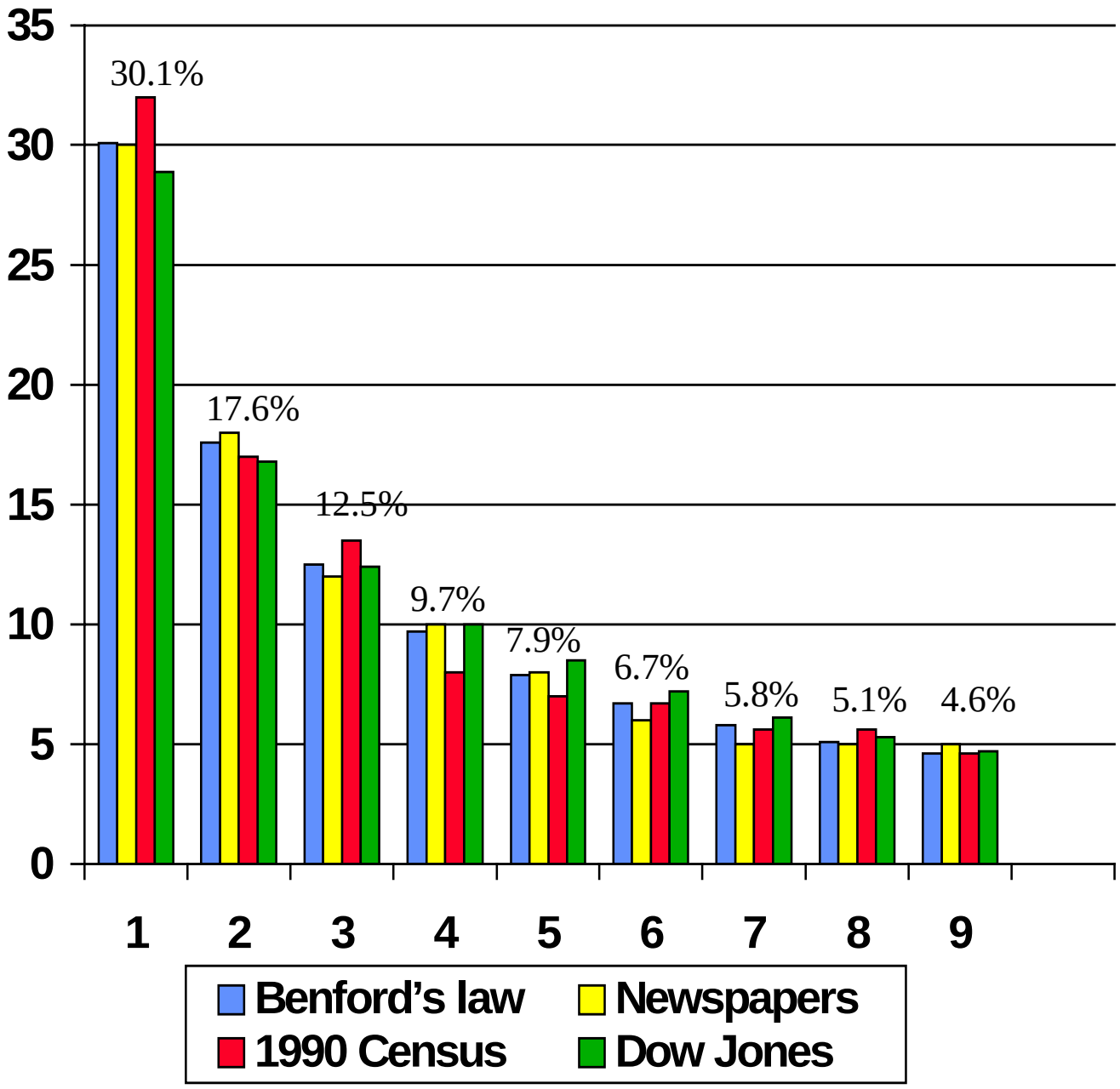

35
30
25
20
15
10
5
0
30.1%
17.6%
12.5%
9.7%
7.9%
6.7%
5.8%
5.1%
4.6%
1
2
3
4
5
6
7
8
9
Benford's law
Newspapers
1990 Census
Dow Jones

FIGURE 2

“Scientific notation” of numbers

$$144 = 1.44 \times 10^{2}$$

$$.0144 = 1.44 \times 10^{-2}$$

$$\frac{1}{4} = 2.5 \times 10^{-1}$$

$$\pi = 3.1416\ldots \times 10^{0}$$

The **first significant digit** is the single digit to the left of the decimal point (never 0).

The set of all positive numbers with first significant digit 1 is the collection of all numbers in the infinite sets of ranges

$$\cdots, (.01 \to .01999+), (.1 \to .1999+), (1.0 \to 1.999+),$$
$$(10 \to 19.99+), (100 \to 199.9+), \cdots$$

Stock	$/Stock	Pesos/Stock	Stocks/$
A	11	77	0.091
B	12	84	0.083
C	14	98	0.071
D	16	112	0.063
E	18	126	0.056
F	19	133	0.053
G	21	147	0.048
H	24	168	0.042
I	28	196	0.036
J	33	231	0.030
K	37	259	0.027
L	42	294	0.024
M	47	329	0.021
N	55	385	0.018
O	64	448	0.016
P	71	497	0.014
Q	83	581	0.012
R	96	672	0.010

First Digits

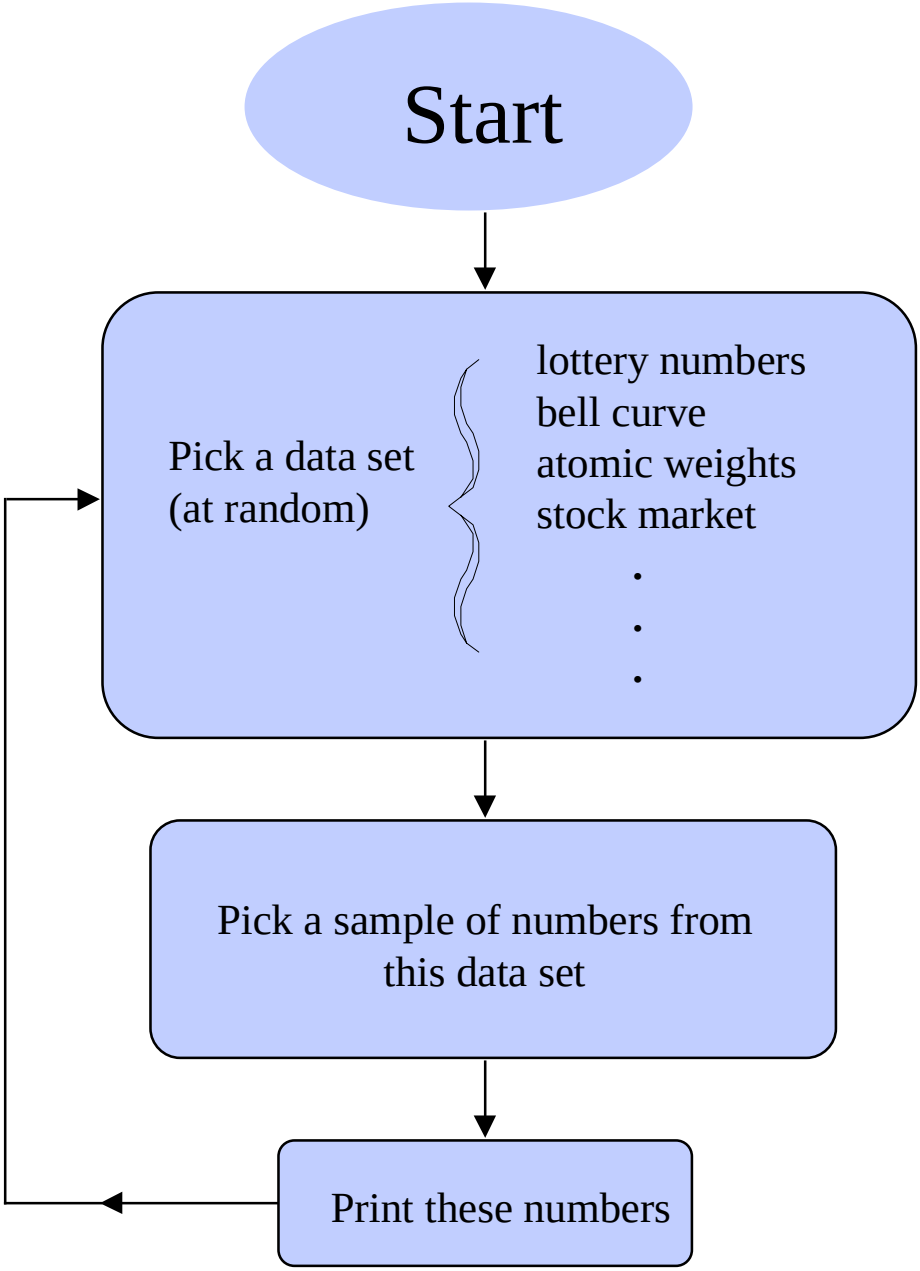
	1	2	3	4	5	6	7	8	9
$/Stock	6	3	2	2	1	1	1	1	1
Pesos/Stock	6	3	2	2	1	1	1	1	1
Stock/$	5	3	2	2	2	1	1	1	1

Stock	$/Stock	Pesos/Stock	Stocks/$
A	10	70	0.100
B	15	105	0.067
C	20	140	0.050
D	25	175	0.040
E	30	210	0.033
F	35	245	0.028
G	40	280	0.025
H	45	315	0.022
I	50	350	0.020
J	55	385	0.018
K	60	420	0.017
L	65	455	0.015
M	70	490	0.014
N	75	525	0.013
O	80	560	0.013
P	85	595	0.012
Q	90	630	0.011
R	95	665	0.011

First Digits

	1	2	3	4	5	6	7	8	9
$/Stock	2	2	2	2	2	2	2	2	2
Pesos/Stock	3	3	3	3	3	2	1	0	0
Stock/$	10	4	1	1	1	1	0	0	0

FIGURE 4



Log Distribution (Benford's Law)

The "random samples from random distributions" theorem says that if distributions are selected at random (in a neutral way), and samples are taken from each of these distributions, then the resulting data set will have digital frequencies approaching Benford's Law (see Figure 5).

	First Digits								
	1	2	3	4	5	6	7	8	9
Benford's Law	30.1%	17.6	12.5	9.7	7.9	6.7	5.8	5.1	4.6
Lottery	11%	11	11	11	11	11	11	11	11
Bell Curve	40%	13	8	8	7	7	6	6	5
Atomic Wts.	41%	28	5	4	7	3	4	4	5

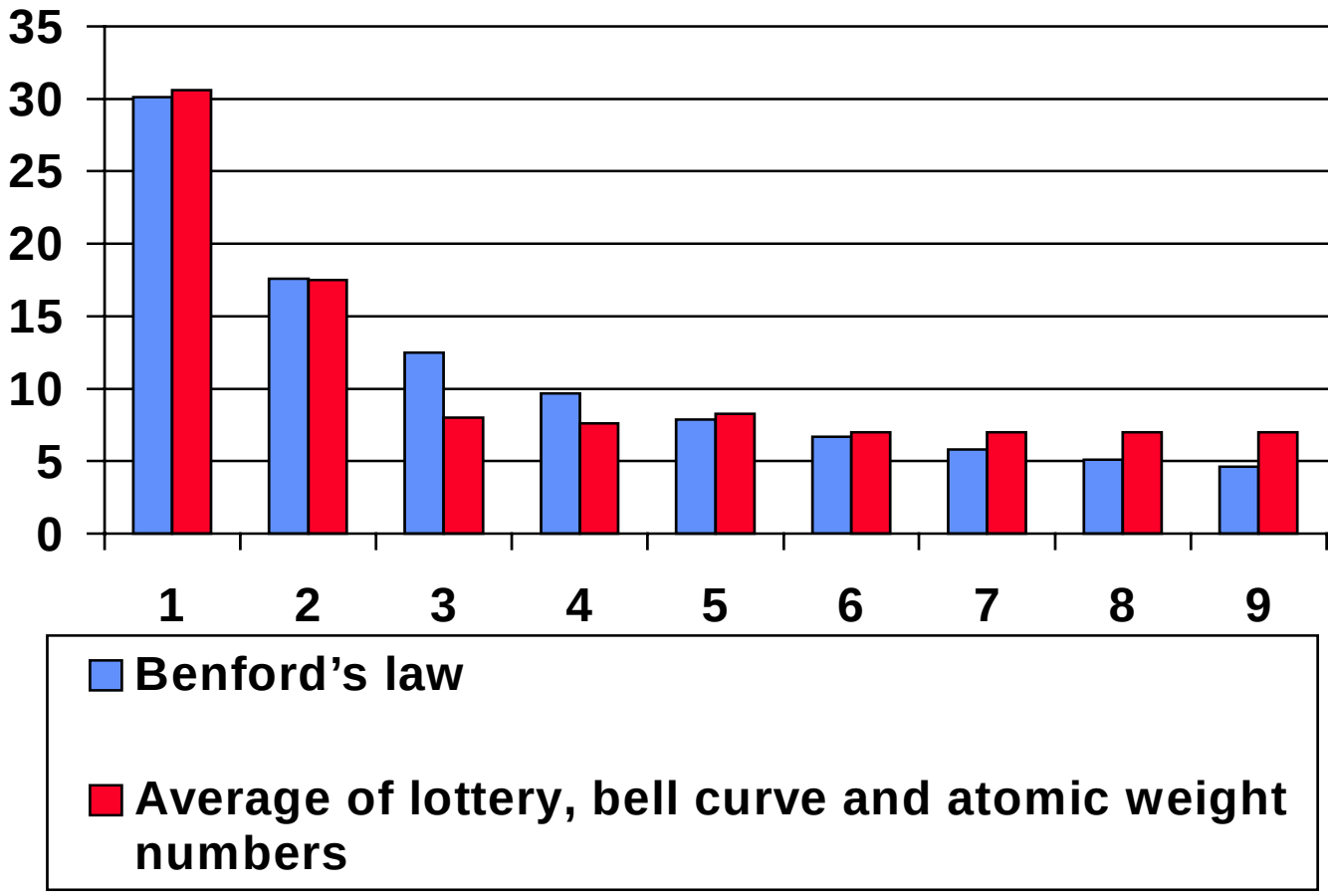
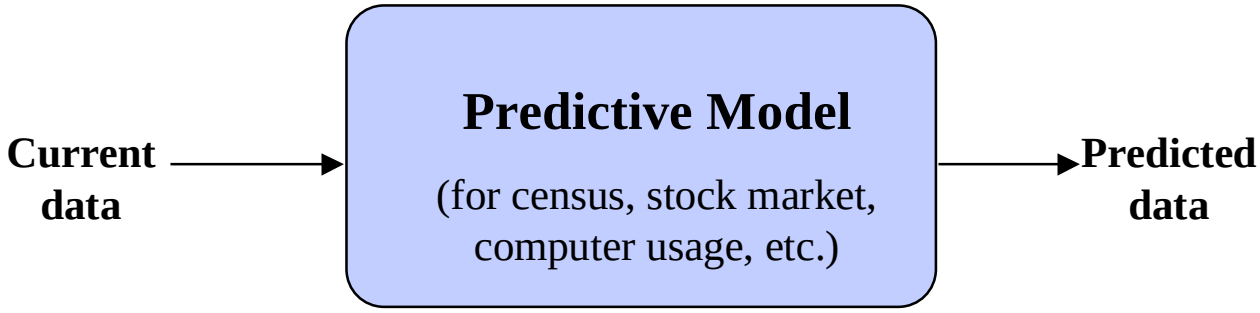


FIGURE 6



Model Testing

Benford-in, Benford-out test says that if current data follows the Log Law closely (as do census, stock market, computer usage, newspaper article numbers, etc.), then a good model will also generate predicted data closely following that law.

Benford's law test for fraud or concocted data.

First Digit Frequencies

	1	2	3	4	5	6	7	8	9
Benford's Law	30.1%	17.6	12.5	9.7	7.9	6.7	5.8	5.1	4.6
True Tax Data	30.5	17.8	12.6	9.6	7.8	6.6	5.6	5.0	4.5
Fraudulent Data	0	1.9	0	9.7	61.2	23.3	1.0	2.9	0
Random-Guess Data	14.7	10.0	10.4	13.3	9.7	15.7	12.0	8.4	5.8