Mediation Analysis with Logistic Regression

Mediation is a hypothesized causal chain in which one variable affects a second variable that, in turn, affects a third variable. The intervening variable, *M*, is the mediator. It "mediates" the relationship between a predictor, *X*, and an outcome. Graphically, mediation can be depicted in Figure 1.1 below:

Figure 1.1

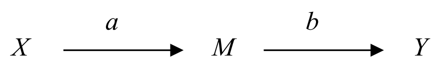


Figure 1.2

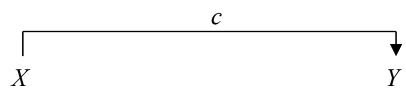
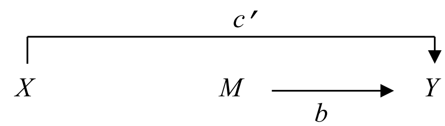


Figure 1.3



Paths a and b are called direct effects. The mediational path, in which X leads to Y through M, is called the *indirect effect*.[1]

Baron and Kenny (1986) proposed a widely cited method of investigating mediation through a series of three simple regression models, establishing a significant relationship for each unstandardized regression coefficient, a, b, and c, depicted in Figures 1.1. and 1.2. Mediation was then indicated by results from a third, multiple regression model, with both X and M predicting Y. *Full mediation* is indicated by the results if the relationship between X and Y was eliminated entirely when M is controlled (i.e., c' is non-significant), and partial mediation is indicated by the reduction but not the elimination of the X to Y relationship. This approach is a less precise and potentially inaccurate method of testing for mediation, because a true mediational relationship may exist if some of these relationships are not found or if there is a more complex set of associations among the three variables (MacKinnon, Fairchild, & Fritz, 2007).

The current approach (e.g., MacKinnon, 2008) involves the computation of an *indirect effect coefficient* that is a function of the compound pathway a and b from Figure 1.3 together (Note: the b path in the product is the partial regression coefficient of Y regressed on X when M is also in the model). In ordinary least squares regression, the difference between the direct effect of X on Y with and without M, $c - c'$ from separate regression models depicted in Figures 1.2 and 1.3 (Judd & Kenny, 1981), and the product of the two paths from the model shown in Figure 1.3, ab (Sobel, 1962), are equivalent. In either case, the indirect effect coefficient gives the change in Y for each unit change in X as mediated through M. A significant indirect effect coefficient is evidence consistent with a mediational hypothesis, but it does not "prove" that the causal pathway is the reason for the associations. Combined with experimental, longitudinal, or other design features, one may eliminate some of the alternative explanations, however, and strengthen the case for a possible causal pathway.

Mediation Analysis with Logistic Regression

Because of the nonlinear nature of logistic regression, the two methods for calculating the indirect effect, ab and $c - c'$, are no longer equivalent (Winship & Mare, 1983) if either *M* or *Y* are binary, particularly with rarer outcomes. The discrepancy between the methods can also vary with the presence of covariates in the model. There are several possible solutions to the problem (see also Iacobucci, 2012a; Imai, Keele, & Tingley, 2010; Kenny, 2013; MacKinnon, Lockwood, Brown, & Wang, 2007; Valeri & VanderWeele, 2013), one of which is to standardize the variables, and then compute the ab product. Special attention needs to be paid to the variance of the binary dependent variable when standardizing either path coefficient. Standardizing the a path, for example, would use the usual formula for converting an unstandardized coefficient to a standardized coefficient using the standard deviations of X and M for the first path, $a^* = a(S_x/S_M)$, but using a special computation of the standard deviation of a binary M variable, $s_M = \sqrt{a^2 s_x^2 + \pi^2/3}$. The mathematical constant π divided by 3 is an estimate of the binomial distribution variance. If Y is also binary, an analogous standardization of the second path, b, would be

---

[1] This is perhaps the most common notation. MacKinnon and colleagues (e.g., Mackinnon 2008) use α and β for the direct paths between the predictor X and mediator M and between the mediator and the outcome Y and τ and τ' for the paths between X and Y.

used. The standardization method nearly eliminates the discrepancy between ab and the $c - c'$ indirect coefficient computations (MacKinnon & Dwyer 1993; MacKinnon et al., 2007). The product of the standardize paths, $(a^*)(b^*)$, is then divided by a standard error estimate computed as a function of the standard deviations of the two variables (Sobel, 1962) or computed from a bootstrap resampling approach.

Using Odds Ratios or Proportion Mediated for the Magnitude of the Indirect Effect
An odds ratio for the indirect effect can be defined, but its interpretation becomes considerably more complex. For a simpler case, when both M and Y are binary, the odds ratio for the indirect effect represents the odds of Y given X for the value of M_1 as compared with the value M_0 (VanderWeele & Vandsteelandt, 2010). VanderWeele & Vandsteelandt also suggest corrections to the odds ratio for rarer outcomes (generally with outcome $\hat{\pi}$ < .10). If the predictor is continuous in the a or b path, the odds ratio involves a unit change interpretation that adds additional complexity. The odds ratio may also be unstable if the relationships among the variables are weak (MacKinnon, Warsi, & Dwyer, 1995). One can estimate the *proportion mediated,* $ab/(c' + ab)$, as a way to gauge the magnitude of the indirect effect, but MacKinnon and colleagues (2007) find that it also is unstable with sample sizes less than 500. Vanderweele (2016) suggests a computation of the proportion mediated using odds ratios for the direct (θ_{direct}) and indirect ($\theta_{indirect}$) effects, where proportion mediated = $\theta_{direct}(\theta_{indrect} - 1)/(\theta_{direct} * \theta_{indirect} - 1)$, although this approach has not been evaluated extensively.

Probit Models
An additional proposed solution is to estimated the models involving a binary outcome with probit analysis. Probit regression, discussed in the next section of the course, is an alternative to logistic regression that uses an assumed normal error distribution and lends itself to standardized coefficients more readily. The probit mediational approach still performs better with the product approach than the difference approach ($c - c'$) to the indirect coefficient, but it also requires rescaling. Coefficient standardization is a bit more straightforward with probit than logit, so rescaling prior to computation of the indirect path is potentially clearer. The probit method appears to perform relatively well with sample sizes of 200 or more given the correct model and when distributional assumptions are met (MacKinnon et al., 2007).

Software Examples
There are several ways to test mediation in current software programs, [2] although I will only illustrate use of Hayes' PROCESS macro in SPSS, R and SAS[3] and the probit (WLSMV) approach in Mplus and lavaan in R. The PROCESS macro and the Mplus methods allows the user to specify more than one mediator (as well as combinations of moderators and mediators) as well as covariates, but I will keep the illustrations simple here. Structural equation modeling packages, can also be used with some greater flexibility including multiple predictors and mediators and latent variables. Some packages, such as Mplus (Muthén & Muthén, 1998-2012; see also Chapter 8 in Muthén, Muthén, & Asparouhov, 2017 for detail and illustrations), have incorporated Sobel and bootstrap approaches to the standard errors and statistical tests. The Mplus approach can be used with the diagonal weighted least squares approach (estimator=WLSMV), which is a probit analysis and for which standardized coefficients are available (addressing the scaling issue described above).

The examples below use negative exchanges (`w1neg`), depression (`w1cesd9`), and heart disease (`w1hheart`) from the LLSSE study (also used in the "Logistic Regression" handout). The hypothesized mediational model is that negative exchanges lead to depression which, in turn, lead to heart disease, `w1neg` → `w1cesd9` → `w1hheart`.

[2] There are other software approaches available including macros developed by Valeri and Vanderweele (2013). The `mediations` function from the `mediation` package in R is yet another possibility (and there are several other R functions that simulate the PROCESS macro or domediation).
[3] Warning: I am not clear on the approach of the PROCESS macro to standardization for the computation of the product *ab* in the logistic case (or whether there is one).

SPSS

```
*I set the temporary directory here to make sure temporary files have a legitimate place to go.
cd "c:\jason\temp".

insert file='C:\Jason\SPSSWIN\macros\process.sps'.
execute.
process y = w1hheart
 / x = w1neg
 / m = w1cesd9
 /total=1
 /boot=10000
 /seed=10000
 /model=4
 /stand=1.
execute.
```

At the bottom of the output, the "Indirect effect of X on Y" gives the indirect effect and its confidence limits (BootLLCI,BootULCI) using the bootstrap standard error method.

```
 Run MATRIX procedure:

***************** PROCESS Procedure for SPSS Version 3.4 *****************

          Written by Andrew F. Hayes, Ph.D.       www.afhayes.com
    Documentation available in Hayes (2018). www.guilford.com/p/hayes3

**************************************************************************
Model  : 4
    Y  : w1hheart
    X  : w1neg
    M  : w1cesd9

Sample
Size:  692

Custom
Seed:     10000

**************************************************************************
OUTCOME VARIABLE:
 w1cesd9

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .3404      .1159    19.7753    90.4255     1.0000   690.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     3.4324      .2083    16.4783      .0000     3.0235     3.8414
w1neg        2.8439      .2991     9.5092      .0000     2.2567     3.4311

**************************************************************************
OUTCOME VARIABLE:
 w1hheart

Coding of binary Y for logistic regression analysis:
  w1hheart  Analysis
       .00       .00
      1.00      1.00

Model Summary
       -2LL    ModelLL         df          p   McFadden   CoxSnell   Nagelkrk
   623.6956     2.0224     2.0000      .3638      .0032      .0029      .0049

Model
              coeff         se          Z          p       LLCI       ULCI
constant    -1.7485      .1495   -11.6926      .0000    -2.0416    -1.4554
w1neg         .0282      .1853      .1523      .8789     -.3349      .3913
w1cesd9       .0279      .0215     1.2978      .1944     -.0142      .0701
```

```
These results are expressed in a log-odds metric.

***************** DIRECT AND INDIRECT EFFECTS OF X ON Y *****************

Direct effect of X on Y
     Effect         se          Z          p       LLCI       ULCI
      .0282      .1853      .1523      .8789     -.3349      .3913

Indirect effect(s) of X on Y:
            Effect     BootSE   BootLLCI   BootULCI
w1cesd9      .0794      .0580     -.0229      .2050

********************** ANALYSIS NOTES AND ERRORS ************************

Level of confidence for all confidence intervals in output:
  95.0000

Number of bootstrap samples for percentile bootstrap confidence intervals:
  1000

NOTE: Total effect model not available with dichotomous Y

NOTE: Effect size option not available with dichotomous Y

NOTE: Direct and indirect effects of X on Y are on a log-odds metric.

------ END MATRIX -----
```

R with PROCESS

I omit the output from the R run here, because it is virtually identical in appearance to the SPSS output.

```
source('c:/jason/R/macros/process.R',echo=FALSE)
process(data=d,y="w1hheart",x="w1neg",m="w1cesd9",total=1,boot=1000,seed=10000,model=4,stand=1)
```

R lavaan (probit)

The lavaan package in R is a structural equation modeling package. The `estimator='dwls'` option gives a robust diagonal weighted least squares approach and gives probit estimates (same as WLSMV in Mplus). The `ab` coefficient in the output is the indirect effect coefficient.

```
library(lavaan)

model = '
       w1cesd9 ~ a*w1neg
  w1hheart ~ b*w1cesd9 + c*w1neg

#indirect effect
       ab := a*b

#total effect
       total := c + (a*b)
'

#fit = sem(model, data = mydata, missing = 'listwise', se = 'bootstrap', estimator='ml',
#             link='probit')

fit = sem(model, data = mydata, missing = 'listwise', se = 'bootstrap',
          estimator='dwls', ordered=c("w1hheart"), parameterization="theta")
summary(fit,fit.measures=TRUE, rsquare=TRUE, standardized=TRUE)

parameterestimates(fit)Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws             1000
  Number of successful bootstrap draws            1000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  w1cesd9 ~
    w1neg      (a)    2.844    0.423    6.721    0.000    2.844    0.341
```

```
  w1hheart ~
    w1cesd9    (b)    0.016    0.011    1.487    0.137    0.016    0.077
    w1neg      (c)    0.015    0.101    0.146    0.884    0.015    0.008

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .w1cesd9          3.432    0.210   16.310    0.000    3.432    0.727
   .w1hheart         0.000                               0.000    0.000

Thresholds:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    w1hheart|t1      1.048    0.087   12.026    0.000    1.048    1.045

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .w1cesd9         19.718    1.743   11.313    0.000   19.718    0.884
   .w1hheart         1.000                               1.000    0.994

Scales y*:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    w1hheart         0.997                               0.997    1.000

R-Square:
                   Estimate
    w1cesd9          0.116
    w1hheart         0.006

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    ab               0.047    0.034    1.388    0.165    0.047    0.026
    total            0.061    0.104    0.588    0.557    0.061    0.035

> parameterestimates(fit)
        lhs  op      rhs label    est    se      z pvalue ci.lower ci.upper
1   w1cesd9   ~    w1neg     a  2.844 0.423  6.721  0.000    2.061    3.724
2  w1hheart   ~  w1cesd9     b  0.016 0.011  1.487  0.137   -0.006    0.037
3  w1hheart   ~    w1neg     c  0.015 0.101  0.146  0.884   -0.203    0.199
4  w1hheart   |       t1        1.048 0.087 12.026  0.000    0.874    1.218
5   w1cesd9  ~~  w1cesd9       19.718 1.743 11.313  0.000   16.295   23.152
6  w1hheart  ~~ w1hheart        1.000 0.000     NA     NA    1.000    1.000
7  w1hheart ~*~ w1hheart        0.997 0.000     NA     NA    0.997    0.997
8   w1cesd9  ~1                 3.432 0.210 16.310  0.000    2.997    3.848
9  w1hheart  ~1                 0.000 0.000     NA     NA    0.000    0.000
10       ab  :=      a*b    ab  0.047 0.034  1.388  0.165   -0.015    0.118
11    total  :=   c+(a*b) total 0.061 0.104  0.588  0.557   -0.169    0.251
```

SAS

I omit the output from the PROCESS macro in the SAS run here, because it is virtually identical in appearance to the the SPSS output.

```
OPTIONS MSTORED SASMSTORE=macros;
%include"c:\jason\sas\macros\process.sas";
%process(data=one, y=w1hheart,x=w1neg,m=w1cesd9,total=1,boot=1000,seed=10000,model=4,stand=1);
run;
```

Mplus

Without going into great detail on setting up models in Mplus, I will point out that the `on` statement is for variable *Y* regressed on *X*. I have omitted some of the output and the `model indirect` command that specifies the indirect effect. The analysis type by default when any categorical variables are declared is WLSMV which is a robust diagonal weighted least squares approach and gives probit estimates.

```
title:  Data from social exchanges study;
data:  file=heart.dat;  format=free;
    listwise=on;
variable:  names = w1hheart w1neg w1cesd9;
    missing = all (-99);
   categorical = w1hheart;
analysis:  type=general;
    bootstrap = 1000;
      ! at least 500 bootstrap samples are recommended;
```

```
model:  w1cesd9 on w1neg;
       w1hheart on w1neg w1cesd9;

! the following command gives the indirect path coefficient test;
Model indirect:  w1hheart ind w1neg;

output:  stdyx cinterval(bootstrap);
! cinterval (bootstrap) gives bootstrap confidence intervals.

MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 W1CESD9  ON
    W1NEG             -0.042      0.135     -0.313      0.754

 W1HHEART ON
    W1NEG              0.020      0.040      0.493      0.622
    W1CESD9            0.020      0.011      1.915      0.056

 Intercepts
    W1CESD9            4.412      0.188     23.434      0.000

 Thresholds
    W1HHEART$1         1.057      0.080     13.216      0.000

 Residual Variances
    W1CESD9           22.177      1.967     11.276      0.000


STANDARDIZED MODEL RESULTS


STDYX Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 W1CESD9  ON
    W1NEG             -0.014      0.040     -0.343      0.731

 W1HHEART ON
    W1NEG              0.031      0.059      0.520      0.603
    W1CESD9            0.096      0.050      1.922      0.055

 Intercepts
    W1CESD9            0.937      0.034     27.942      0.000

 Thresholds
    W1HHEART$1         1.056      0.080     13.269      0.000

 Residual Variances
    W1CESD9            1.000      0.002    435.185      0.000


R-SQUARE

    Observed                                        Two-Tailed   Residual
    Variable        Estimate       S.E.  Est./S.E.    P-Value    Variance

    W1HHEART           0.010      0.011      0.886      0.376      0.991
    W1CESD9            0.000      0.002      0.084      0.933


TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Effects from W1NEG to W1HHEART

  Total                0.019      0.040      0.470      0.638
  Total indirect      -0.001      0.003     -0.271      0.787
```

```
  Specific indirect 1
    W1HHEART
    W1CESD9
    W1NEG              -0.001      0.003     -0.271      0.787

  Direct
    W1HHEART
    W1NEG               0.020      0.040      0.493      0.622

STANDARDIZED TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS


STDYX Standardization

                                                     Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Effects from W1NEG to W1HHEART

  Total                 0.029      0.059      0.498      0.618
  Total indirect       -0.001      0.004     -0.304      0.761

  Specific indirect 1
    W1HHEART
    W1CESD9
    W1NEG              -0.001      0.004     -0.304      0.761

  Direct
    W1HHEART
    W1NEG               0.031      0.059      0.520      0.603



CONFIDENCE INTERVALS OF TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS


                  Lower .5%  Lower 2.5%    Lower 5%    Estimate    Upper 5%  Upper 2.5%   Upper .5%

Effects from W1NEG to W1HHEART

  Total              -0.122      -0.084      -0.057       0.019       0.072       0.081       0.104
  Total indirect     -0.007      -0.005      -0.004      -0.001       0.006       0.008       0.015

  Specific indirect 1
    W1HHEART
    W1CESD9
    W1NEG            -0.007      -0.005      -0.004      -0.001       0.006       0.008       0.015

  Direct
    W1HHEART
    W1NEG            -0.122      -0.083      -0.058       0.020       0.070       0.080       0.100
```

Sample Write-up (based on the process results; odds ratios were computed by hand)
Two regression models were tested to investigate whether the association between negative social exchanges and heart disease is mediated by depression symptomatology. In the first ordinary least squares regression model, negative social exchanges were significantly related to higher depression scores, b = 2.844, SE = .299, p <.001, 95% CI = 2.257, 3.431. In the second logistic regression model, which included negative social exchanges and depression as predictors of heart disease, neither negative exchanges, b = .028, SE =.185, ns, OR = 1.03, 95% CI -.335, .391, nor depression, b = .028, SE = .022, ns, OR = 1.028, 95% CI = -.014, .070, was significantly independently associated with heart disease. [I would interpret the ORs here if they had been significant]. The bootstrap confidence intervals derived from 1000 samples indicated that the indirect effect coefficient was not significant, b = .079, SE = .058, 95% CI = -.021,.205, which did not support the hypothesis that the relation between negative social exchanges and heart disease is mediated by depression

References and Further Reading

Iacobucci, D. (2012a). Mediation analysis and categorical variables: The final frontier. *Journal of Consumer Psychology, Forthcoming*.
Iacobucci, D. (2012b). Mediation with categorical variables compleat. *Journal of Consumer Psychology*, *22*(4), 603-604.
Kenny, D. A. (2013). Mediation with dichotomous outcomes. Research Note, University of Connecticut. http://davidakenny.net/doc/dichmed.pdf.
Imai, K., Keele, L., & Tingley, D. (2010). A general approach to causal mediation analysis. *Psychological methods*, *15*(4), 309.
MacKinnon, D.P. (2008). *Introduction to Statistical Mediation Analysis*. Mahwah, NJ: Taylor & Francis.
MacKinnon, D. P., & Cox, M. C. (2012). Commentary on "Mediation analysis and categorical variables: The final frontier" by Dawn Iacobucci. *Journal of consumer psychology: the official journal of the Society for Consumer Psychology*, *22*(4), 600.
MacKinnon, D. P., Lockwood, C. M., Brown, C. H., Wang, W., & Hoffman, J. M. (2007). The intermediate endpoint effect in logistic and probit regression. *Clinical Trials, 4*(5), 499-513.
MacKinnon, D. P., & Dwyer, J. H. (1993). Estimating mediated effects in prevention studies. *Evaluation review*, *17*(2), 144-158.
MacKinnon, D. P., Fairchild, A. J., & Fritz, M. S. (2007). Mediation analysis. *Annual review of psychology*, *58*, 593.
MacKinnon, D. P., Warsi, G., & Dwyer, J. H. (1995). A simulation study of mediated effect measures. *Multivariate behavioral research*, *30*(1), 41-62.
Muthén, L. K., & Muthén, B. O. (1998–2012). *Mplus User's Guide, Seventh Edition*. Los Angeles: Muthén & Muthén.
Muthén, B. O., Muthén, L. K., & Asparouhov, T. (2017). *Regression and mediation analysis using Mplus.* Los Angeles, CA: Muthén & Muthén.
Judd CM, Kenny DA. 1981. Process analysis: estimating mediation in treatment evaluations. *Eval. Rev.* 5:602–19
Baron, R. M., & Kenny, D. A. (1986). The moderator–mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of personality and social psychology*, *51*(6), 1173.
Sorkin, D. H., & Rook, K. S. (2004). Interpersonal control strivings and vulnerability to negative social exchanges in later life. *Psychology and Aging, 19*, 555–564.
Valeri, L., & VanderWeele, T. J. (2013). Mediation analysis allowing for exposure–mediator interactions and causal interpretation: Theoretical assumptions and implementation with SAS and SPSS macros. *Psychological methods*, *18*(2), 137.
Winship, C., & Mare, R. D. (1983). Structural equations and path analysis for discrete data. *American Journal of Sociology*, 54-110.