Random Walks, Polyhedra and Hamiltonian Cycles

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Complexity

Definition

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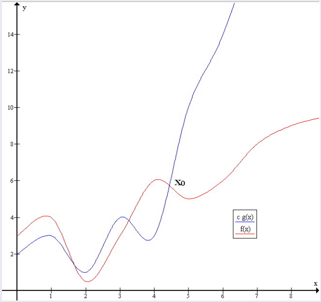
- We say that the function $f(x)$ is $\mathcal{O}(g(x))$, if and only if there exists a positive constant c and a real number x_0 such that for all $x \geq x_0$, $f(x) \leq c\,g(x)$.



Linear Programming

The $\mathcal{LP}$ Problem

$$
\begin{array}{rrcl}
\text{maximize} & \sum_{i=1}^{n} c_i x_i & & \\
\text{subject to} & \sum_{i=1}^{n} a_{1\,i} x_i & = & b_1 \\
 & \cdots & = & \cdots \\
 & \sum_{i=1}^{n} a_{m\,i} x_i & = & b_m \\
\text{and} & x_1, \ldots, x_n & \geq & 0
\end{array}
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Algorithms to Solve $\mathcal{LP}$

Algorithm	Developed by	Year	The Complexity
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Interior Point	Karmarkar	1984	$\mathcal{O}(n^{3.5})$

Comparison

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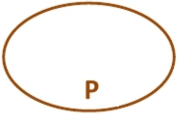
	Polynomial	**Exponential**

Complexity Classes



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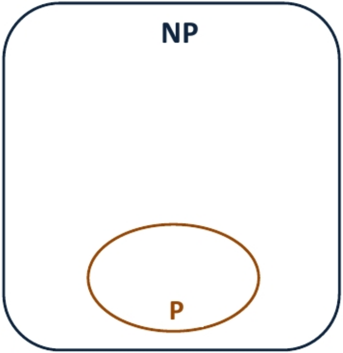

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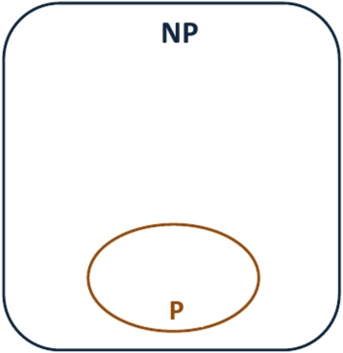

A Challenging Question

$$P \stackrel{?}{=} NP$$

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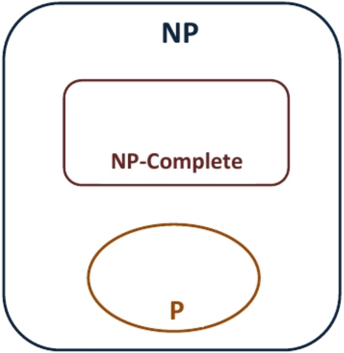

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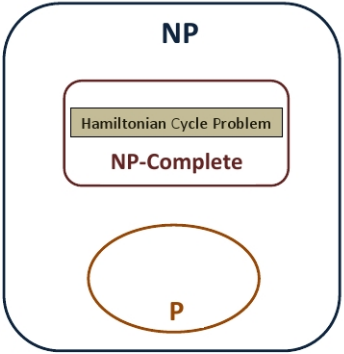

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Given a graph **G**, a simple path that starts from an arbitrary node, visits all nodes exactly once and returns to the initial node is called a **Hamiltonian cycle** or a **tour**.

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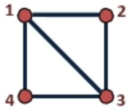
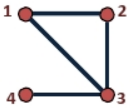


Hamiltonian Graph



non-Hamiltonian Graph

Embedding in Markov Decision Processes

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- In 2000, Feinberg converted the HCP to a class of Markov decision processes, the so-called **weighted discounted Markov decision processes**.
- MDP embedding implies that you can search for a Hamiltonian cycle in a nicely structured **polyhedral domain of discounted occupational measures**.

Domain of Discounted Occupational Measures

$\mathcal{H}_\beta$ Polytope Associated with the Graph G on n Nodes; $\beta \in (0,1)$

$$\begin{aligned}
\sum_{a \in \mathrm{O}(1)} x_{1a} - \beta \sum_{b \in \mathrm{I}(1)} x_{b1} &= 1 - \beta^n \\
\sum_{a \in \mathrm{O}(i)} x_{ia} - \beta \sum_{b \in \mathrm{I}(i)} x_{bi} &= 0 \; ; \; i = 2, 3, \ldots, n \\
\sum_{a \in \mathrm{O}(1)} x_{1a} &= 1 \\
x_{ia} \geq 0 \quad &; \quad \forall\, i \in \mathcal{S} \; , \; a \in \mathrm{O}(i)
\end{aligned}$$

Hamiltonian Extreme Points

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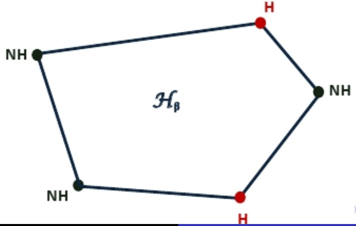

Illustration

Example

$$
\begin{aligned}
x_{12} + x_{13} + x_{14} - \beta x_{21} - \beta x_{41} &= 1 - \beta^4 \\
x_{21} + x_{23} - \beta x_{12} - \beta x_{32} &= 0 \\
x_{32} + x_{34} - \beta x_{13} - \beta x_{23} - \beta x_{43} &= 0 \\
x_{41} + x_{43} - \beta x_{14} - \beta x_{34} &= 0 \\
x_{12} + x_{13} + x_{14} &= 1 \\
x_{ia} &\geq 0 \;\; ; \; i = 1, 2, 3, 4 \; , \;\; a \in O(i)
\end{aligned}
$$

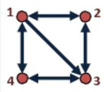

Illustration (Cont.)

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- **One particular basic feasible solution**:

$$x_{12} = 1\,,\ x_{23} = \beta\,,\ x_{34} = \beta^2\,,\ x_{41} = \beta^3\,,$$

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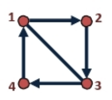
- It traces out the **standard Hamiltonian cycle** $1 \to 2 \to 3 \to 4 \to 1$.



Short and Noose Cycles [Ejov et al. 2009]

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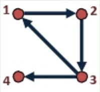
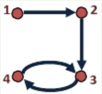


A short cycle



A noose cycle

Hamiltonian and non-Hamiltonian Extreme Points of $\mathcal{H}_\beta$

Theorem (Ejov et al., 2009)

Consider a graph G and the corresponding polytope $\mathcal{H}_\beta$. Any extreme point $\mathbf{x}$ corresponds to either a **Hamiltonian cycle** *or a* **combination of a short cycle and a noose cycle**.

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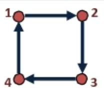
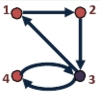


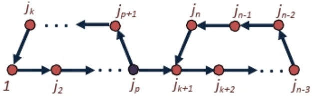
A Hamiltonian extreme point



A combined extreme point

non-Hamiltonian Extreme Points [Eshragh and Filar, 2011]

1. Type I (Binocular)



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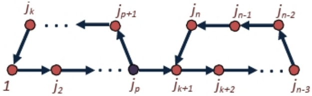
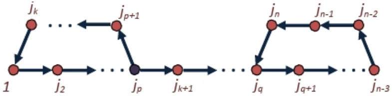


2. Type II



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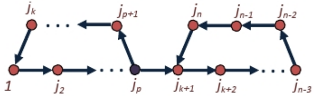

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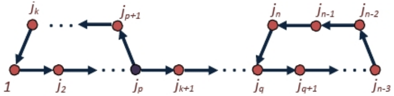
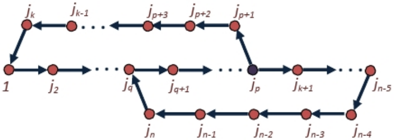


3. Type III



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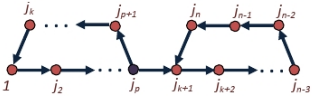

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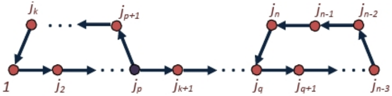

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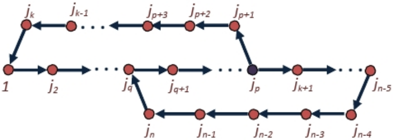
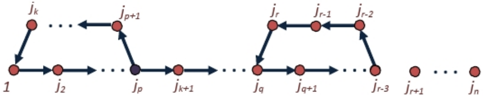


4. Type IV



The Prevalence of Hamiltonian Extreme Points

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- We utilized **Erdös-Rényi Random Graphs $G_{n,p}$**.

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2. $$\frac{E[\#\text{ of Hamiltonian Extreme Points}]}{E[\#\text{ of NH Extreme Points Types II \& III}]} = \frac{6n^2-12n}{2n^3-9n^2+7n+12}$$

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Reducing the Feasible Region

The Wedged Hamiltonian Polytope $\mathcal{WH}_\beta$ [Eshragh et al. 2009]

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and

$$\mathbf{A}(\beta)\,\mathbf{x} \;\leq\; \mathbf{b}(\beta)$$

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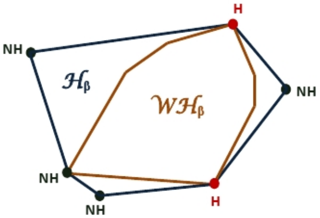

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Consider the graph G and polytopes $\mathcal{H}_\beta$ and $\mathcal{WH}_\beta$. For $\beta \in \left((1 - \frac{1}{n-2})^{\frac{1}{n-2}}, 1 \right)$, the intersection of extreme points of these two polytopes can be partitioned into two disjoint (possibly empty) subsets:

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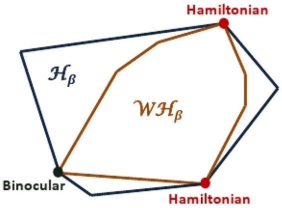
(ii) **binocular extreme points**.

Further Developments



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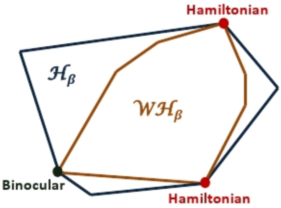

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(i) *There exists a* **polynomial-time algorithm** *to generate extreme points of the polytope* $\mathcal{WH}_\beta$*, uniformly, at random.*

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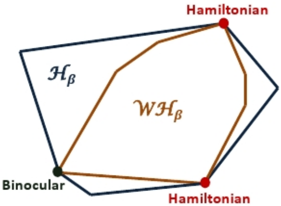

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(i) *There exists a* **polynomial-time algorithm** *to generate extreme points of the polytope* $\mathcal{WH}_\beta$*, uniformly, at random.*

(ii) *For large values of* β*, the proportion of Hamiltonian extreme points in the the polytope* $\mathcal{WH}_\beta$ *is bounded below by* $\frac{1}{\rho(n)}$*, where* $\rho(n)$ *is a polynomial function of* n*.*

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- For a given Hamiltonian graph G,

$$\mathbf{Pr(Required\ number\ of\ iterations} > \tau) \leq \mathbf{e}^{-\frac{\tau}{\rho(n)}};$$

Exploiting the Conjecture

A New Algorithm for the HCP

1. Construct the polytope $\mathcal{WH}_\beta$ corresponding to a given graph G, set β large enough and $t = 1$;
2. Generate an extreme point of polytope $\mathcal{WH}_\beta$, say $\mathbf{x}_t$, uniformly, at random. If $\mathbf{x}_t$ is a Hamiltonian extreme point, then **STOP** and claim that G is Hamiltonian;
3. If $t > \alpha\rho(n)$, then **STOP** and claim that **with high probability**, G is non-Hamiltonian. Otherwise, set $t = t + 1$ and return to Step 2.

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Reducing the Feasible Region

The Convex Body $\mathcal{CH}_\beta$ [Borkar and Filar 2013]

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and

$$\mathbf{g}(\mathbf{x};\beta) \;\leq\; \mathbf{c}(\beta)$$

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Theorem (Borkar and Filar 2013)

Consider the graph G and construct the corresponding polytope $\mathcal{H}_\beta$ and the convex body $\mathcal{CH}_\beta$. The graph G is **non-Hamiltonian**, *if and only if $\boldsymbol{\mathcal{H}_\beta = \mathcal{CH}_\beta}$. Otherwise, that is if G is* **Hamiltonian**, $\boldsymbol{\mathcal{CH}_\beta \subset \mathcal{H}_\beta}$.

Further Developments

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- In 1991, Dyer er al. developed a **probabilistic** algorithm to approximate the volume a convex body with a desired level of precision in **polynomial-time**.
- Developing a **polynomial-time probabilistic algorithm** to compare the volumes of the polytope $\mathcal{H}_\beta$ and the convex body $\mathcal{CH}_\beta$.

Quotation

Albert Einstein

"You **can't** solve a problem with the **same mind** that created it."

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- Prof. Martin Dyer, University of Leeds, UK

End

Thank you $\cdots$ Questions?