of nodes. Regions of lower correlation and steep slopes have a denser triangle density compared with flat high correlation areas. Patching is another approach to

overcome the problem, but the optimization area is reduced to the patch size.

3 CONCLUSIONS

In this paper we have reviewed several practical approaches to phase unwrapping that take into the account the characteristics of SAR interferograms including regions with very high noise and phase discontinuities. The introduction of the MCF and triangulation paradigms significantly improves the robustness and applicability of SAR interferometry to geophysical research. The commercial availability of these algorithms is leading to wide spread application in the user community.

4 REFERENCES

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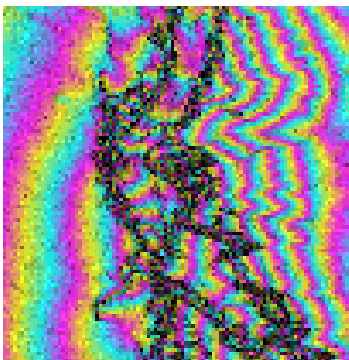
Figure 3a: Interferogram showing interferometric phase intensity weighted by the interferometric correlation. Areas of layover have low correlation and significant phase discontinuities. Phase display is 2π per color cycle.
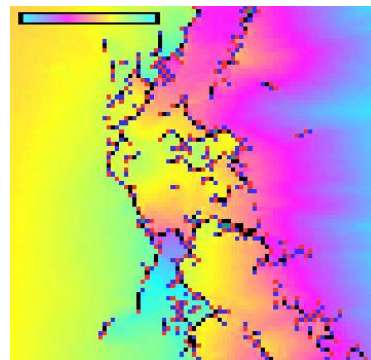

Figure 3b: Unwrapped interferometric phase showing residues (+: red, -: blue) and branch cuts (black). Note that most residues come in +/- pairs. Chains of residues occur along regions of lay-over. Phase display is 10π per color cycle.
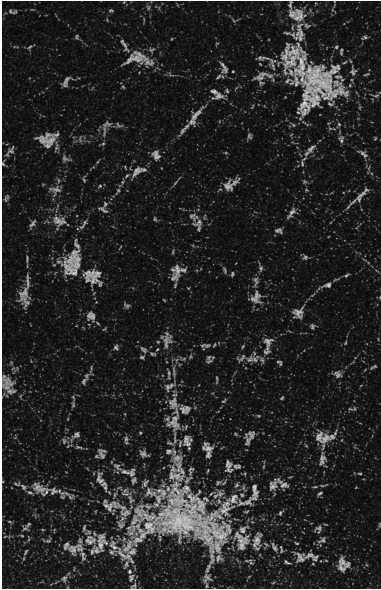
Figure 4a: Correlation map for an urban differential interferogram after filtering. Correlation displayed from 0.0 to 1.0 as linear intensity.
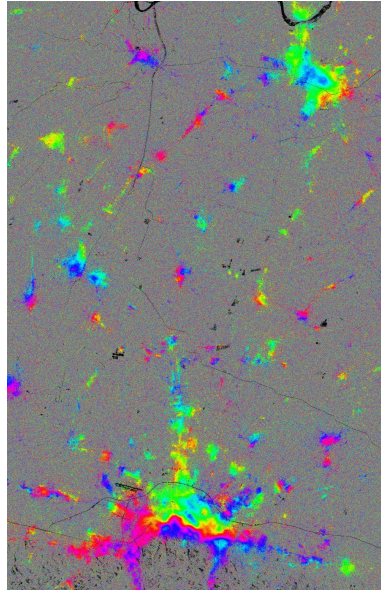

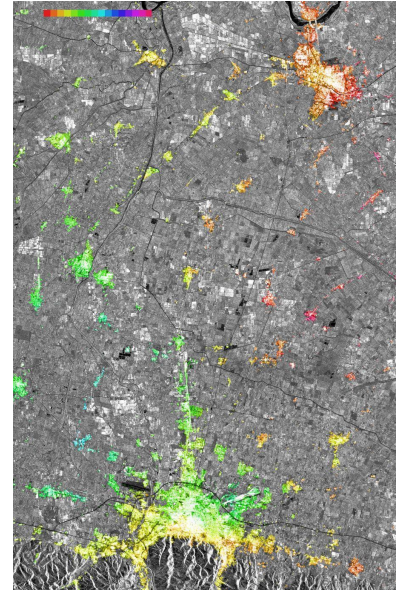**Figure 4b:** Interferometric phase after adaptive filtering. Phase displayed as 2π per color cycle.

Figure 4c: Unwrapped interferometric phase using triangulation to connect regions. Phase displayed as 10π per color cycle.

Figure 5a: Correlation over Las Vegas ERS Tandem data, 5 looks, 20 meter azimuth spacing. Correlation displayed from 0.0 to 1.0 as intensity.
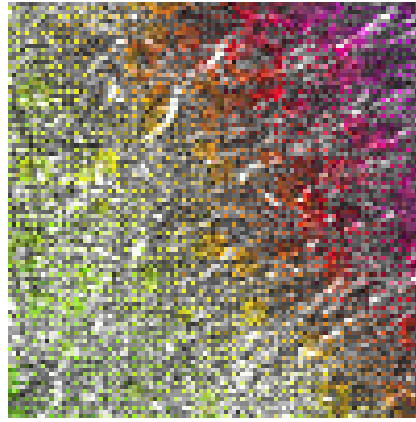

Figure 5b: Sampled unwrapped phase showing adaptively thinned unwrapped phase mesh. Phase displayed as 20π per color cycle.
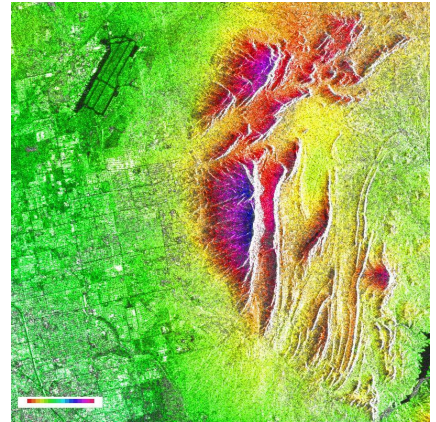

Figure 5c: Full unwrapped phase using sampled unwrapped data as model for the full interferogram. Phase displayed as 20π per color cycle.