Estimation of daily yield from AM/PM milkings

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February 27, 2007

Introduction

A method is specified to estimate 24-hour milk volume, fat and protein yields based on partial yields from either morning (AM) or evening (PM) milkings.

Method

Let y_{24} refer to 24-hour yield and y_{am} and y_{pm} denote the corresponding AM and PM yields. Daily yield can be estimated from a single AM milking as

$$\hat{y}_{24} = \hat{F}_{am} y_{am}$$

where $\hat{F}_{am}$ is an estimated expansion factor. If we think of the above as a regression through the origin, $y = bx + e$, the regression coefficient can be estimated using least squares by

$$\hat{b} = \begin{cases} \sum xy / \sum x^2 \\ \sum y / \sum x \\ \sum (y/x)/n \end{cases}$$

the first being the simple regression estimator and second and third being estimators based on weighted least squares where the weights are inversely proportional to the variance which is assumed proportional to the mean or the square of the mean respectively (Delorenzo & Wiggans, 1986). Delorenzo and Wiggans (1986) used the second method, citing its statistical properties and variance mean relationship, and then fitted a linear model to the inverse of F, the portion of daily yield, to account for milking interval and stage of lactation. Note that

$$F_{am}^{-1} = 1 - F_{pm}^{-1} \tag{1}$$

a property satisfied by the second estimator.

The model used to estimate F is

$$y_{24} = F_{am} y_{am} + e$$

where the portion of daily yield, F_{am}^{-1}, is specified as a linear model to account for sources of variation such as age of cow, days in milk (DIM) and milking interval in hours (INT). The above equation is easily rearranged so that the parameter estimates can be based on a generalised linear model with Gaussian error and inverse link. In keeping with the second estimator above, y_{am}^{-1} is used as a weighting variable to account for proportionality between variance and mean.

Milk component percentages at AM and PM were recorded in SPS2002 herds milking in the 2005/6 season. The data included 63144 cows of all age groups from 176 herds with an average of 3.1 tests per cow. The SPS herds included in this analysis were those using Self Sample Assist so that milking interval could be determined, at the herd level, from the assist start times. The linear model included 5 classes for age of cow (2yr, 3yr, 4yr, 5 to 9 yr, >9yr), and covariates for DIM and INT. Herd tests beyond day 305 were not included. The traits used in the analyses include milk volume and the fat and protein yields.

Results

Figure 1 shows the boxplot of the portion of AM milk volume, y_{am}/y_{24}, for individual herd tests and for herd averages derived from all 378 SPS herds. The distribution for individuals has long tails and, with consideration given to the variation in herd average, data outside the range (0.15,0.9) (0.014% of the data) were rejected as recording errors for the purposes of parameter estimation .

Table 1 shows the frequency distribution for the AM milking interval by 30 minute intervals. The average AM interval was 14.3 hours. Figure 2 shows herd-test-date averages for portion of AM volume against milking interval. A linear fit for INT is recommended until better information suggests otherwise, higher order polynomials or fitting as a factor gave questionable estimates at the extremes. Estimates of effects for AM testing are shown in Table 2. In all cases there is a trend with age. Breed was also tested but had little influence on RSD. As an example, the AM scale factor is estimated to be $\hat{F}_{am} = 1/0.5844 = 1.71$ for milk volume at day 120 in 2-yr olds with a 14 hour milking interval and the corresponding PM scale

Table 1: Frequency distribution for AM milking interval

INT (hr)	no. herd-test-dates	%
< 13.5	17	2.8
13.5 -	87	14.2
14.0 -	260	42.6
14.5 -	172	28.1
15.0 -	61	10.0
>= 15.5	14	2.3

Table 2: Parameter estimates for portion of total yield $\hat{F}_{am}^{-1}$

Effect	milk	fat	protein
2-yr old	0.5844	0.5612	0.5814
3-yr old	0.5811	0.5542	0.5800
4-yr old	0.5782	0.5441	0.5782
5 to 9-yr old	0.5768	0.5367	0.5778
>9-yr old	0.5750	0.5322	0.5770
$(DIM - 120)/100$	0.00368	0.01936	0.00253
$(DIM - 120)^2/10000$	-0.00148	-0.01162	-0.00235
$INT - 14$	0.02385	0.01469	0.02496

factor $\hat{F}_{pm} = 1/(1 - 0.5844) = 2.41$. For fat yield the corresponding figures are 1.78 and 2.28. The parameters for protein in Table 2 are similar to the volume parameters indicating little difference between AM and PM protein percentage. Figures 3 and 4 illustrate AM scale factors for milk volume and fat yield.

The parameter estimates for PM testing can be obtained from those in Table 2 but replacing the milking interval covariate by $10 - INT$ and then using equation 1. Consider a 2-yr old tested AM at day 100 with a milking interval of 14.5 hours. The AM volume conversion factor is then

$$\begin{aligned}\hat{F}_{am} &= \frac{1}{0.5844 + 0.00368 * (-0.2) - 0.00148 * (-0.2)^2 + 0.02385 * (0.5)} \\ &= 1.68\end{aligned}$$

and the PM conversion factor for a 4-yr old at day 150 and milking interval of 11 hours would be

$$\hat{F}_{pm} = \frac{1}{1 - [0.5782 + 0.00368 * (0.3) - 0.00148 * (0.3)^2 + 0.02385 * (-1)]}$$
$$= 2.25$$

The prediction error for 24-hour yield, expressed as a percentage, had an estimated standard deviation of 9.1% for milk volume and 14.0% for fat yield when AM testing. These values may be inflated because of the long tails of the distribution, 99% of the errors were within ±20% for milk volume and ±27% for fat yield.

Accumulated yield

The accumulated volume yield calculation was carried out for the SPS data. AM+PM, AM only, PM only and alternate AM/PM schemes were considered. In the case of single testing, the parameters in Table 2 were used to adjust to a 24-hour basis. The prediction error standard deviation, relative to the AM+PM standard, is presented in Figures 5 and 6. After 4 herd tests the prediction error is about 5.0% for milk volume and 8.5% for fat yield when AM testing only and is lower in the alternate AM/PM scheme for fat yield. The prediction error for protein yield is similar to that for volume.

Figure 1: Boxplots of portion of AM milk volume

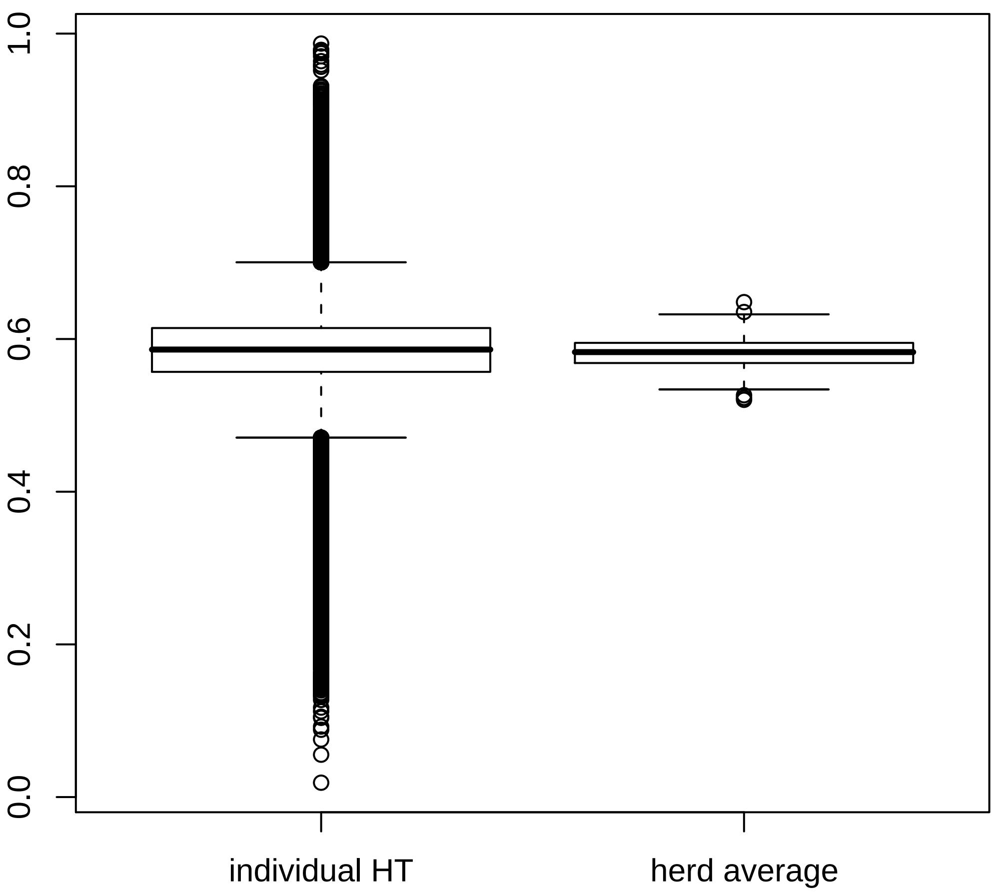

Figure 2: Portion of AM milk volume by milking interval

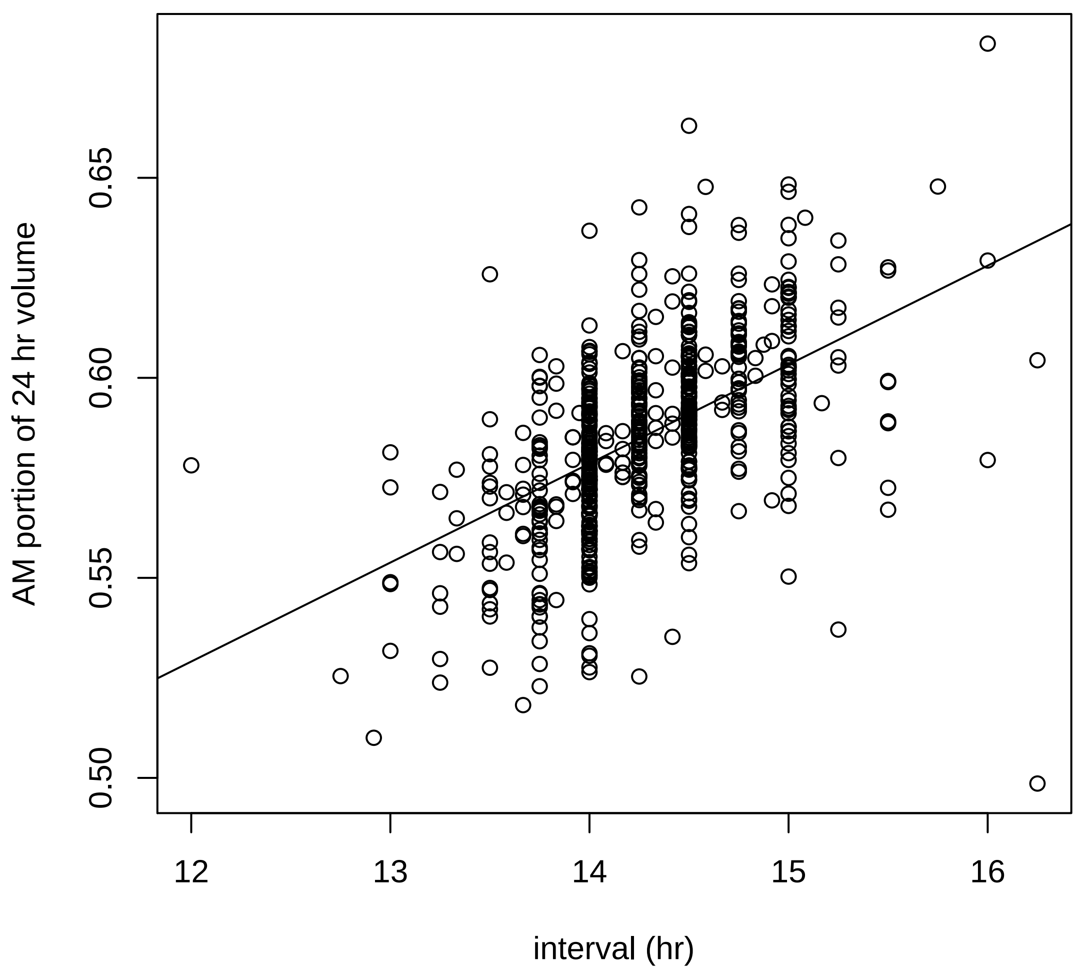

Figure 3: Factor to predict 24-hour milk volume from AM volume at 14 hour milking interval

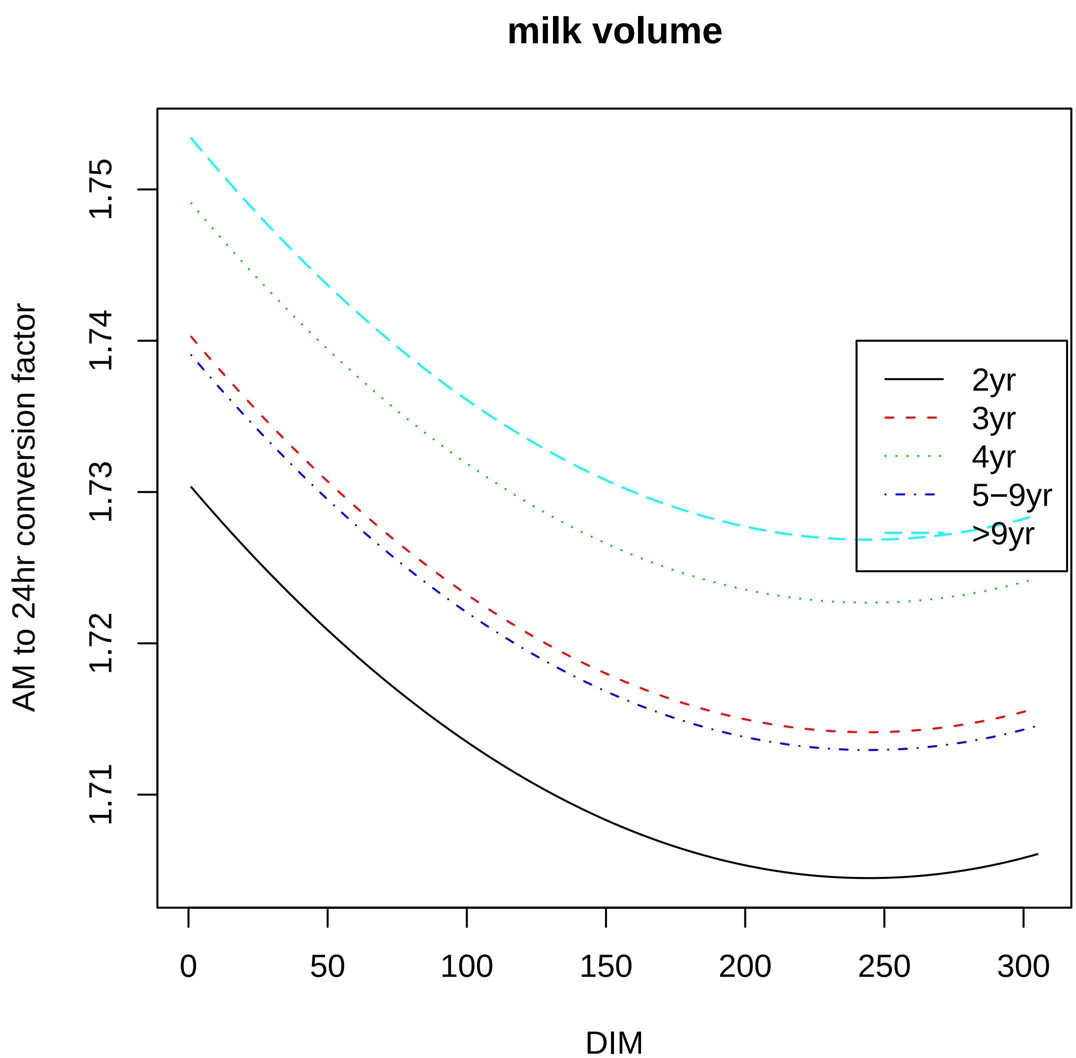

Figure 4: Factor to predict 24-hour fat yield from AM yield at 14 hour milking interval

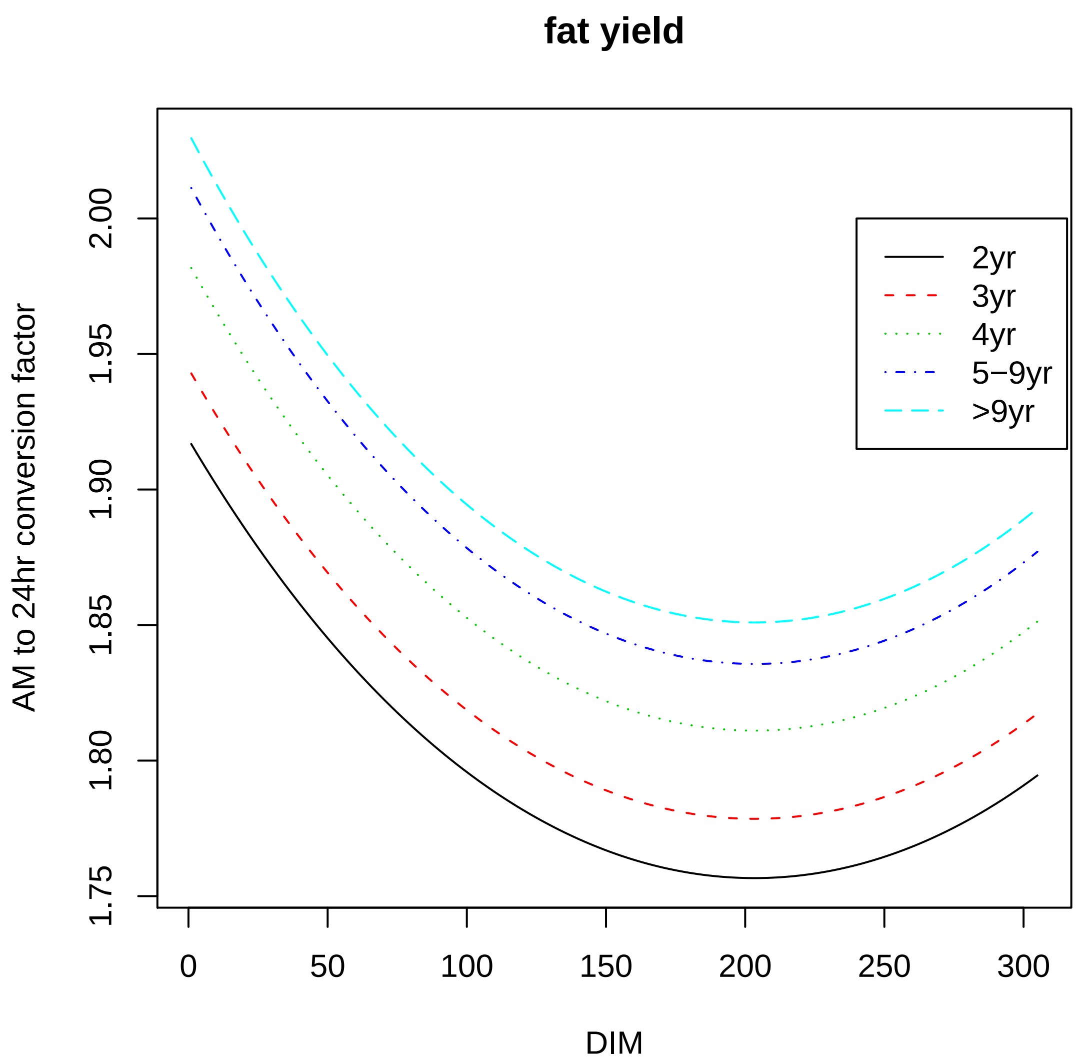

Figure 5: Prediction error for milk volume accumulated yield based on single testing relative to AM+PM

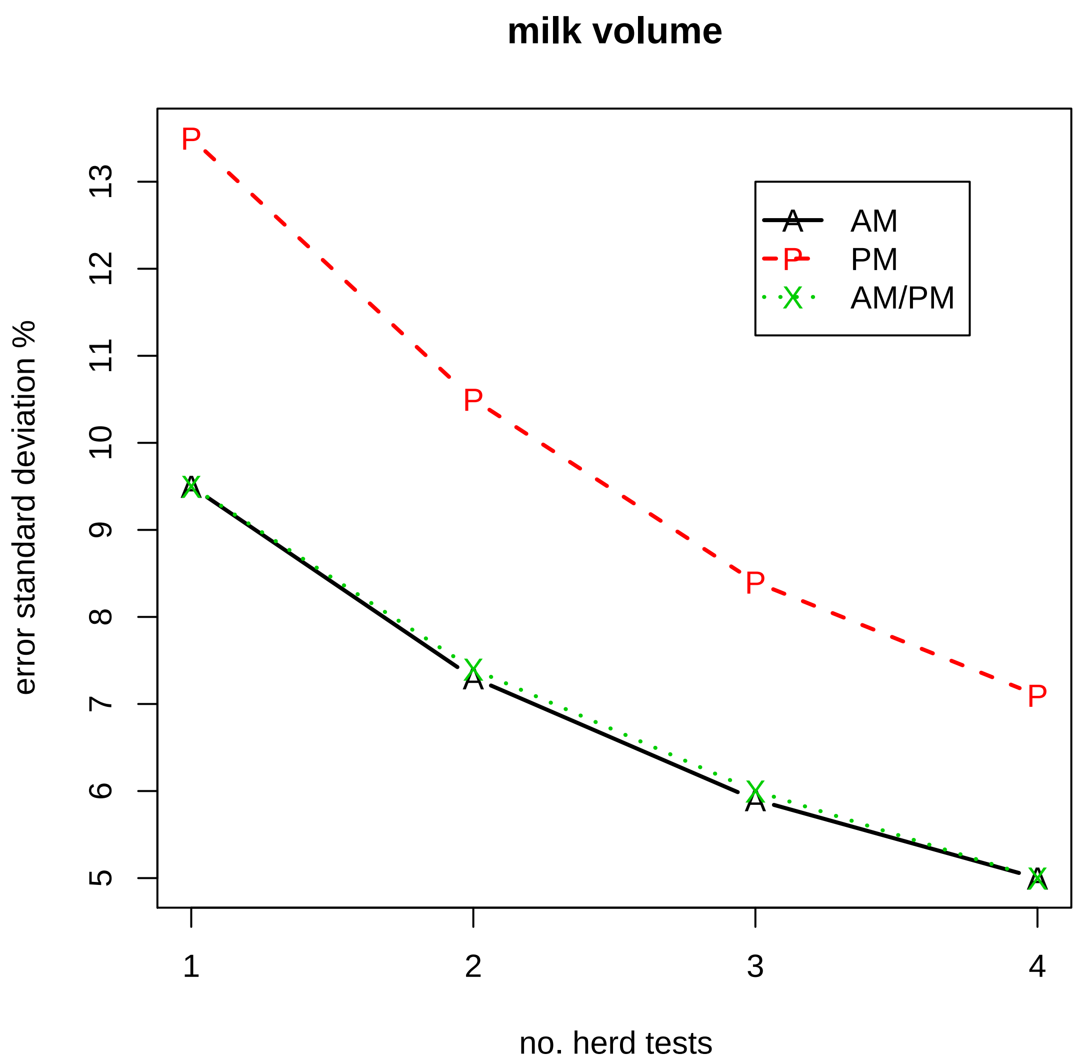

Figure 6: Prediction error for fat accumulated yield based on single testing relative to AM+PM