

# Control of a subsonic electropneumatic acoustic generator with dynamic recurrent neural networks

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**Abstract.** A dynamic recurrent neural network model is discussed, which presents two types of adaptive parameters : the classical weights between the units and the time constant associated with each artificial neuron. This dynamic neural model with recurrent connections can deal with time-varying input and output tasks in nontrivial ways. It is successfully used for linearising a subsonic electropneumatic acoustic generator, that is demonstrated to be an efficient but nonlinear acoustic generator.

## 1. Introduction

Control theory deals with the process of influencing the behaviour of a dynamical system so as to achieve a desired objective. The objective is in general to maintain the output(s) of a system (e.g. altitude of an aircraft, glucose level in blood, ...) around prescribed constant levels (*regulation*) or to track predetermined time functions, e.g. the trajectory of a rocket in space (*tracking or servo-control*). The control of linear time-invariant systems was widely developed in the last decade and design methods are now well established. Linear models are in general not adequate for modelling nonlinear systems, and although much effort has been produced on the mathematical properties of nonlinear systems, very few procedures currently exist for designing controllers for such systems. This paper discuss the control of a subsonic electropneumatic acoustic generator. Dynamic recurrent neural networks are shown to be quite efficient in controlling these highly nonlinear systems. The paper is arranged as follows. Section 2 deals with the architecture of recurrent networks, whereas Section 3 is devoted to a brief presentation of the subsonic electropneumatic acoustic generator. The method used for controlling this generator is discussed in Section 4, and last section presents and discuss the results achieved.

## 2. Recurrent neural models

Considerable efforts were made in the past few years in exploring the theory, the architectures and the applications of artificial neural networks for system identification and control. These efforts were essentially focused on feedforward networks, which were shown to be powerful tools for approximating functions,

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for executing classification tasks or to act as an associative memory. Feedforward neural networks essentially perform tasks that can be considered as *static*: recognition of characters, patterns, images, sequences, etc. Feedforward networks merely transform representations. The real power of these parallel distributed representation networks is to select vector representations that embody the desired topological relationships. Problems solved by feedforward networks have a common constraint: they are temporally independent, i.e. the "what" of current input unambiguously determines the current output independently of "when" it occurs.

A class of neural networks known as *recurrent neural networks* is often brought to bear in situations for which time is the important parameter. In recurrent networks, the current activation of the network not only depends on the current input of the system but also on previous inputs. These models exhibit important features not found in feedforward networks, including attractor dynamics and the ability to store information for latter use.

The neural model considered here works in continuous-time space and each neuron-like unit is governed by the following equation [6]:

$$T_i \frac{dy_i}{dt} = -y_i + F(x_i) \quad \text{with} \quad x_i = \sum_j w_{ji} y_j \quad (1)$$

where  $y_i$  is the state or activation level of unit  $i$ ,  $F(\alpha)$  is the squashing function (sigmoid-like function) and  $x_i$  is the total input of the neuron. The model presents two types of adaptive parameters: the classical weights between the units and the time constants  $T_i$ . These ones act as relaxation processes. The correction of the time constants is included in the learning process in order to increase the dynamics of the model. It was indeed demonstrated that these adaptive time constants have a positive influence on the network frequential behaviour, on its dynamical features and on its long-term memory capacities [3], [4].

The network consists of a series of neurons possibly organized in layers. All the possible connections are allowed (feedback, feedforward, self connection and even feedforward and feedback connections between two identical neurons).

Such a network can be trained using a learning algorithm called *Time-Dependent Recurrent Backpropagation* [4].

### 3. Subsonic electropneumatic acoustic generators

Electropneumatic acoustic generators are device which operate by the release of compressed air through an aperture, the area of which is made to vary with time. The device considered in this paper is shown on Figure 1. It consists of a plenum chamber that is supplied with compressed air and that is separated from the source output section by a valve that modulates the airflow. The movement of this valve is controlled by an electrodynamic shaker.

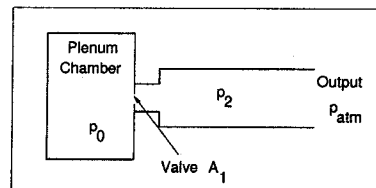


Figure 1: Simplified model of the electropneumatic transducer as suggested by Meyer [5]

Depending on the Mach number  $M_1$  at the throat, the device is said to be sonic ( $M_1 = 1$ ), or subsonic ( $M_1 \ll 1$ ). Subsonic sources were studied in [2] and [1],

and were demonstrated to be one order of magnitude more efficient than common electrodynamic loudspeakers. Moreover, unlike these loudspeakers, electropneumatic generators are, because of their mechanical design, able to resist extreme environments (hot, humid, corrosive, ...). Under some assumptions (the most important being that the system is output memoryless), the fundamental equation of subsonic sources can be written as follows [1] :

$$u_2(t) = \frac{A_1(t)}{A_2} \sqrt{\frac{C_d (p_0 - p_2(t))}{\rho}} \quad (2)$$

where  $u_2(t)$  is the particle velocity at the source output,  $A_1(t)$  is the instantaneous value of the throat area,  $A_2$  is the radial area of the source output section,  $p_0$  is the pressure in the plenum chamber,  $p_2(t)$  is the acoustic pressure at the source output,  $\rho$  is the density of air and  $C_d$  is the discharge coefficient of the orifice formed by the valve, that was experimentally demonstrated to be reasonably constant and equal to 2 throughout the cycle of operation of the source.

Equation (2) clearly shows that provided that the pressure  $p_2(t)$  at the source output is close to the plenum chamber (which is achieved when the source sees an acoustic load resistance  $R_l$  large compared to the source internal impedance), the source behaves nonlinearly. This nonlinear behaviour is even more apparent in equation (3) [1] that shows that the acoustic pressure at the source output  $\delta p_2(t)$  is nonlinearly related to the throat area :

$$\delta p_2(t) = A_2 R_l \left[ \frac{\bar{A}_1 + \delta A_1}{A_2} \sqrt{\frac{2(p_0 - \bar{p}_2 - \delta p_2)}{\rho}} - \bar{u}_2 \right] \quad (3)$$

Equation (3) was directly derived from equation (2), writing all the time-varying quantities as  $p_2 = \bar{p}_2 + \delta p_2$ ;  $u_2 = \bar{u}_2 + \delta u_2$ ;  $A_1 = \bar{A}_1 + \delta A_1$ , where, for example,  $\bar{p}_2$  is the average value of  $p_2$  and  $\delta p_2$  is the time-varying pressure at station 2. Equation (3) gives the solution of the *direct* problem, since it allows to compute the acoustic pressure variations  $\delta p_2(t)$  from the throat area variations  $\delta A_1(t)$ . For control purposes, what must be assessed is the throat area variations required for producing the desired acoustic pressure waveform at the source output. The following nonlinear equation must be therefore be solved, that represents the *inverse* problem :

$$\bar{A}_1 + \delta A_1(t) = \frac{\bar{A}_1 \sqrt{\frac{C_d}{\rho} (p_0 - \bar{p}_2)} + \frac{\delta p_2(t)}{R_l}}{\sqrt{\frac{C_d}{\rho} (p_0 - \bar{p}_2 - \delta p_2(t))}} \quad (4)$$

Note that when solving this equation, one must account for two constraints :

$$A_1(t) \geq 0 \quad \text{and} \quad A_1(t) \leq A_{1,max} \quad (5)$$

where  $A_{1,max}$  is the area of the valve when it is fully open.

The mean pressure  $\bar{p}_2$  in equation (4) can be computed using the assumption according to which the system is output memoryless : if  $A_1(t) = \bar{A}_1 + \delta A_1(t) = 0$ , then  $p_2(t) = p_{atm}$ , where  $p_{atm}$  is the atmospheric pressure. After a little algebra, mean pressure  $\bar{p}_2$  is shown to be the solution of the following equation :

$$\frac{\bar{p}_2^2}{R_l^2} + \bar{p}_2 \left( \frac{C_d}{\rho} \bar{A}_1^2 - \frac{C_d p_{atm}}{R_l^2} \right) + \frac{p_{atm}^2}{R_l^2} - \bar{A}_1^2 \frac{C_d p_0}{\rho} = 0 \quad (6)$$

This equation in  $\bar{p}_2$  has two solutions; the one we are interested in is such that  $\bar{p}_2 > p_{atm}$ .

Analytically solving equation (4) is not a trivial task, hence the interest of using a neural network for solving the inverse problem.

#### 4. Methods

The neural model used is based on the principle of the *inverse identification* (Figure 2). The objective is to identify the system whose output is the throat area of the subsonic source and whose input is the desired pressure (target). The input signal of the neural device is thus this desired acoustic pressure (in Pa) and the output signal is the throat area.

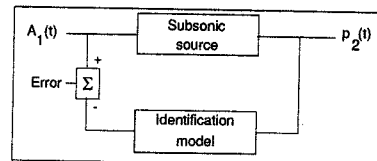


Figure 2: Inverse identification problem

The interest of inverse identification is that it provides the command signal to be applied to the generator in order to get the desired acoustical output : the controller exhibits an open-loop structure and therefore avoids instabilities.

The solution of an inverse identification problem is not always unique. To illustrate this problem, consider for example a particular direct transformation  $f(u)$  whose inverse transformation  $f^{-1}[f(u)]$  leads, for a particular point  $f(u^*)$ , to three different solutions :  $u_1$ ,  $u_2$  and  $u_3$ . Using a dynamic system for the inverse identification is a good way for resolving the potential ambiguity, i.e. for choosing the right possibility, since such a system learns the temporal evolution of the inverse transformation.

For identification purposes, a fully-connected recurrent neural networks was used. The training phases consisted in the identification of different couples of signals (desired pressure as output and surface area of the valve as input) computed by solving equation (4). Several other couples of signals were also generated to investigate the efficiency of the training phase. These signals last for 0.05 s and used a time-step of 0.2 ms (i.e. 250 points). This guaranteed that at least five periods were present in every sampled signal. Moreover, a time-step of 0.2 ms satisfied the *Asymptotic Consistency Criterion* (see [4]).

#### 5. Results

The inverse identification of the subsonic generator was simulated first for sinusoidal acoustic pressures between 50 Hz and 400 Hz. Different mean values  $\bar{A}_1$  of the throat area were chosen.

Two different sets of data were generated : one for the training of the network and the other one for the validation of the identification. The values of  $\bar{A}_1$  for the training process ranged from 0.4 to 4.0 cm<sup>2</sup> by step of 0.4 cm<sup>2</sup> and for the validation process from 0.2 to 3.6 cm<sup>2</sup> by step of 0.4 cm<sup>2</sup>.

Conditions were as follows : atmospheric pressure  $p_{atm}$  : 102,100 Pa; plenum pressure  $p_0 = p_{atm} + 1,100$  Pa; air density  $\rho$  : 1.2 kg/m<sup>3</sup>;  $A_2$  : 16 cm<sup>2</sup>. After several tests, we found that the best architecture for the network was a 20 fully-connected neuron model.

The results show that after the training phase, the network is able to generalize for unknown values of  $\bar{A}_1$  (see Table 1). For illustration purposes, Figure 3(left)

Training set		Validation set	
#	RMS Error	#	RMS Error
1	$6.41 \cdot 10^{-4}$	1	$6.47 \cdot 10^{-4}$
2	$6.29 \cdot 10^{-4}$	2	$6.35 \cdot 10^{-4}$
3	$6.12 \cdot 10^{-4}$	3	$6.21 \cdot 10^{-4}$
4	$5.92 \cdot 10^{-4}$	4	$6.03 \cdot 10^{-4}$
5	$5.65 \cdot 10^{-4}$	5	$5.79 \cdot 10^{-4}$
6	$5.31 \cdot 10^{-4}$	6	$5.49 \cdot 10^{-4}$
7	$4.89 \cdot 10^{-4}$	7	$5.11 \cdot 10^{-4}$
8	$4.32 \cdot 10^{-4}$	8	$4.62 \cdot 10^{-4}$
9	$3.62 \cdot 10^{-4}$	9	$3.99 \cdot 10^{-4}$
10	$2.72 \cdot 10^{-4}$	10	$3.21 \cdot 10^{-4}$
Mean	$5.12 \cdot 10^{-4}$	Mean	$5.32 \cdot 10^{-4}$

Table 1: Root mean square errors for the ten different training and validation signals. The last line gives the mean error over the 10 signals of the set.

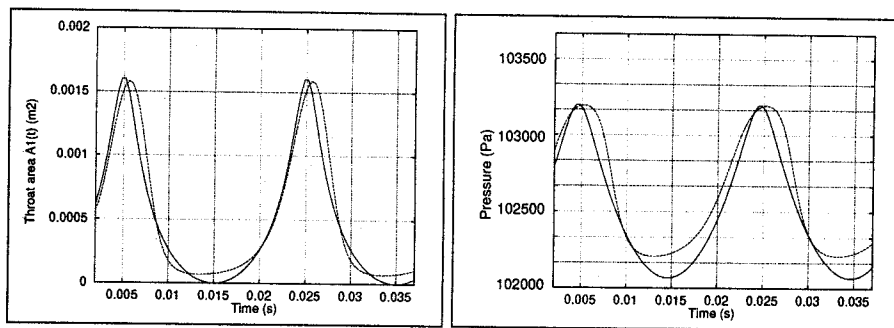


Figure 3: Left : comparison between the throat area  $A_1(t)$  generated by the network (dashed line) and the desired one (solid line) for the validation set 1. Right : comparison between the output pressure resulting from the DRNN command (dashed line) and the desired one (solid line) for the validation set Nr 1,  $\bar{A}_1 = 0.2 \text{ cm}^2$ .

depicts a comparison between the throat area  $A_1(t)$  generated by the network (dashed line) and the desired one (solid line) for the validation set 1.

Using the command signal  $A_1(t)$  generated by the network, the direct problem was solved to get the output pressure. Figure 3(right) compares the pressure at the output of the generator (i.e. the generated sound). Harmonics in this signal were at least 20 dB below the fundamental which shows that the system is quite efficient in producing sinusoidal acoustic pressures.

We also tested the robustness of the dynamic recurrent model to generate periodic but nonsinusoidal signals. For this test, the network was trained with ten different signals based on different combinations of sinusoidal signals whose fundamental frequencies were 50 Hz, 100 Hz, 150 Hz. The network was then validated with a signal based on a combination of sinusoidal signals with fundamental frequencies of 75 Hz and 125 Hz. Figure 4 shows the comparison between the output pressure resulting from the DRNN command (dashed line) and the desired one (solid line) for this validation signal. The shape of the acoustic pressure is reasonably close to the desired pressure.

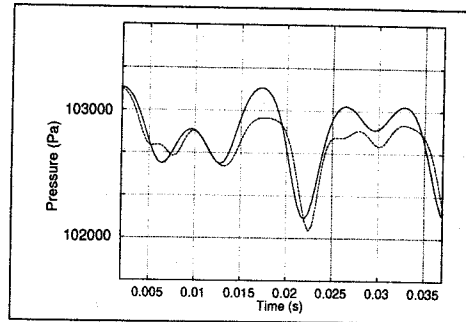


Figure 4: Test of robustness of the system. This plot shows the comparison between the output pressure resulting from the DRNN command (dashed line) and the desired one (solid line).

## 6. Conclusion

Dynamic recurrent neural networks can provide a fruitful method for controlling complex temporal systems. The success of the neural computing solution can be judged from the previous results: the network converges to a plausible dynamical behaviour and different validation tests prove the efficiency of the control. Moreover, the system is able to extrapolate command signals with various fundamental frequencies.

Furthermore, due to their dynamical features and to their features, dynamic recurrent neural networks can be applied to several other research fields. We are currently studying some other applications of dynamic neural networks in the fields of mathematics (such as interpolation tasks i.e., for the forecasting of stock market value) and of engineering.

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