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On-off intermittency in small neural networks with synaptic noise

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Abstract. Numerical evidence is presented for the occurence of on-off intermittency (OOI) and attractor bubbling (AB) in the time series of synaptic potentials of analog neurons with time-dependent synaptic noise. The results were obtained for a single neuron with synaptic self-connection and a network of two neurons with various weights of synaptic connections.

1. Introduction

Discrete-time chaotic neurons and neural networks (NNs) of chaotic neurons with continuous output function are a subject of intensive research [16,5,2,1], as they are both complex enough to model chaotic behaviour of real neurons and simple enough to allow efficient numerical modelling. The effect of noise on the dynamics of such neurons is usually considered for the case of threshold noise. Recently it was pointed out that another biologically significant source of noise arises from the random release of chemical transmitters into synapses [7,6,8]. This kind of multiplicative noise, which we call here time-dependent synaptic noise, may be modelled by updating weights of synaptic connections at every time step according to a certain probability distribution. Analysis of quantal synaptic noise has been carried out only in binary NN [7,6], though it was suggested that application of this concept to analog neurons is straightforward [7]. In this paper we consider one of many possible effects which appear in analog NNs due to this kind of noise, namely OOI and AB.

A chaotic system exhibits OOI when in the observed signal a sequence of laminar phases, during which the signal is almost constant, and chaotic bursts occurs [13,9]. The simplest model capturing essential features of OOI is the linear map with the control parameter varying randomly in time [9]

$$y_{n+1} = a x_n y_n + \xi x'_n \qquad (1)$$

Here, y_n is the observed variable, x_n, x'_n are two non-correlated random variables with uniform distribution at the interval [0,1), a is the system control parameter and ξ is the amplitude of additive noise. If $\xi = 0$ and $a < e = 2.718\ldots$

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then y_n converges to zero as n increases, independently on its initial value. If $\xi = 0$ and a slightly exceeds $a_c = e$ then a sequence of quiescent phases, when $y_n \approx 0$, and chaotic bursts, when $y_n \sim O(1)$ is observed. The distribution of laminar phase durations obeys the scaling law $P(\tau) \propto \tau^{-3/2}$ and their mean duration decreases with a as $\langle\tau\rangle \propto |a - a_c|^{-1}$ [9]. If $\xi > 0$ then bursts appear already if $a > a_b = 1$ and the transverse instability of the attractor $y_n = 0$ caused by noise or other factors is called AB [12,3] Recently, scaling laws e.g. for the maximum burst amplitude and variance of y_n as a function of a, ξ were obtained [15,4].

OOI and AB are ubiquitous in many physical systems, e.g. in systems with symmetry, in the chaotic synchronization problem etc. [15], but to our knowledge they have not been reported in NNs. Only a related problem of attractors with riddled basins has been discussed for the case of back-propagation learning of chaotic time series by NN [11]. In the following, small analog NNs are considered in which OOI and AB occur due to the presence of time-dependent synaptic noise. For simplicity we consider only the uniform and continuous synaptic noise, i.e. at every time step n the synaptic connection weights are chosen as $w_{ij}(n) = w_{ij}x_n$, where w_{ij} is a constant connection strength and x_n is a random variable with uniform distribution at [0,1). The results for quantal noise (which occurs in biological neurons [7,6,8]) differ from this case only quantitatively. The results below are also discussed as a special case of OOI in coupled map lattices (CMLs).

2. On-off intermittency in a single neuron

A trivial example of OOI in a single neuron is obtained by considering analog neuron with synaptic self-connection and zero threshold, in the absence of external inputs and threshold noise

$$y(n+1) = f_\mu(w_{11}x_n y(n)) \tag{2}$$

where $y(n)$ is the neuron output at time n, $f_\mu(z)$ is a neuron activation function and μ is a neuron gain. If we assume a symmetric activation function $f_\mu(z) = \tanh(\mu z)$ with $\mu = 1$ then there is always a fixed point $y_n = 0$ of (2). In its neighbourhood the map (2) may be linearized to yield $y(n+1) = w_{11}x_n y(n)$ which is exactly the map (1) with $a = w_{11}$ and $\xi = 0$. Then if $|w_{11}| > w_{11,c} = e$ a sequence of chaotic bursts in the neuron output is observed, separated by laminar phases during which $y_n \approx 0$. The actual behaviour of neuron output depends on the sign of w_{11} and the initial condition $y(0)$ (Fig.1a). If $w_{11} > 0$ then during the bursts the neuron is either inactive ($y(n) < 0$ for $y(0) < 0$) or active ($y(n) > 0$ for $y(0) > 0$). If $w_{11} < 0$ the bursts consist in rapid switching between the active and inactive state (Fig.1a). The duration of laminar phases obeys the statistics $P(\tau) \propto \tau^{-3/2}$ (Fig.1b).

It should be noted that for $f_\mu(z) = 1/(1 + \exp(-\mu z))$ the map (2) has not a fixed point $y_n = 0$ and thus OOI does not occur.

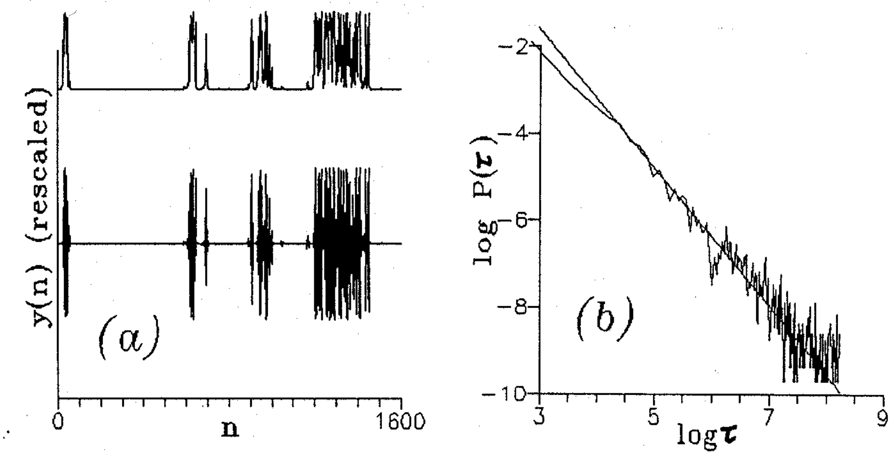

Figure 1: a) Time series of $y(n)$ for $w_{11} = 2.8 > 0$, $y(0) > 0$ (top) and $w_{11} = -2.8 < 0$ (bottom); b) log-log plot of $P(\tau)$ vs. τ for $w_{11} = 2.75$, straight line has a slope -3/2

A less trivial example may be obtained in a model of chaotic neuron with symmetric activation function and refractoriness [2,1]. Our investigation showed that stronger refractoriness increased the OOI threshold while introducing the dependence of the neuron state on its history, in general, decreased the threshold.

3. On-off intermittency and attractor bubbling in two-neuron neural networks with symmetric activation function

A two-neuron NN with parallel updating may be described by a map

$$\begin{aligned} y_1(n+1) &= f_\mu\left(w_{11}x_n^{(11)}y_1(n) + w_{12}x_n^{(12)}y_2(n)\right) \\ y_2(n+1) &= f_\mu\left(w_{21}x_n^{(21)}y_1(n) + w_{22}x_n^{(22)}y_2(n)\right) \end{aligned} \quad (3)$$

where $x_n^{(ij)}$, $i, j = 1, 2$ are non-correlated random variables. In this form we deal with a two-dimensional CML. We start with the case $f_\mu(z) = \tanh(\mu z)$ and $\mu = 1$.

A simplest case to analyze is $w_{11} = a$, $w_{12} = 0$, $w_{21} = \varepsilon$, $w_{22} = b$, with $a, b, \varepsilon > 0$, $\varepsilon \ll a, b$. Then the neurons have strong self-connections and neuron 2 is weakly connected to neuron 1. As neuron 1 is independent of 2, y_1 decreases to zero if $a < e$ and shows OOI if $a > e$. If $a < e$ and $b > e$ then neuron 2

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shows OOI. However when $a > e$ and $1 < b < e$ we observe AB in the time series of y_2 — the dynamics of neuron 1 influences that of neuron 2 in a similar manner as a weak external noise with amplitude ξ. If $a < e$ the maximum burst amplitude $y_{2,max}$ of the output of neuron 2 suddenly increases with b at $b = e$, and if $a > e$ it rises more gradually. For $a < e$ the variance var (y_2) of the output of neuron 2 increases linearly with $b - e$. For $a > e$, the variance var (y_2) has a tail for $b \ll e$, where the dynamics of neuron 2 is fully determined by the influence of neuron 1, and gradually increases when $b \approx e$ (Fig.2a). This behaviour of $y_{2,max}$ and var (y_2) is typical of OOI and AB [15,4]. From the point of view of CMLs we observed here AB in one map triggered by a second map with OOI.

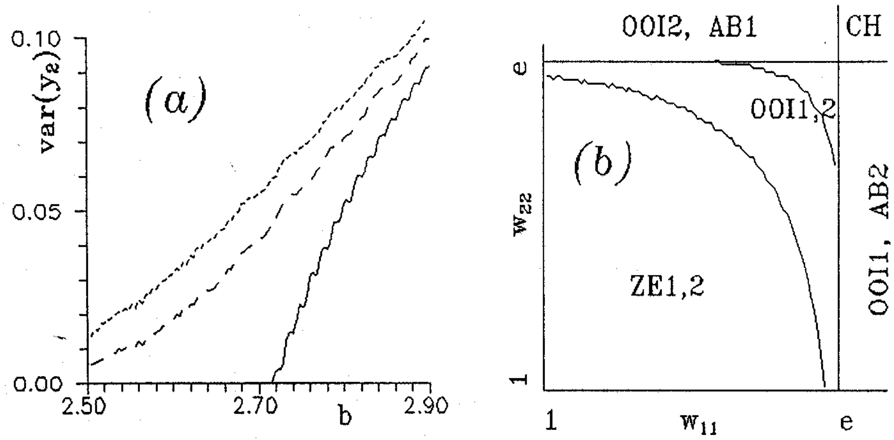

Figure 2: a) var (y_2) vs. b for one-way coupled two-neuron NN, $\varepsilon = 1.0 \cdot 10^{-5}$ (dots), $\varepsilon = 1.0 \cdot 10^{-7}$ (dashes), $\varepsilon = 0$ (solid line), $a = 3.0$; b) type of activity of individual neurons in a two-way coupled two-neuron NN, numbers $i = 1, 2$ denote the neurons, ZEi — $y_i(n)$ approaches zero with rising n, CH — chaotic bursting

A more complex case is with $w_{11} = a$, $w_{22} = b$, $w_{12} = w_{21} = \varepsilon \ll a, b$. The "phase diagram" for two different values of ε is shown in Fig.2b. E.g. for given a and $\varepsilon = 0.001$ three different routes to chaos are possible. If $a < 2.04$, then if $b < e$ both y_1, y_2 decay to zero and if $b > e$ neuron 2 shows OOI and neuron 1 shows AB. If $2.1 < a < e$ both neurons exhibit OOI as b is increased above some critical value $b_c < e$. An important feature is the decrease of the OOI threshold with ε, an effect already reported in big CMLs of equivalent maps [14]. If $a > e$ then neuron 1 shows OOI and neuron 2 shows AB. Both transitions to chaos via OOI and AB have similar properties as in the case of $w_{12} = 0$. However, b_c is now a function of a, ε. As we could see coupling different maps with OOI may lead to coexistence of OOI and AB in one system, a property, to our knowledge, not reported in the literature on CMLs.

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The two examples above are interesting as they have much in common with the current interest in OOI in CMLs, but connections among different neurons are usually stronger than self-connections. In this case OOI is also possible, e.g. in a system without self-connections and a symmetric weight matrix $w_{11} = w_{22} = a$. The OOI threshold in this case is again $a_c = e$ and it is decreased by adding non-zero self-connections. The individual neurons exhibit interesting behaviour: if at time step n e.g. $y_1(n) \sim O(1)$ then $y_1(n+1) \approx 0$ and neuron 2 has exactly the same dynamics shifted in phase by one time step. Thus there are plenty of laminar phases of unit length; this does not influence the probability distribution of longer laminar phase durations and OOI characteristics are still observed. OOI in similar CMLs with strong self-connections has not been investigated yet.

4. On-off intermittency and attractor bubbling in two-neuron neural networks with logistic activation function

If in Eq.(3) $f_\mu(z) = 1/(1+\exp(-\mu z))$ OOI can appear only under very restrictive conditions. E.g. it was shown in Ref.[15] that in a NN with $w_{11} = -5$, $w_{12} = 5$, $w_{21} = -25$, $w_{22} = 25$ the period-doubling route to chaos is observed with a stable fixed point $y_1 = y_2 = 0.5$ for $\mu < 0.21$. Treating μ as the control parameter and increasing it above $\mu_c \approx 0.61$, or, equivalently, keeping μ constant and proportionally increasing all synaptic weights, it is possible to obtain OOI with laminar phases y_1, $y_2 \approx 0.5$ in the above-mentioned NN with uniform time-dependent synaptic noise. However, the noise in all connections must be perfectly correlated, i.e. at every n all $x_n^{(ij)}$ must be equal. Any small lack of correlation between noise in different synapses or small deviation from the exact values of synaptic weights results in AB rather than OOI. Thus NN of logistic neurons are not expected to exhibit OOI in real experiments.

5. Conclusions

In this paper we have shown that OOI and AB can appear in analog NN with time-dependent synaptic noise. The mechanism of their appearance is exactly the same as in Eq.(1). This picture may be obscured in real neurons by the influence of external inputs and threshold noise, but we think that these phenomena should be considered as a possible cause of large-scale bursts of neural activity.

The results presented here are also important in the investigation of OOI in high-dimensional complex systems. So far, only CMLs of identical maps have been considered in this context (see [14] and references therein). NN form a natural basis for the investigation of OOI in systems consisting of different coupled subsystems; some research in this line has been done only in magnetic

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systems so far [10]. As we have shown in such systems different parts of them may exhibit OOI, AB or remain quiescent. We are going to extend the present results to the case of large NN and to investigate the possible peculiarities of scaling laws characteristic of OOI and AB in such systems. These results, and the ones concerning NN with biologically motivated quantal time-dependent synaptic noise will be published separately.

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