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Improved generalization ability of neurocontrollers by imposing NL_q stability constraints

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Abstract. In this paper we demonstrate how the generalization ability of neural controllers can be improved by imposing an additional stability constraint for the closed-loop system. The generalization is improving with respect to process noise and measurement noise. In order to achieve this, conditions for global asymptotic stability from NL_q theory are used in order to constrain the dynamic backpropagation procedure.

1. Introduction

Thanks to the universal approximation ability, multilayer perceptrons are powerful tools to parametrize general nonlinear models and controllers. In a recurrent neural network context, a classical method for training neural controllers, based upon identified neural network models for the plant, is dynamic backpropagation [3, 4]. Because of the finite time horizon that has to be chosen for tracking specific reference inputs, it may lead to bad generalization of the controller to future data points. This problem has been observed on a real-life example of controlling a ball and beam system [9]. In this case even instabilities were observed for the control scheme, while the error on the training set was low for a step or sinusoidal reference input.

An approach to overcome these problems is to apply NL_q theory [6, 7, 8], which is a neural control framework with global asymptotic stability criteria. The stability criteria can be used in order to constrain the dynamic backpropagation algorithm, i.e. searching for optimal performance within a class of stabilizing neural controllers. In this way the ball and beam system was successfully controlled [9]. Often the neural controller is trained based upon a deterministic nonlinear model and applied afterwards to a possibly noisy nonlinear plant.

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The aim of this paper is to show that by imposing the NL_q stability constraint to dynamic backpropagation, the generalization ability of the neural controller improves with respect to process noise and measurement noise on the system. Also for system identification and classification problems it is known that a modification of the original mean squared error cost function (regularization in this case) leads to improved generalization [1, 5] of the model. Here we observe an improved generalization by means of stabilization in the context of control problems.

This paper is organized as follows. In Section 2 we present the neural control scheme. In Section 3 we derive the NL_q representation for the closed-loop system. In Section 4 the modified dynamic backpropagation with NL_q stability constraint is discussed. The improved generalization ability is illustrated on an example in Section 5.

2. Neural control scheme

Consider a nonlinear plant $\mathcal{P}$ for which we assume the following true model

$$\mathcal{P}: \left\{ \begin{array}{rcl} x_{k+1} & = & W_{AB} \tanh(V_A x_k + V_B u_k + \beta_{AB}) + w_k \\ y_k & = & W_C \tanh(V_C x_k) + v_k \end{array} \right. \quad (1)$$

with state $x_k \in \mathbb{R}^n$, input $u_k \in \mathbb{R}^m$, output $y_k \in \mathbb{R}^l$, zero mean white Gaussian process noise $w_k \in \mathbb{R}^n$ and measurement noise $v_k \in \mathbb{R}^l$. The state and output equation are parametrized by one-hidden layer multilayer perceptrons with interconnection matrices W_*, V_* and bias vectors β_* of appropriate dimension. The number of hidden units is equal to n_{h_x} and n_{h_y} respectively.

We assume that the following deterministic model $\mathcal{M}$ has been identified for the plant and a neural controller $\mathcal{C}$ has to be designed based upon $\mathcal{M}$ in order to control the plant $\mathcal{P}$:

$$\begin{array}{l} \mathcal{M}: \left\{ \begin{array}{rcl} \hat{x}_{k+1} & = & W_{AB} \tanh(V_A \hat{x}_k + V_B u_k + \beta_{AB}) \\ y_k & = & W_C \tanh(V_C \hat{x}_k) \end{array} \right. \\ \mathcal{C}: \left\{ \begin{array}{rcl} z_{k+1} & = & W_{EF} \tanh(V_E z_k + V_F y_k + V_{F_2} d_k + \beta_{EF}) \\ u_k & = & W_G \tanh(V_G z_k) \end{array} \right. \end{array} \quad (2)$$

where $\hat{x}_k \in \mathbb{R}^n$ is the estimated state vector and d_k is the reference input of the control scheme. The number of hidden units for $\mathcal{C}$ is equal to n_{h_z} and n_{h_u} respectively.

3. Closed-loop scheme in NL_q form

The state equation for the closed loop system $\{\mathcal{M}, \mathcal{C}\}$ is

$$\left\{ \begin{array}{rcl} \hat{x}_{k+1} & = & W_{AB} \tanh(V_A \hat{x}_k + V_B W_G \tanh(V_G z_k) + \beta_{AB}) \\ z_{k+1} & = & W_{EF} \tanh(V_E z_k + V_F W_C \tanh(V_C \hat{x}_k) + V_{F_2} d_k + \beta_{EF}). \end{array} \right. \quad (3)$$

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By introducing additional state variables $\xi_k = \tanh(V_C \hat{x}_k)$ and $\eta_k = \tanh(V_G z_k)$ one obtains

$$\left\{ \begin{array}{lcl} \hat{x}_{k+1} & = & W_{AB} \tanh(V_A \hat{x}_k + V_B W_G \eta_k + \beta_{AB}) \\ z_{k+1} & = & W_{EF} \tanh(V_E z_k + V_F W_C \xi_k + V_{F_2} d_k + \beta_{EF}) \\ \xi_{k+1} & = & \tanh(V_C W_{AB} \tanh(V_A \hat{x}_k + V_B W_G \eta_k + \beta_{AB})) \\ \eta_{k+1} & = & \tanh(V_G W_{EF} \tanh(V_E z_k + V_F W_{CD} \xi_k + V_{F_2} d_k + \beta_{EF})) \end{array} \right. \quad (4)$$

with state vector $p_k = [\hat{x}_k; z_k; \xi_k; \eta_k]$.

The latter equation is in NL_q form [6, 7]. Since we will focus on internal stability in the sequel, we write down the equation for the autonomous NL_q system ($q = 2$):

$$p_{k+1} = \Gamma_1(V_1 \Gamma_2(V_2 p_k)) \quad (5)$$

with

$$V_1 = \begin{bmatrix} W_{AB} & & & \\ & W_{EF} & & \\ & & V_C W_{AB} & \\ & & & V_G W_{EF} \end{bmatrix}, V_2 = \begin{bmatrix} V_A & 0 & 0 & V_B W_G \\ 0 & V_E & V_F W_C & 0 \\ V_A & 0 & 0 & V_B W_G \\ 0 & V_E & V_F W_C & 0 \end{bmatrix}$$

and Γ_1, Γ_2 are diagonal matrices with diagonal elements depending on p_k but belonging to $[0, 1]$ for all values of p_k (which corresponds to the fact that tanh(.) belongs to sector $[0, 1]$). The matrices V_1 and V_2 are of dimension $n_1 \times n_2$ and $n_2 \times n_1$ respectively, with $n_1 = n + n_z + n_{h_y} + n_{h_u}$ and $n_2 = 2n_{h_x} + 2n_{h_z}$. For NL_q systems, conditions are available for global asymptotic stability, input/output stability with finite L_2-gain and robust stability [6, 7]. The criteria have been expressed in terms of diagonal scaling, diagonal dominance and condition number factors. It has been shown how these stability criteria can be used in order to control nonlinear behaviour with a unique or multiple equilibria, periodic behavior and chaos [6].

4. Dynamic backpropagation with NL_q stability constraints

A classical method for training recurrent neural networks is Narendra's dynamic backpropagation procedure [3, 4]. In order to learn the neural controller to track a specific reference input d_k, one formulates the optimization problem

$$\min_{\theta_c} J(\theta_c) = \frac{1}{2N} \sum_{k=1}^{N} \|d_k - y_k(\theta_c)\|_2^2 \quad (6)$$

where θ_c denotes the neural controller parameter vector and N is the finite time horizon for the training data. In order to impose global asymptotic stability of the closed-loop scheme $\{\mathcal{M}, \mathcal{C}\}$, dynamic backpropagation can be modified with NL_q stability constraints [6, 7]. We use here the criteria which are expressed

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in terms of condition numbers:

$$\min_{\theta_c, P_{tot}} J(\theta_c) \quad \text{such that} \quad \begin{cases} \|P_{tot} V_{tot} P_{tot}^{-1}\|_2 < c_1 < 1 \\ \kappa(P_{tot}) < c_2 \end{cases} \tag{7}$$

with

$$V_{tot} = \begin{bmatrix} 0 & V_2 \\ V_1 & 0 \end{bmatrix}, \; P_{tot} = \begin{bmatrix} P_2 & 0 \\ 0 & P_1 \end{bmatrix}.$$

The positive real constants c_1, c_2 are user defined and $\kappa(.)$ denotes the condition number of a matrix. The first constraint imposes local stability of the origin while minimizing the upper bound c_2 enlarges the basin of attraction of the origin. Strictly speaking global asymptotic stability is only guaranteed if $\kappa(P_1)\kappa(P_2)c_1 < 1$, but simulation results on several types of nonlinear systems strongly suggest that $c_1 < 1$ together with a minimization of these condition numbers can be sufficient in order to obtain global asymptotic stability, even for chaotic systems [6].

Now many simulation results show that controllers, obtained as solution to (7), also have an improved generalization with respect to (6), when process noise and measurement noise are added to the model as for the true model (1). This phenomenon is illustrated on an example in the next Section.

5. Simulation example

In this example we study the following model $\mathcal{M}$

$$W_{AB} = \begin{bmatrix} 0.0541 & -0.0088 & -0.3261 \\ 0.0053 & -0.4749 & 0.1917 \\ -0.4819 & 0.1042 & -0.2935 \end{bmatrix}, V_A = \begin{bmatrix} -0.2587 & -0.5950 & -0.3103 \\ 0.4379 & 1.2163 & -0.3023 \\ -0.3008 & -0.2314 & 0.4797 \end{bmatrix},$$

$$W_C = \begin{bmatrix} 0.6250 & 0.4344 & 1.2744 \\ -1.0473 & -1.9171 & 0.6385 \\ 1.5357 & 0.4699 & 1.3808 \end{bmatrix}, V_C = \begin{bmatrix} 1.3198 & 1.7887 & -0.4060 \\ -0.9094 & 0.3908 & -1.5349 \\ -2.3056 & 0.0203 & 0.2214 \end{bmatrix} \tag{8}$$

and $V_B = I_3$ where $n = 3, n_{h_x} = 3, n_{h_y} = 3, m = 3, l = 3$. Simulation results suggest that it is globally asymptotically stable.

We consider the problem of tracking a sinusoidal reference input $d_k = \sin(0.5\,k)$. We take a third order neural controller with $n_z = 3, n_{h_z} = 3, n_{h_u} = 3$. A time horizon of $N = 30$ has been taken in order to compare classical dynamic backpropagation (6) and modified dynamic backpropagation (7). The optimization has been done by means of a quasi-Newton method and sequential quadratic programming respectively [2] (Matlab's *fminu* and *constr*). As starting points, random controller parameter vectors θ_c have been taken (zero mean normal distribution with standard deviation 0.1) and P_1, P_2 identity matrices. For $c_1 = 0.95$ and $c_2 = 100$ two good local minima are compared by simulating the closed-loop system of the neural controllers $\mathcal{C}$ with the plant $\mathcal{P}$ (1) for different noise levels σ. The process noise and measurement noise was chosen with zero mean and equal variance σ. In order to check the generalization ability of the two controllers the cost functions $J(\theta_c)$ were calculated

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for a larger time horizon $N = 100$ (including the first 30 data points from the training and 70 points of test data). The values of $J(\theta_c)$ were averaged over 50 independent simulation results at each of the different noise levels for σ values ranging in the interval $[0, 0.2]$. As a result from these experiments one clearly sees that dynamic backpropagation with stability constraint has an improved generalization ability when noise starts playing a significant role in degrading the tracking performance (Fig.1).

6. Conclusions

We illustrated on a simulation example of training neural controllers, that imposing NL_q stability constraints to dynamic backpropagation improves the generalization ability with respect to process noise and measurement noise. The combination of dynamic backpropagation with NL_q theory offers a more reliable controller design procedure.

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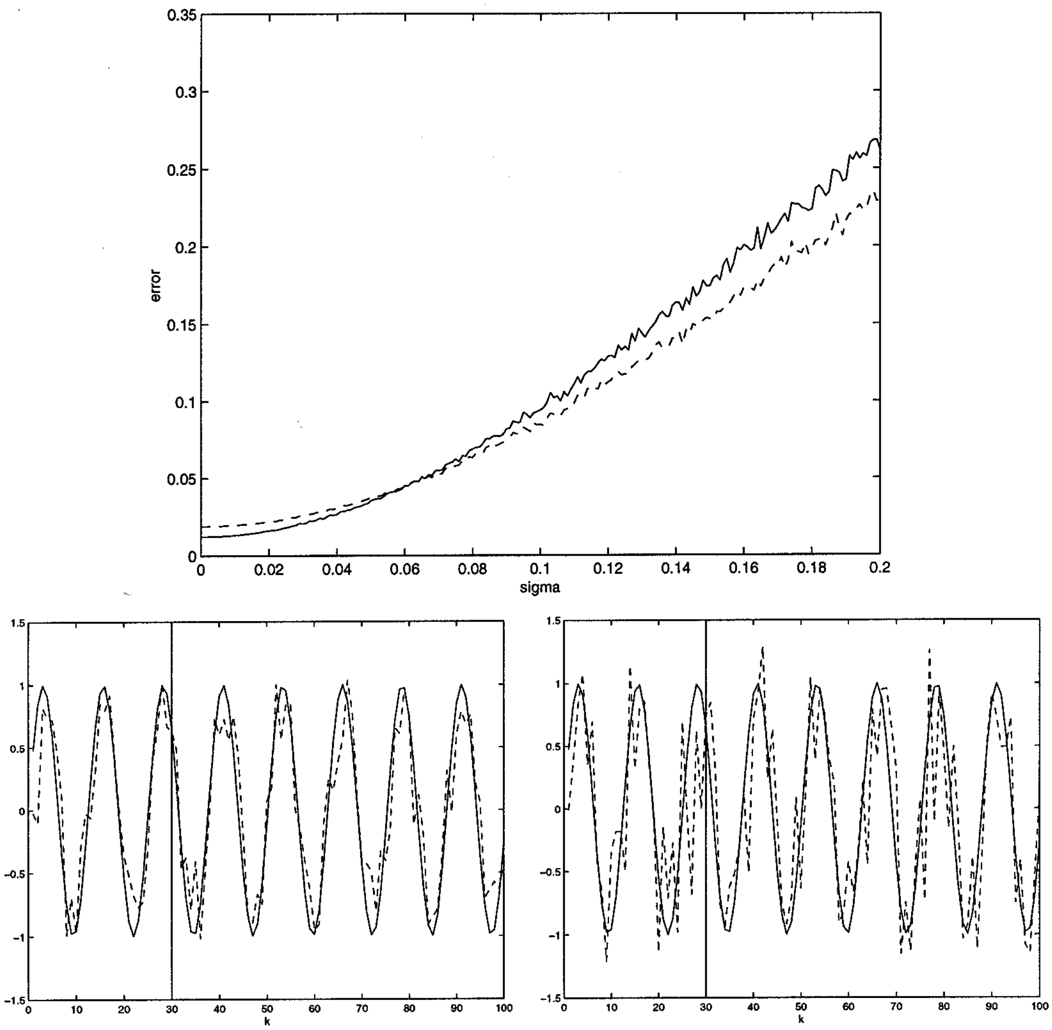

Fig. 1: *(Top) Comparison between dynamic backpropagation (full line) and modified dynamic backpropagation with NL_q internal stability constraint (dashed line). The generalization for tracking error is shown with respect to the noise level σ of equal process noise and measurement noise, averaged over 50 simulations for each noise level. (Bottom-right) result for modified dynamic backpropagation at $\sigma = 0.06$: 30 training data before the vertical line and 70 points of test data; sinusoidal reference input (Full line) and output of the plant (dashed line). (Bottom-left) $\sigma = 0.12$.*