

PHYLLOTAXIS

Sister Mary de Sales McNabb
Georgetown Visitation Preparatory School

When Nehemiah Grew remarked in 1682 that "from the contemplation of plants, men might be invited to Mathematical Enquirys," [5] he might not have been thinking of the amazing relationship between phyllotaxis and Fibonacci numbers, but he could well have been; for the phenomenon of phyllotaxis, literally "leaf arrangement," has long been a subject of special investigation, much speculation, and even heated debate among mathematicians and botanists alike.

By right it is the botanists who deserve the credit for bringing to light the discovery that plants of every type and description seem to have their form elements, that is, their branches, leaves, flowers, or seeds, assembled and arranged according to a certain general pattern; but surely even the old Greek and Egyptian geometers could not have failed to observe the spiral nature of the architecture of plants. Many and varied and even contradictory are the theories on this fascinating phenomenon of phyllotaxis, but it would be beyond the scope of this paper to investigate them here; instead we shall simply try to describe the manifestation of it in the interval-spacing of leaves around a cylindrical stem, in the florets of the sunflower and, finally, in the scales of fir cones and pineapples.

Before we proceed to consider the actual arrangement of the form elements, however, it is interesting to note the relationship between the number of petals of many well-known flowers and the Fibonacci numbers. Two-petalled flowers are not common but enchanter's nightshade is one such example. Several members of the iris and lily families have three petals, while five-petalled flowers, including the common buttercup, some delphiniums, larkspurs and columbines, are the most common of all. Other varieties of delphiniums have eight petals, as does the lesser celandine, and in the daisy family, squalid and field senecio likewise have eight petals in the outer ring of ray florets. Thirteen-petalled flowers are quite common and include the globe flower and some double delphiniums as well as ragwort, corn marigold, mayweed, and several of the chamomiles. Many garden and wild flowers, including some heleniums and asters, chicory, doronicum, and some hawk-bits, have twenty-one petals, while thirty-four is the most common number in the daisy family and is characteristic of the field daisies, ox-eye daisies, some heleniums, gaillardias, plantains, pyrethrums, and a number of hawk-bits and hawkweeds. Some field daisies have fifty-five

petals, and Michaelmas daisies often have either fifty-five or eighty-nine petals. It is difficult to trace this relationship much further, but it must be remembered that this number pattern is not necessarily followed by every plant of a species but simply seems to be characteristic of the species as a whole.

Fibonacci numbers occur in other types of patterns too. The milkwort will commonly be found to have two large sepals, three smaller sepals, five petals and eight stamens, and Frank Land [4] reports that he found a clump of alstroemerias in his garden in which one plant had two flowers growing on each of three stalks and that, where the three stalks grew out from the top of the main stem, a whorl of five leaves grew out radially; while another plant had three flowers on each of five stems with a whorl of eight leaves at the base of the flower stalks.

The Fibonacci number pattern, however, which has received the most attention is that associated with the spiral arrangement of the form elements of the plants. In its simplest manifestation it may be observed in plants and trees which have their leaves or buds or branches arranged at intervals around a cylindrical stem. If we should take a twig or branch of a tree, for instance, and choose a certain bud, then by revolving the hand spirally around the branch until we came to a bud directly above the first one counted, we would find that the number of buds per revolution as well as the number of revolutions itself are both Fibonacci numbers, consecutive or alternate ones depending on the direction of revolution, and different for various plants and trees. If the number of revolutions is m and the number of leaves or buds is n , then the leaf or bud arrangement is commonly called an m/n spiral or m/n phyllotaxis. Hence in some trees, such as the elm and basswood, where the leaves along a twig seem to occur directly opposite one another, we speak of $1/2$ phyllotaxis, whereas in the beech and the hazel, where the leaves are separated by one-third of a revolution, we speak of $1/3$ phyllotaxis. Likewise, the oak, the apricot, and the cherry tree exhibit $2/5$ phyllotaxis, the poplar and the pear $3/8$, while that of the willow and the almond is $5/13$. Much investigation along these lines seems to indicate that, at least as far as leaves and blossoms are concerned, each species is characterized by its own particular phyllotaxis ratio, and that almost always, except where damage or abnormal growth has modified the arrangement, the ratios encountered are ratios of consecutive or alternate terms of the Fibonacci sequence.

When the form elements of certain plants are assembled in the form of a disk rather than along a cylindrical stem, we have a slightly different form of phyllotaxis. It is best exemplified in the head of a sunflower, which consists of a number of tightly packed florets, in reality the seeds of the flower. Very clearly the seeds can be seen to be distributed over the head in

two distinct sets of spirals which radiate from the center of the head to the outermost edge in both clockwise and counterclockwise directions. These spirals, logarithmic in character, are of the same nature as those mentioned earlier in plants with cylindrical stems, but in those instances, the adjacent leaves being generally rather far apart along the stem, it is more difficult for the eye to detect the regular spiral arrangement. Here in the close-packed arrangement of the head of the sunflower, we can see the phenomenon in almost two-dimensional form. As was the case with the cylindrical-stemmed plants, the number pattern exhibited by the double sets of spirals is intimately bound up with Fibonacci numbers. The normal sunflower head, which is about five or six inches in diameter, will generally have thirty-four spirals winding in one direction and fifty-five in the other. Smaller sunflower heads will commonly exhibit twenty-one spirals in one direction and thirty-four in the other or a combination of thirteen and twenty-one. Abnormally large heads have been developed with a combination of fifty-five and eighty-nine spirals and even a gigantic one at Oxford with eighty-nine spirals in one direction and a hundred and forty-four in the other. In each instance the combination of clockwise and counterclockwise spirals consists of successive terms of the Fibonacci sequence.

One other interesting manifestation of phyllotaxis and its relation to the Fibonacci numbers is observed in the scales of fir cones and pineapples. These scales are really modified leaves crowded together on relatively short stems, and so, in a sense, we have a combination of the other two forms of the phenomenon; namely, a short conical or cylindrical stem and a close-packed arrangement which easily enables us to observe that the scales are arranged in ascending spirals or helical whorls called parastichies. In the fir cone, as in the sunflower head, two sets of spirals are obvious, and hence in many cones, such as those of the Norway spruce or the American larch, five rows of scales may be seen to be winding steeply up the cone in one direction while three rows wind less steeply the other way; in the common larch we usually find eight rows winding in one direction and five in the other, and frequently the two arrangements cross each other on different parts of the cone. In the pineapple, on the other hand, three distinct groups of parastichies may be observed; five rows winding slowly up the pineapple in one direction, eight rows ascending more steeply in the opposite direction, and, finally, thirteen rows winding upwards very steeply in the first direction. The fact that pineapple scales are of irregular hexagonal shapes accounts for the three sets of whorls, for three distinct sets of scales can consequently be contiguous, and hence, constitute a different formation. Moreover, Fibonacci numbers manifest themselves in still another way in connection with the scales of the pineapple. If the scales should be numbered successively around the fruit from the bottom

to the top, the numbering being based on the corresponding lateral distances of the scales along the axis of the pineapple, we would find that each of the three observable groups of parastichies winds through numbers which constitute arithmetic sequences with common differences of 5, 8, and 13, the same three successive Fibonacci numbers observed above. Thus a spiral of the first group would ascend through the numbers 0, 5, 10, ...; one of the second group through the numbers 0, 8, 16, ...; and, finally, a spiral of the third group would wind steeply up the pineapple through the numbers 0, 13, 26,

In all these many and varied ways, then, in the number of petals possessed by different species of flowering plants, in the interspacing of leaves or buds around a cylindrical stem, in the double spirals of the close-packed florets of sunflowers, and in the ascending spirals or parastichies of the fir cone and the pineapple, we have encountered number patterns which again and again involve particular terms of the Fibonacci sequence. These Fibonacci number patterns or combinations occur so continually in the varied manifestations of phyllotaxis that we often hear of the "law" of phyllotaxis. However, it must be admitted that not all four-petalled flowers are so rare as the four-leaf clover is reputed to be, and that other combinations also occur, notably in those species exhibiting symmetrical arrangements. Moreover, in the cases of fir cones and some large sunflowers, where the spiral pattern can be verified more carefully, deviations, sometimes even large ones, from the Fibonacci pattern have been found. If this is at all disturbing to the modern botanist, it is not at all so to the Fibonacci devotee, for whom the whole phenomenon, if not a "law", is at least, in the words of H. S. M. Coxeter [1], a fascinatingly prevalent tendency!

REFERENCES

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V. E. Hoggatt, Jr., raised a large sunflower in Santa Clara, California, which exhibited Lucas numbers 76 and 123 in its spiral arrangements. The Lucas numbers 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, ..., which have the same rule of formation as the Fibonacci numbers, seem to occur when Fibonacci numbers do not in phyllotaxis, at least frequently enough to be interesting.