

Contents

<i>Preface</i>	<i>page xi</i>
1 The role of gravity	1
2 Differential geometry	10
2.1 Manifolds	11
2.2 Vectors and tensors	15
2.3 Maps of manifolds	22
2.4 Exterior differentiation and the Lie derivative	24
2.5 Covariant differentiation and the curvature tensor	30
2.6 The metric	36
2.7 Hypersurfaces	44
2.8 The volume element and Gauss' theorem	47
2.9 Fibre bundles	50
3 General Relativity	56
3.1 The space–time manifold	56
3.2 The matter fields	59
3.3 Lagrangian formulation	64
3.4 The field equations	71
4 The physical significance of curvature	78
4.1 Timelike curves	78
4.2 Null curves	86
4.3 Energy conditions	88
4.4 Conjugate points	96
4.5 Variation of arc-length	102
5 Exact solutions	117
5.1 Minkowski space–time	118
5.2 De Sitter and anti-de Sitter space–times	124
5.3 Robertson–Walker spaces	134
5.4 Spatially homogeneous cosmological models	142

CONTENTS

5.5 The Schwarzschild and Reissner–Nordström solutions	<i>page</i> 149
5.6 The Kerr solution	161
5.7 Gödel’s universe	168
5.8 Taub–NUT space	170
5.9 Further exact solutions	178
6 Causal structure	180
6.1 Orientability	181
6.2 Causal curves	182
6.3 Achronal boundaries	186
6.4 Causality conditions	189
6.5 Cauchy developments	201
6.6 Global hyperbolicity	206
6.7 The existence of geodesics	213
6.8 The causal boundary of space–time	217
6.9 Asymptotically simple spaces	221
7 The Cauchy problem in General Relativity	226
7.1 The nature of the problem	227
7.2 The reduced Einstein equations	228
7.3 The initial data	231
7.4 Second order hyperbolic equations	233
7.5 The existence and uniqueness of developments for the empty space Einstein equations	244
7.6 The maximal development and stability	249
7.7 The Einstein equations with matter	254
8 Space–time singularities	256
8.1 The definition of singularities	256
8.2 Singularity theorems	261
8.3 The description of singularities	276
8.4 The character of the singularities	284
8.5 Imprisoned incompleteness	289
9 Gravitational collapse and black holes	299
9.1 Stellar collapse	299
9.2 Black holes	308
9.3 The final state of black holes	323

CONTENTS

10	The initial singularity in the universe	<i>page</i> 348
10.1	The expansion of the universe	348
10.2	The nature and implications of singularities	359
Appendix A:		
	Translation of an essay by P. S. Laplace	365
Appendix B:		
	Spherically symmetric solutions and Birkhoff's theorem	369
	<i>References</i>	373
	<i>Notation</i>	381
	<i>Index</i>	385

