The Fibonacci series

In the 12th century, Leonardo Fibonacci took an interest in the number of rabbits that could be raised in a year if one began with a single pair. Fibonacci reasoned that, if the rabbits reached maturity two months after birth and produced an additional pair every month thereafter, the total population of rabbit pairs would increase monthly according to the series

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

This deceptively simple-looking series is obtained merely by sequentially adding numbers which are the sum of the preceding two.

The ratio of each successive pair of numbers in the series approximates Phi or Φ (where Φ = 1.618…), as 5 divided by 3 is 1.666..., and 8 divided by 5 is 1.60. The ratios of the successive numbers in the Fibonacci series quickly converge on Phi, where Phi is called the **Golden Ratio**. After the 40th number in the series, the ratio is accurate to 15 decimal places.

$$\Phi = \mathbf{1.618033988749895, \ldots}$$

The Golden Ratio consists of the two numbers 1.618034 and 0.618034, each of which is the reciprocal of the other. Rectangles with sides proportioned 0.618034 to 1 (or 1 to 1.618034) are often the shape taken by such commonplace items as picture frames and playing cards.

1.618034 and 0.618034

These are remarkable numbers. Not only are the figures after the decimal point identical in both, but each is the reciprocal of the other (that is, the number 1 divided by either yields the other). These are the only two numbers that demonstrate this property. Unlike pi, another fundamental constant in which the decimals extend to infinity (π = 3.14159…), these factors are exact after the first six decimals.

Thus, the shape seems to be subliminally pleasing to the human eye, as witnessed by the many ways in which it is used in art and in construction. It is also found in nature, reflected in essentially every spiral form from a snail shell to the arms of a galaxy.

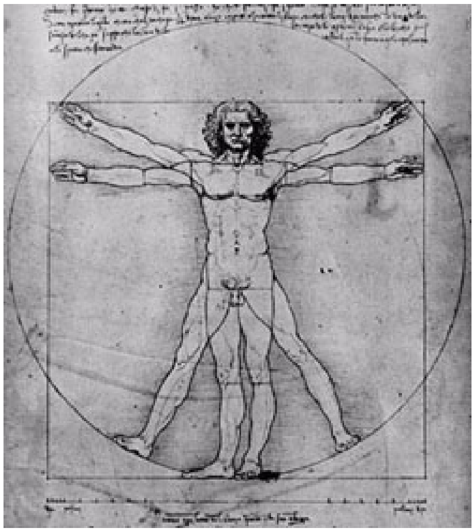
Leonardo Fibonacci

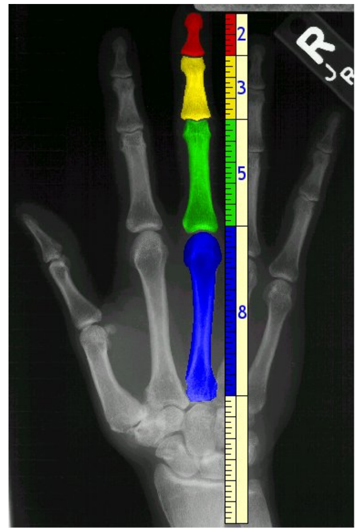
Examples of the occurrence of the Golden Ratio in nature and civilization

The earliest evidence of human appreciation for the pleasing qualities of these proportions is found in the pyramids at Giza, which appear to have been built with a 5 to 8 ratio between height and base. This is a close approximation (0.625) to the "perfect" ratio, although scholars disagree over whether the Egyptians were actually aware of it.

Even if it is not certain that the Egyptians knew of the ratio, there is no question that the Greeks had been able to calculate it. They called it the "Golden Ratio". Not understanding why, they knew that it felt good and it looked good, and they incorporated it into much of their art and into many of their buildings (including the Parthenon, which is generally considered to be antiquity's most perfect structure)

The secret was lost with the fall of Greece, but it began to resurface in the 16th century when Leonardo da Vinci utilized it in his painting and sculpture. Soon, many of the masters began to proportion their canvases according to the Golden Ratio, and it is still the shape most preferred today for anything from window blinds to table tops.





But it is not just the artistic eye that appreciates the golden ratio (for the musically inclined person, it should be noted that musical harmonics are also based on the Golden Ratio). Its expression is found almost any place we look in nature.

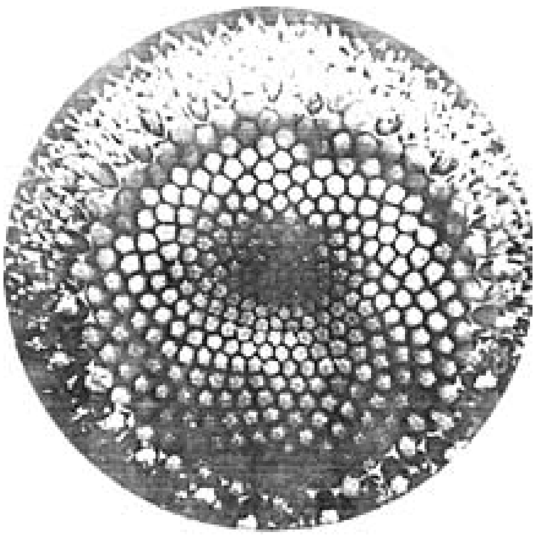
Among other things, there is a common growth pattern in which the sizes of additional units are governed by a proportionality relationship to units that have been formed earlier. The same golden ratio that appears as a rectangle in human constructs often is expressed in nature forms as an elegant spiral. The chambered nautilus expresses this principle as it outgrows its old "living quarters" and sequentially builds roomier ones in a spiral pattern whose dimensions are determined by the golden ratio. When a new apartment is finished, the animal crawls into it and slams the door on the old one with a lid of mother-of-pearl. The old chambers are then used for flotation.

Spirals emerge from the golden ratio in the following manner: If a golden rectangle is subdivided by a line which forms a square out of one end, the remainder is another, smaller golden rectangle. Repeated subdivision results in a series of squares and golden rectangles which spiral into the center of the pattern. Connecting the centers of all the squares formed in this manner gives rise to a series of "legs," each of which is relative in length to the one preceding or following by the golden ratio. Together, these all form a "logarithmic spiral," which is so rampant in nature.

Science cannot number the many applications that the Golden Ratio finds in nature. But to the person on the street, as it was to the ancient Greeks, maybe the best thing about it is that it looks nice.

A Daisy head reveals two sets of opposing spirals formed by individual florets. The clockwise spiral contains 21 arms; the counter-clockwise spiral contains 34. These are two adjacent numbers in the Fibonacci series.

Other examples of the Golden Ratio include the spiral arms of galaxies, e.g. the well-known galaxy M51. The length-to-width ratio of regular playing cards also closely matches the Golden Ratio.



References:

Larry Gedney, Geophysical Institute, University of Alaska Fairbanks, in cooperation with the UAF research community (2003)

NASA (pictures from Hubble space telescope), 2003

www.eatonhand.com/hw/fibonacci1.jpg (July 2003)

www.gi.alaska.edu/ScienceForum/ASF7/716.html (July 2003)

www.allaboutcards.powerup.com.au/Forcb.htm (July 2003)