Multi-echelon Inventory optimization under supply and demand uncertainty

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Multi-echelon Inventory Optimization under Uncertainty

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Titre : Optimisation multi-échelon du stock avec incertitude sur l'approvisionnement et la demande

Résumé :

La gestion de la chaîne logistique (Supply Chain Management, SCM) est un élément important de la plupart des entreprises et l'application de la stratégie appropriée est essentielle pour les gestionnaires de secteurs et de marchés concurrentiels. Dans ce contexte, la gestion des stocks joue un rôle crucial. Il est fondamentalement difficile dôoptimiser les décisions concernant la gestion des stocks, en particulier dans les réseaux multi-échelons. Un défi clé dans la gestion des stocks est de faire face aux incertitudes de lôapprovisionnement et de la demande. La diminution simultanée du taux de service et de l'augmentation des coûts liés aux stocks sont les effets les plus significatifs de ces incertitudes. Pour faire face à cette situation, les responsables de la chaîne dôapprovisionnement doivent établir des stratégies dôapprovisionnement et de distribution plus efficaces et plus souples. Dans cette thèse, un modelé pour optimiser les décisions dans les réseaux de distribution multi-échelons avec incertitude sur lôapprovisionnement et la demande est proposé.

Dans la première partie des travaux de recherche, les systèmes de distribution multi-échelons, avec incertitude sur la demande, sont étudiés. Ces systèmes de distribution font partie des topologies de réseau d'inventaire les plus difficiles à analyser. Les politiques de stock optimales pour ces systèmes ne sont pas encore connues. Nous considérons un type de réseau de distribution de base avec un seul type de produit dans le cadre dôune révision périodique. Sur la base de cette propriété, une approche de programmation en nombres entiers mixtes en deux étapes est proposée pour trouver les décisions optimales liées aux stocks en tenant compte du modèle de demande non stationnaire. Le modèle, qui repose sur une approche de planification des besoins de distribution (DRP), minimise le coût total prévu composé des coûts dôallocation fixe, de stockage, dôapprovisionnement, de transport et de retard. Des modèles alternatifs d'optimisation des stocks, comprenant la stratégie de lateral transshipment et multi-sourcing, sont ainsi construits et les programmes stochastiques correspondants sont résolus à l'aide de la méthode d'approximation de la moyenne de l'échantillon (SAA). Plusieurs exemples de problèmes sont générés pour valider l'applicabilité du modèle et pour évaluer l'avantage des lateral transshipment et de multi-sourcing en termes de réduction des coûts totaux attendus du réseau de distribution. Une enquête empirique est également menée pour valider les résultats numériques en utilisant le cas du réseau de distribution dôun grand distributeur français.

La deuxième partie du travail de recherche porte sur la structure de la politique de stock optimale qui fait lôobjet dôune enquête en cas de rupture dôapprovisionnement. Un modèle stochastique en deux étapes est proposé pour résoudre un problème d'optimisation des stocks multi-échelons prenant en compte une demande stochastique ainsi qu'une capacité de débit incertaine et des pertes d'inventaire possibles, dues à des perturbations. Le modèle minimise le coût total, composé des coûts dôallocation fixes, des coûts de conservation des stocks, de transport et de réapprovisionnement, en optimisant les décisions en matière de politique et de flux des stocks. Le niveau du stock est contrôlé selon une stratégie d'ordre des points de réapprovisionnement (s, S). Afin de faire face aux incertitudes, plusieurs échantillons de scénarios sont générés par la méthode de Monte Carlo. Les programmes d'approximations moyennes des échantillons correspondants sont résolus pour obtenir la politique de réponse adéquate au système d'inventaire en cas de perturbation. De plus, de nombreuses expériences numériques sont menées. Les résultats permettent de mieux comprendre l'impact des perturbations sur le coût total et le taux de service du réseau.

Mots clés : Chaîne d'Approvisionnement Multi-échelons; L'incertitude; Demande Non-Stationnaire; Perturbation; Programmation Stochastique

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Title : Multi-echelon Inventory Optimization under Uncertainty

Abstract :

Supply Chain Management (SCM) is an important part of most companies and applying the appropriate strategy is essential for managers in competitive industries and markets. In this context, Inventory Management plays a crucial role. Different inventory systems are widely used in practice. However, it is fundamentally difficult to optimize, especially in multi-echelon networks. A key challenge in managing inventory is dealing with uncertainties in supply and demand. The simultaneous decrease of customer service and increase of inventory-related costs are the most significant effects of such uncertainties. To deal with this pattern, supply chain managers need to establish more effective and more flexible sourcing and distribution strategies. In this thesis, a ñframework to optimize inventory decisions in multi-echelon distribution networks under supply and demand uncertaintyò is proposed.

In the first part of the research work, multi-echelon distribution systems, subject to demand uncertainty, are studied. Such distribution systems are one of the most challenging inventory network topologies to analyze. The optimal inventory and sourcing policies for these systems are not yet unknown. We consider a basic type of distribution network with a single family product through a periodic review setting. Based on this property, a two-stage mixed integer programming approach is proposed to find the optimal inventory-related decisions considering the non-stationary demand pattern. The model, which is based on a Distribution Requirements

Planning (DRP) approach, minimizes the expected total cost composed of the fixed allocation, inventory holding, procurement, transportation, and back-ordering costs. Alternative inventory optimization models, including the lateral transshipment strategy and multiple sourcing, are thus built, and the corresponding stochastic programs are solved using the sample average approximation method. Several problem instances are generated to validate the applicability of the model and to evaluate the benefit of lateral transshipments and multiple sourcing in reducing the expected total costs of the distribution network. An empirical investigation is also conducted to validate the numerical findings by using the case of a major French retailerôs distribution network.

The second part of the research work is focused on the structure of the optimal inventory policy which is investigated under supply disruptions. A two-stage stochastic model is proposed to solve a capacitated multi-echelon inventory optimization problem considering a stochastic demand as well as uncertain throughput capacity and possible inventory losses, due to disruptions. The model minimizes the total cost, composed of fixed allocation cost, inventory holding, transportation and backordering costs by optimizing inventory policy and flow decisions. The inventory is controlled according to a reorder point order-up-to-level (*s*, *S*) policy. In order to deal with the uncertainties, several scenario samples are generated by Monte Carlo method. Corresponding sample average approximations programs are solved to obtain the adequate response policy to the inventory system under disruptions. In addition, extensive numerical experiments are conducted. The results enable insights to be gained into the impact of disruptions on the network total cost and service level.

Keywords : Multi-echelon Supply Chain; Uncertainty ; Non-Stationary Demand ; Disruption; Stochastic Programming

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A mes parents Davoud et Shirin et à mon frère Hadi qui ont consacré beaucoup d’efforts et sacrifié de leur temps à mon éducation et ma réussite.

A Rohangiz qui a si chaleureusement supporté mon indisponibilité.

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Introduction

Nowadays there is a growing pressure for managers to improve the supply chain performance of their companies. The most challenging issue which impacts the performance of the supply chain is matching supply and demand. Supply and demand uncertainties are two key sources of this issue, which may lead to simultaneous decrease of service level and increase of inventory costs. An effective inventory management can contribute to tackle these issues. It is important to note that the main concern when managing the supply chain inventories is to find the optimal replenishment policy which determines when, where, from which supplier, and how much to order. Under this context, the main challenge is the anticipation of the future demand and supply in order to improve the quality of the inventory decisions. In order to efficiently mitigate supply and demand uncertainty, sourcing and inventory decisions should proactively take the risk exposure into account. Multi-sourcing and lateral transshipment could be considered as the potential sourcing options to increase the network flexibility.

Hence, the main objective of this thesis is to examine different impacts of supply and demand uncertainty on the inventory related decisions. In order to do so, an optimization approach for multi-echelon distribution network is proposed to minimize the total cost including the fixed allocation cost, the transportation cost, the backorder cost, the holding cost and the fixed procurement cost. The effect of different sourcing options is also evaluated in a multi-echelon distribution network under supply and demand uncertainty.

This thesis is made up of five chapters. Chapter one presents an overview of the research path conducted in the thesis. The problem context, the research question, the thesis scope, the research methodology and the contributions are detailed.

Chapter 2 aims to provide an overview of the literature on multi-echelon inventory optimization. An analysis of the literature is provided in the last section of this chapter. This enables to identify the gaps in the literature that the research work in this thesis attempts to bridge.

Chapter 3 develops a scenario-based modeling approach that is used to solve a two-stage multi-echelon inventory optimization problem considering a non-stationary demand. The model is based on a distribution requirements planning (DRP) approach and minimizes the expected total operational and tactical cost. Multi-sourcing and lateral transshipment in a periodic review inventory setting, are considered in this modeling approach. A European retailer case study and managerial insights is provided in the last section of this chapter.

Chapter 4 proposes a two-stage stochastic model to solve a capacitated multi-echelon inventory optimization problem considering a stochastic demand as well as uncertain throughput capacity and possible inventory loss, due to disruptions. The model minimizes the expected total operational and tactical cost. The inventory is controlled according to a reorder point order-up-to-level (s, S) policy and lateral transshipments in the network are considered. In order to deal with the uncertainties, several scenario samples are generated by Monte Carlo and corresponding sample average approximations programs are solved to obtain the adequate response policy to the inventory system under disruptions. Extensive numerical experiments are conducted and the results enable insights to be gained on the impact of disruptions on the network total cost and service level.

Finally, we get our conclusion and discuss the future work in Chapter 5.

Chapter 1. Context, Motivations and Problem description

1.1 Introduction

This chapter provides the general academic perspective, the objectives of this work, and the steps required to conduct the research to meet the objectives. First, a general introduction is presented to define the problem context. Then, the problem statement including the objectives and contributions of this thesis are briefly presented. Afterward, the methodological approach employed for the purposes of this work is discussed. The structure of this PhD thesis is presented at the end of the chapter.

1.2 Context

1.2.1 Supply Chain and Supply Chain Network

Nowadays, there is an increase of interest on the part of managers to select a decent strategy to improve the flows of products and information between the suppliers and customers due to the competitive market.

The alignment of partnerships that bring products or services to market is defined as a *supply chain* (Lambert et al., 1998). More specifically, a *Supply Chain* is a system of suppliers, manufacturers, warehouses, transportation modes, distributors, and retailers. The key purpose of this structure is to transform raw materials to final products and supply those products to customers in order to make profit for its entities.

In reality, a manufacturer may receive material from several suppliers and then supply several distributors. Thus, most supply chains are actually *networks*. It may be more accurate to use the term *supply chain network* (SCN) to describe the structure of most of the supply chains (Chopra et al., 2007). A supply chain network is a set of facilities and related links that join the facilities together to bring a product from one echelon of the supply chain to the other. These echelons may be from a production center to a distribution center, from a production center to a retailer, or from a distribution center to a retailer. Figure 1 illustrates a general supply chain network.

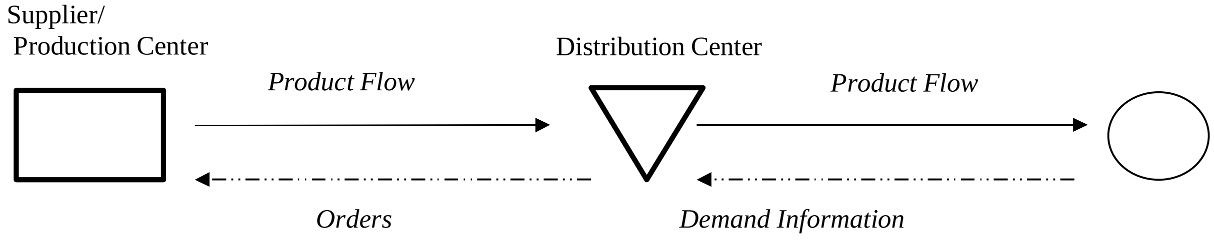


Figure 1. A Simple Supply Chain Network

Each stage in a supply chain is connected through the flow of products and information. Information & product flows should be mapped to get a comprehensive picture of supply chain network. *Product flow* is defined as the movement of goods from the raw material to a finished product. Information flow is the demand from the end-customer to preceding platforms in the network.

1.2.2 Multi-Echelon Supply Chain Network (ME-SCN)

As companies are progressively localizing their suppliers and markets all over the world, supply chain networks have become more widely spread around the world. A Multi-Echelon Supply Chain Network (ME-SCN) is a common network structure for large-scale companies that are deployed globally and have to manage a high number of products and large market zones with many suppliers and subcontractors. More specifically, such a network is composed of suppliers, production, distribution centers and the channels between them to acquire raw materials, convert the raw materials to finished products, and distribute final products in an efficient way to customer zones.

In Figure 2, a ME-SCN that includes three various levels of enterprises is demonstrated. The information and product flows are distinguished with different types of arrows. These flows could occur in both directions. In this figure each node characterizes either a production (supplier, manufacturer) or a distribution center. Inventory can be implemented in each node.

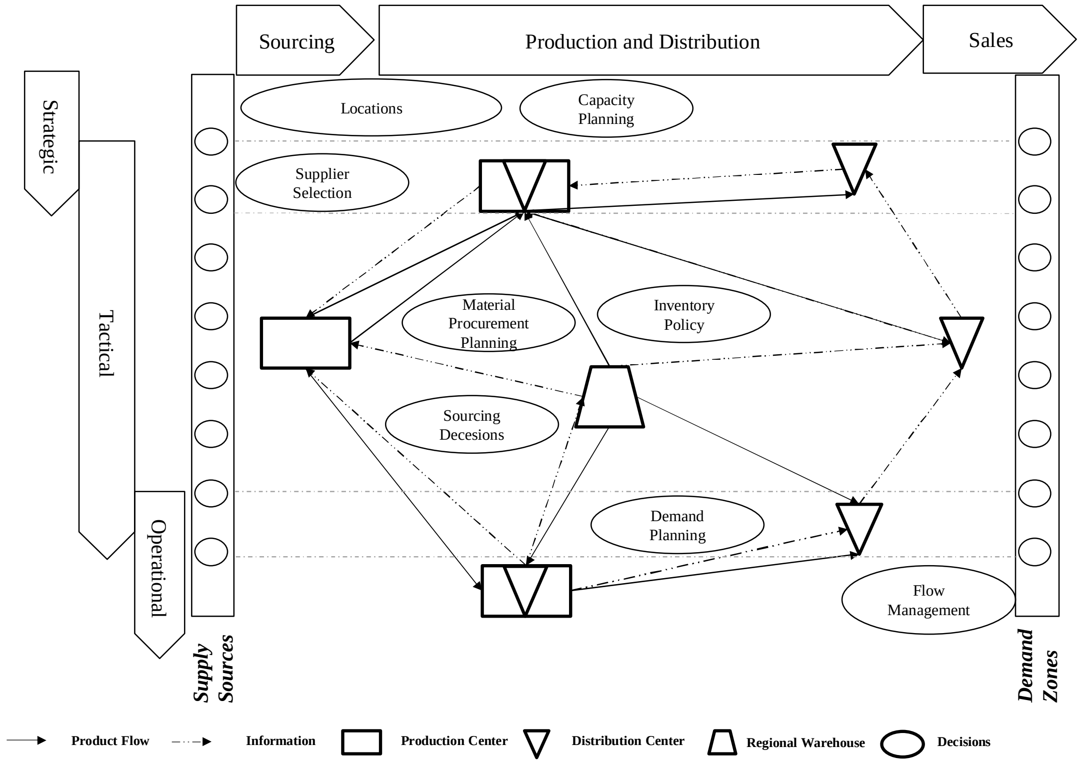


Figure 2. Supply Chain Network and Decisions

1.2.3 Supply Chain Management

Supply chain management (SCM) is an effective method to integrate both information and material flows seamlessly across the supply chain. Simchi-Levi et al. (2000) define supply chain management as ñthe integration of key business processes among a network of interdependent suppliers, manufacturers, distribution centers, and retailers in order to improve the flow of goods, services, and information from original suppliers to final customers, with the objectives of reducing system-wide costs while maintaining required service levelsò. In other words, supply chain management is a global approach to integrate the strategy of suppliers, manufacturers, warehouses, distributors, and stores in order to distribute the products in the right quantities, to the right locations, at the right time while minimizing related costs. This global approach is involved with different types of decisions. Supply chain management plays a significant role in the success or failure of a business. Supply chain management is even more important and more challenging in ME-SCN ((Tompkins and Harmelink, 1994).

The results of a decent supply chain management strategy are impressive. Amazon is a great example of an efficient supply chain management on its ME-SCN. The revenue of *Amazon* has reached to almost $136 billion in 2016. In fact, Amazon is the fastest company to reach $100 billion in sales revenue, taking only 20 years. The combination of sophisticated information technology, an extensive network of warehouses and excellent transportation makes Amazonȏs supply chain one of the most efficient among in the world (Leblanc, 2017)

1.2.4 Decision Levels in SCM

One way to organize SCM decisions is by sorting them into strategic, tactical, and operational decisions (see a simplified supply chain decisional structure in Figure 2). The strategic level deals with the goals to reach, long-term decisions and objectives. It decides what the chain's configuration will be, how resources will be allocated, and what processes each stage will perform. Since supply chain strategic decisions are typically made for the long term (a matter of years), they are very expensive to alter on short notice. Thus, when companies make these decisions, they must take into account uncertainty in anticipated market conditions over the next few years. Examples of this category include facilities location, supplier selection, etc. Tactical decisions are concerned with mid-term decisions. They usually emphasis on planning of supply chain functions such as markets which will be supplied from which locations, and inventory policies to be followed, among others. Operational SC decisions have a time horizon of a week or day and focus on making decisions regarding individual customer orders, such as daily flow management between the different distribution centers, generating pick lists at a warehouse or setting delivery schedules for trucks**.**

At tactical level, once the network is set, it gives rise to the resource planning problem. One of the most important parts of resource planning is the inventory management, which deals with the positioning of inventories in time and space.

1.2.5 Inventory Management at Tactical level

Inventory related problems have a significant effect on the total performance of supply chain. Changing inventory policies can intensely alter the efficiency and responsiveness of the supply chain. An important role of inventory in the supply chain is to increase the service level.

Inventory is kept all over the supply chain in different forms: raw materials, work in process, and finished goods (Silver et al., 1998).

The most challenging matter in inventory management is finding the fundamental trade-off between responsiveness and efficiency when making inventory decisions. Increasing inventory generally makes the supply chain more responsive to the customer. A higher level of inventory also facilitates a reduction in production and transportation costs because of improved economies of scale in both functions. This choice, however, increases inventory holding cost (Chopra et al., 2007).

1.2.6 Issues of Inventory Management in ME-SCN

Nowadays there is a growing pressure to improve supply chain cost performance for many companies. An efficient inventory management can contribute to this. Customerõs dissatisfaction, excess inventory in the network, the lead-times could be mentioned as costly issues in inventory management. These issues are even more visible in a company with operations in numerous locations, and particularly in ME-SCN where the locations are located in different tiers or echelons of the companyõs supply chain network.

Demand uncertainty and *supply disruption* are two important sources of the above issues. For years, Operations managers have recognized that the matching of supply and demand is one of their most challenging problems in Inventory Management.

1.1.6.1 Demand Uncertainty

In inventory management, demand uncertainty is one of the important issues which add complexity to the system. It fits cases in which the demand is affected by a trend, seasonal factors, or cyclical behaviors. An inappropriate demand planning approach could result into a devastating situation.

It is therefore substantial to have solid information on the demand pattern when negotiating contracts with suppliers and while deciding the branding strategy for each product. This reflects the real-world setting in which demand for fast moving consumer food goods is highly variable and often with non-stationary patterns. Non-stationary is a pattern in which demand is not constant for each time period but varies due to seasonality, trend or other factors.

For instance, H&M, the famous Swedish fashion retailer with over than 4,700 stores around the world, has recently reported that there are 4.3 Billion dollars of unsold clothes in its warehouses (Paton, 2018). Although, the companyôs chief executive just blamed poor inventory management over this issue, the main problem is a poor level of preparedness against the demand uncertainty. Since H&M was opening 220 new stores and expanding its e-commerce operations, they had decided to increase the inventory level in their warehouses to satisfy the potential demand. Meanwhile, a social media backlash after an advertisement[1] in January 2018 completely destroyed the companyôs image. Consequently, the demand pattern has decreased in Africa and some parts of Europe. A flexible production-distribution planning approach could handle these types of problems by redistributing the stock among the network.

1.1.6.2 Supply Uncertainty

Although the globalization improves the performance of companies in various ways, it also augments the risk of possible supply disruptions (as a type of uncertainty). This could be caused for example by discontinuities in supply, political instability, natural disasters and labor strikes. They could have a severe effect on the supply chain performance.

Natural hazards (e.g. earthquakes, hurricanes and flooding) and man-made mistakes (e.g. fire, explosion, terrorist attacks) are just some examples of disasters that could result in failure of supply chain networks. Disruptions are infrequent and are temporary events. However, they can cause noticeable losses. Potential consequences of disruptions involve great economic losses and dissatisfaction of the clients (Snyder et al., 2015). The severity and frequency of disasters have been increased over past decades, relatively as a result of climate change, urbanization, population growth, and political conflicts.

The Centre for Research on the Epidemiology of Disasters (CREDð www.emdat. be) recorded 983 disasters affecting European countries between the years 2000 and 2018. These disasters have caused more than 200 billion dollars[2] damages.

[1] H&M ran an ad showing a black child model wearing a hooded sweatshirt that said, ñCoolest monkey in the jungle.ò
[2] Appendix A details the disaster types and damages

Figure 3 highlights the number of hazards observed over the past two decades per country in five different levels. These events can paralyze production distribution systems, and even can lead to very serious crashes.

Figure 3. Disasters in European countries between 2000 and 2018[3]

According to a survey in 2012, 63 % of European, Middle East, and African companies were disrupted because of unforeseen events beyond their control linked to the economic context (24 %), natural disasters (19 %), subcontractor difficulties (16 %), and even terrorism (5 %). It is important to know that after an occurrence of a disruptive event, it has taken companies an average of 63 days to get back to business as usual (Martel and Klibi, 2016).

The cascade effects that serious disruptions may have on interconnected companies are also important. The 2011 Japanese tsunami has caused a 1.1 % reduction of world industrial production in the month that followed. In particular, PSA Peugeot Citroën in Europe was severely affected. The temporary shutdown of a Hitachi plant in Japan leads to the interruption of the supply of an electronic component to PSA, which in turn resulted in a 25ï 60 % reduction of

[3] EM-DAT: The Emergency Events Database - Université catholique de Louvain (UCL) - CRED, D. Guha-Sapir - www.emdat.be, Brussels, Belgium

production in eight PSA assembly plants in France, Spain, and Slovakia (Bourgin and Lenoir, 2012). This fact provides a considerable motivation to analyze these events and their impacts in multi-echelon networks.

1.2.7 Thesis Scope

We consider a three-echelon distribution network that includes implicit suppliers, a set of production-distribution centers (PDCs), a set of distribution centers (DCs), and a customer zone (CZ) stage (i.e., consumption points). As illustrated in Figure 4, each stage is fed from the upper echelon and feeds the ones below. The multiple arrows between PDCs and DCs represent the multi-sourcing opportunities with respect to the throughput capacity per period of each platform. A lateral transshipment (LT) option, which allows replenishment flows in the same echelon, is available between DCs. Lateral transshipments are stock movements between distribution centers in the same echelon of a supply chain network (Neale and Willems, 2009).

A tactical planning horizon (e.g., yearly, seasonally) is considered and is partitioned into a set of control periods (e.g., months, weeks, days). At the tactical level of the supply chain, when a make-to-stock policy is considered, a key decision is related to the positioning of inventories in time and space. In other words, the main issue in inventory management problems is to apply the optimal replenishment policy, which specifies when and how much to order.

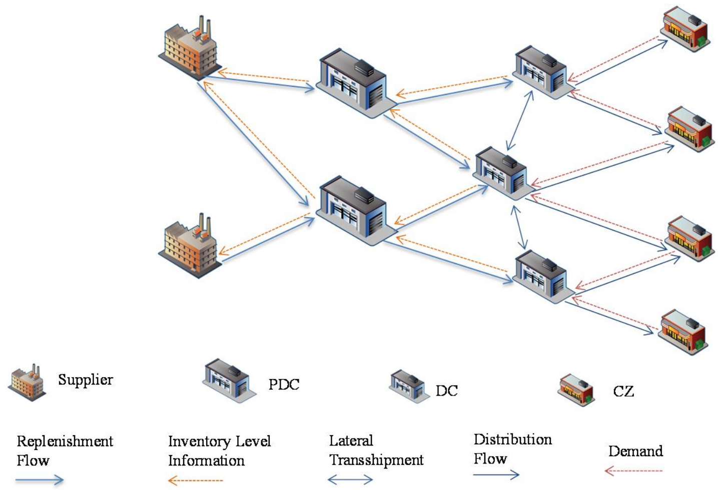


Figure 4. A Multi-Echelon Network with Lateral Transshipment between DCs and Multi-Sourcing

In our problems, the purpose is to minimize the expected total cost considering supply and demand uncertainty. When making the tactical decisions (first-stage decisions in the proposed two-stage model), there are two sources of uncertainty to be considered. The first regards the demand for final products that is unknown with certainty especially in non-stationary patterns. The second one is the occurrence of different disruptions. When a disruption occurs, some PDCs and DCs may lose part of their capacity (space to stock) and their inventory.

1.3 Inventory Optimization Problem

As stated in the context, companies must anticipate demand and supply uncertainties and be prepared against them. Therefore, the risk mitigation policies must be considered. Flexible and robust demand allocation and sourcing strategies are considered as influential risk mitigation policies.

Inventory Optimization is one of the most straightforward approaches for risk mitigation by adding redundancy to the network against supply and demand uncertainty (Snyder and Shen, 2011). The main concern in inventory optimization problems is to find the optimal replenishment policy which determines when, where, from which supplier, and how much to order.

One successful example of how a company can recover quickly from disruptions by robust demand allocation and sourcing strategy techniques is Wal-Mart's performance after the Hurricane Katrina disaster in the Gulf coast. Wal-Mart has employees dedicated to tracking potential disruptions and planning for them. With Katrina upcoming, Wal-Mart overstocked its nearby distribution centers with items that would be needed (such as bottled water, Pop-Tarts, and generators), and after Katrina struck, its solid transportation network allowed it to respond quickly to deliver supplies and reduce the disruption negative impacts to its supply chain. Without this preparation, Wal-Mart's recovery time would have been much longer and much more costly for the company (Leonard, 2005).

Traditional inventory optimization models tend to reduce the problem to a single-echelon setting, that is, a set of independent single-echelon inventory systems, in order to keep the model solvable and thus derive optimal properties (Diks and de Kok, 1998).

The single echelonï based approach entirely neglects interdependencies between the echelons and, consequently, could dismiss some inventory positioning opportunities or inventory coverage strategies. Consequently, the network carries excess inventory in form of redundant safety stock, and end-customer service failure could occur, even when adequate inventory exists in the network (Silver et al., 1998, Snyder and Shen, 2011).

Over the last decade, multi-echelon inventory optimization models have gained more importance mainly since on one hand, the current environment dynamics require looking beyond classical sourcing and distribution strategies (Christopher et al., 2011, Eruguz et al., 2014) and on the other hand, recent Information Technology advances have made the management of such networks feasible in practice (Kalchschmidt et al., 2003).

Hence whenever a disruption happens, it has a major impact on the whole supply chain network. From an academic perspective, to set a robust inventory optimization approach, first we need to model disruption occurrences and impacts.

Among different inventory control policies, several studies have shown that (s, S) policy is optimal under some specific conditions (e.g., in the presence of fixed ordering cost) (Sethi et al., 2003, Fox et al., 2006, Huggins and Olsen, 2010).

However, many works (Firouz et al., 2017, Schmitt and Snyder, 2012) indicate that it is very difficult to find an optimal policy in scenarios where the suppliers could be disrupted.

1.4 Problem Statement and Research Methodology

In this section, the global research questions mentioned in the previous section are detailed into problem statement with devoted approaches. We also detail the methodology adopted for resolving the thesis problematics and achieving the objectives stated (see Figure 6). The main challenge that is addressed in the methodology is the ñanticipation of the future demand variability and disruption occurrencesò. In order to efficiently mitigate supply and demand risks, sourcing and inventory decisions should proactively take that risk into account. Traditional methods do not include the impact of supply and demand uncertainty on multi-echelon inventory optimization decisions. Unlike the traditional modeling approaches, we propose stochastic approaches with different types of uncertainty. In this research, customer demands and network disruptions are modeled as compound stochastic processes.

In our methodology, we consider the effect of uncertain parameters (demand and disruption) on all decisions. We form the non-stationary demand by the historical daily demand information in chapter 3. In parallel (as detailed in chapter 4), a compound stochastic process is defined to describe how disruptions occur in space and time. This process also specifies the eventȏs intensity and duration. The different impacts of disruptions on the network are modeled using mathematical functions. The risk modeling approach used in chapter 4, is an improved form of the framework developed in (Klibi and Martel, 2012).

Considering the demand and disruption uncertainty, two two-stage stochastic mixed integer programming (MILP) formulations are proposed to optimize the tactical (first-stage) and operational (second-stage) decisions in Chapter 3 and Chapter 4. In this sort of mathematical models, the decision variables are divided into first-stage decisions that must be made earlier to the occurrence of the uncertainty (tactical decisions in this research) and second-stage decisions (operational decisions in this research) that are made after the uncertainty is unveiled. In this framework, first-stage decisions is valid for all considered scenarios, such that the costs associated with the first-stage decisions and the expected cost of the second-stage decisions are optimized (Birge and Louveaux, 2011).

For each part, a stochastic scenario-based inventory optimization model is developed to minimize the expected total cost that is composed of the fixed allocation, inventory holding, procurement, transportation, and back-ordering costs. As illustrated in Figure 5, in our proposed two-stage model the allocation and sourcing decisions are optimized at the beginning of the planning horizon (i.e., in the first stage), then in the second stage, the inventory levels, the transportation flow decisions, and the ordered quantities at all echelons are optimized for all periods of the planning horizon.

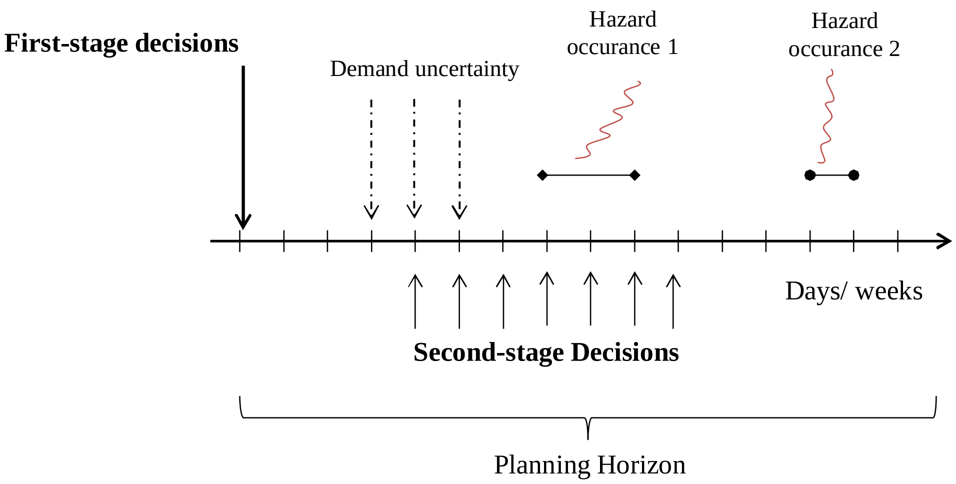


Figure 5. Business environment during a planning horizon

In this thesis, based on the stated thesis problematic and adopted Inventory Optimization approach, six objectives have been formulated:

1- To propose an optimization approach for multi-echelon distribution network to minimize the total cost including (fixed allocation cost, transportation cost, backorder cost, holding cost, fixed procurement cost)
2- To evaluate effect of multi-sourcing and lateral transshipment in multi-echelon distribution network under supply and demand uncertainty
3- To determine the optimal (*s*, *S*) inventory policy parameters in multi-echelon distribution networks
4- To examine different impacts of supply disruptions on demand allocation decisions
5- To analyze the effect of supply disruption on the (s, S) policy parameters.
6- To test the empirical validity and utility of the proposed approach on a large set of real world data.

Our work is built on a stochastic programming approach (Shapiro, 2003, Borodin et al., 2016) with the use of scenarios to shape the demand uncertainty and on the Sample Average Approximation (SAA) method (Shapiro, 2008) to solve a set of equivalent deterministic problems. This scenario-based modeling and solving approach is known for producing ñgood-qualityò solutions based on the best trade-offs between expected cost and service level and the explicit inclusion and evaluation of recourse costs. The plausible future scenario samples required to formulate the stochastic models are generated using Monte-Carlo methods.

The contributions of this thesis are fold 3 principal axes: Uncertainty, Distribution Strategy and Inventory Policy. In chapter 2, we prepared a comprehensive analysis on the literature to detail our proposed contribution. The Figure 7 indicates each chapter contributions separately. In chapter 3, by the motivation of extending the work of Martel (2003), we consider a more flexible sourcing strategy by allowing multi-sourcing and lateral transshipment flows.

For the sake of generality, Distribution Requirements Planning (DRP) approach is applied as the replenishment policy. DRP is a rolling horizon, echelon-by-echelon approach that bases procurement decisions on time-phased expected future site requirements (Martin, 1994). A DRP-based policy is built to evaluate the effect of different distribution strategies on inventory-related decisions. Moreover, disruption and the associated impacts are modeled in chapter 4. Recall that (s, S) policy is one the most practical policies in this network setting, the policy parameters are optimized through the proposed two-stage stochastic model.

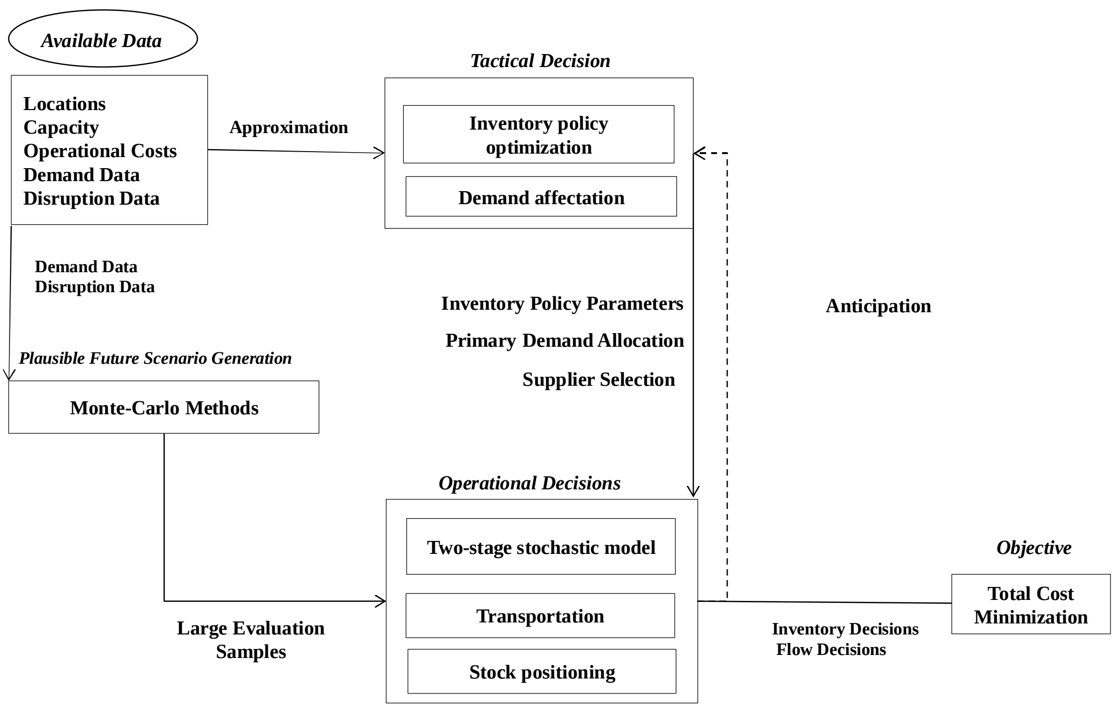


Figure 6. Research methodology

Following the methodology presented above, the achieved contributions can be summarized as:

(1) Proposition of a two-stage multi-echelon inventory optimization model with a DRP approach to handle non-stationary demand processes (Chapter3).

(2) Modeling of lateral transshipments and multi-sourcing strategies in a multi-echelon network in order to improve its flexibility and capabilities to reduce shortages (Chapter 3).

(3) Development of a two-stage multi-echelon inventory optimization model which optimize the (s, S) policy parameters (Chapter 4)

(4) Modeling of lateral transshipments and multi-sourcing strategies in a multi-echelon network in order to improve its flexibility and capabilities to reduce shortages (Chapter 3).

(5) Modeling of two different disruption impacts on multi-echelon networks by considering stochastic throughput capacity and possible inventory loss (Chapter 4).

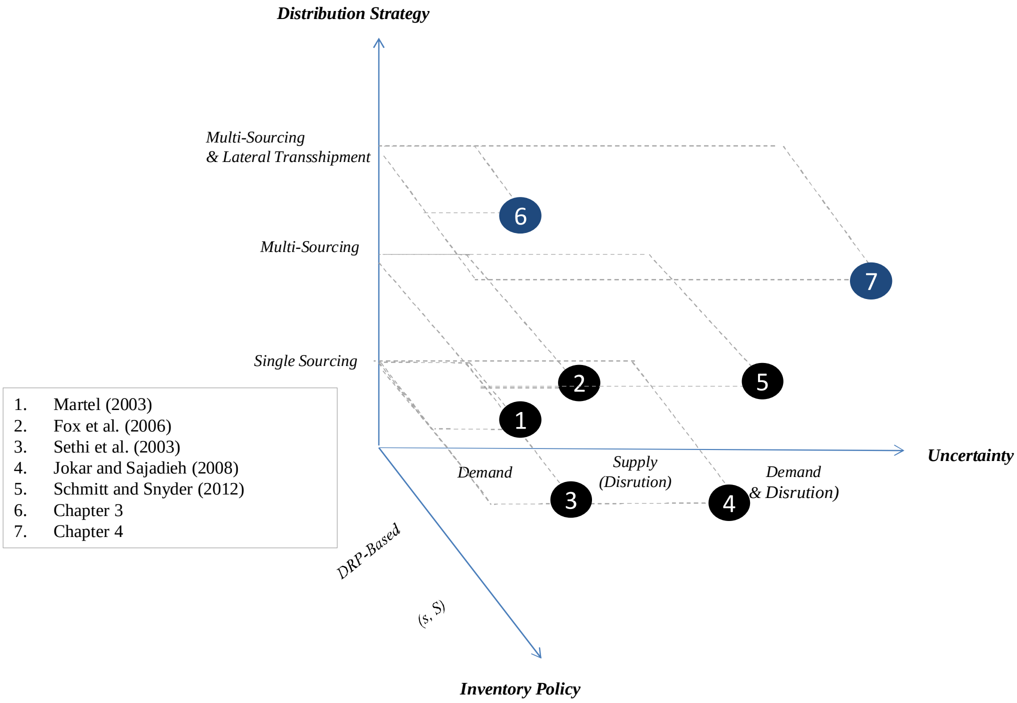


Figure 7. Contributions of this research

1.5 Conclusion

This chapter summarized the research path conducted in this study. First, the problem context containing the basic definitions, business context and thesis perimeter is presented. Then the inventory optimization problem is introduced in the next section. The research methodology and the problem definition are developed in section 4 by presenting the main objectives and contributions.

To attain a unified understanding of the concepts related to this research work, it is necessary to analyze the related research work that has been done in this field. In the next chapter, an overview of the literature on multi-echelon inventory optimization will be presented.

Chapter 2. State of the art

The first chapter summarized the research path conducted in this study. It outlined the research through a summary of the research background, problem context, and designated methodology. This chapter aims to provide an overview of the literature on multi-echelon inventory optimization. An analysis of the literature is provided in the last section of this chapter.

2.1 Introduction

There is a considerable research on multi-echelon distribution planning and inventory optimization problems. Three streams of research are particularly related to our work and will be reviewed in this chapter: uncertainty, distribution strategies and inventory policies. The Figure 8 indicates a summary of reviewed issues in multi-echelon inventory systems. The section numbers in this chapter is indicated on each issue in the presented structure.

The first one relates to multi-echelon inventory optimization problems under different types of uncertainty. We review the methods proposed in the literature for the analysis and optimization of multi-echelon inventory systems, especially for the systems with fixed order costs. First, a general introduction of the studies in multi-echelon inventory management is given. Then, different types of uncertainty in multi-echelon systems are investigated: supply uncertainty and demand uncertainty.

Afterwards, in the second part, different distributions strategies are reviewed. The works which considered multi-sourcing and lateral transshipment in inventory optimization problems are examined in detail.

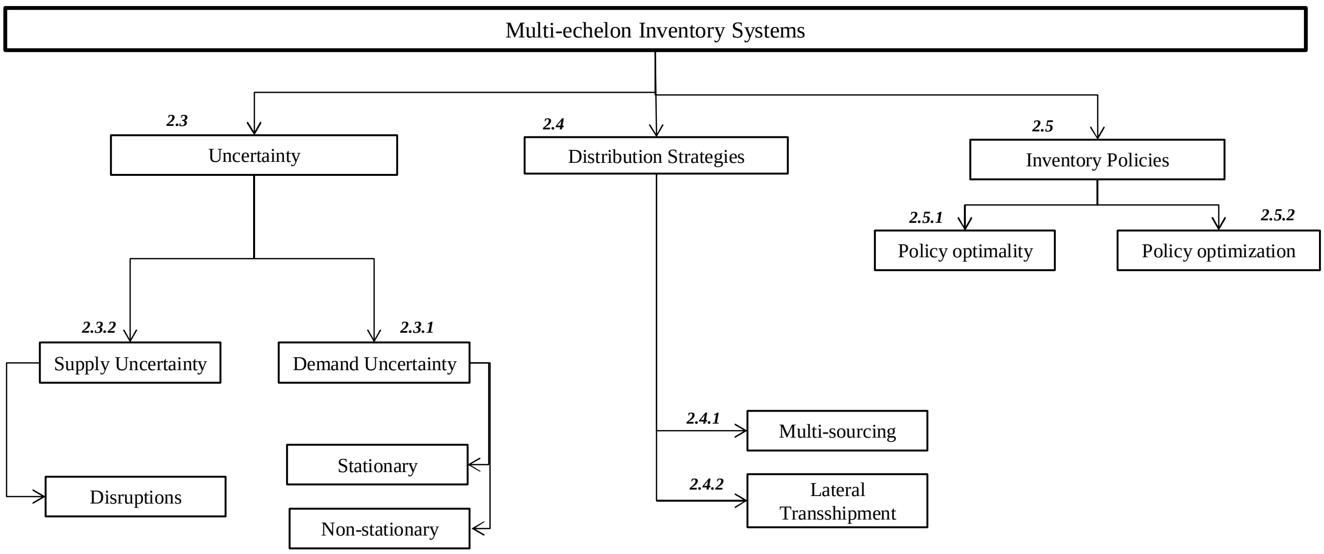


Figure 8. Literature review structure

In the third part, we briefly introduce different inventory policies used in multi-echelon inventory systems. First, the works which prove the optimality of different policies, especially *(s, S)* policy are studied. Then, the design of optimal policy parameters in different setting is reviewed.
A comprehensive discussion on literature review is presented at the end of this chapter. The research gaps are argued to show more clearly the contributions of each chapter separately.

2.2 Multi-echelon Inventory Optimization

Inventory levels can be reduced by as much as 25% due to effective multi-echelon inventory management. That is one of the reasons that multi-echelon distribution planning problems have attracted many researchers in the last decades (Yeong-joon et al., 1997, Martel, 2003, Wang, 2009, Yang et al., 2017).

Inventory optimization approaches are applied to minimize inventory-related cost all over the network. Our modeling approach is cost-oriented. Four cost components are taken into account for developing these models in the literature.

The first one is the holding cost. Storage cost for the unsold items that may be kept through one period or multiple periods will incur and it can be so expensive. Depending on the business, the location, and so on, holding costs vary considerably. It is typically superior to 10%, some high technical items have holding costs greater than 50%.

The second component is the replenishment cost. It consists of the purchasing cost and ordering costs considered for placing orders in most models. This ordering cost is independent from the size of replenishment.

The third component is the shortage cost whenever the customer demand cannot be fulfilled. Shortage costs can be represented by backordering cost, lost sales cost, or just penalty cost in many models. In our approach we only consider backorder cost which is relevant to the retailing business context. The last cost component is the fixed allocation cost. The fixed allocation cost are those expenses associated with assigning a distribution center to a demand zone and are incurred once the product flow between a determined distribution center and a demand zone is allowed. This cost is involved with tactical decisions which determine customer demand allocations.

According to the literature, most of inventory models consider basic cost structures that are linearly proportional to the amount of dependent variables for minimizing the total expected cost when fixed costs are negligible compared to some variable costs, such as procurement cost, inventory holding cost, or backorder cost (e.g.,(Yan et al., 2003, Yang et al., 2013, Amiri-Aref et al., 2018).

From a mathematical point of view, the inventory models used can be classified into deterministic and stochastic inventory models.

A deterministic inventory model assumes that there is no uncertainty in supply and demand. Demand is considered as deterministic and suppliers are 100% reliable. Due to these assumptions, the analysis of the model is considerably simplified. Obviously shortage in a deterministic inventory model is not allowed. Deterministic inventory models can further be divided into static and dynamic models. The static models are generally follow classical economic order quantity (EOQ) which computes an optimal trade-off between fixed order costs and variable inventory carrying costs. Such models can be applied in the situations when the system conditions are stable, suppliers are reliable, and there are no variations in the demand. To deal with the conditions with deterministic time varying demand, several lot sizing models have been developed, which can be used in different situations.

The most common methods for single stock lot sizing are Wagner-Whitin method (Wagner and Whitin, 1958), part period balancing (Callarman and Hamrin, 1984) and Silver meal heuristics (Silver and Pyke, 1998).

Note that these deterministic models provide a basis for handling inventory systems with uncertainty. While deterministic models allow obtaining an optimal solution for a single scenario, stochastic models can consider the stochastic processes in a comprehensive manner which considers many scenarios at once. Therefore, when any parameter (e.g., demand, capacity, lead time) of a problem is subject to uncertainty, stochastic programming becomes more appropriate than traditional deterministic approaches.

Since stochastic inventory models take into account supply and demand uncertainties, they are more realistic compared with their deterministic equivalents.
The analysis of stochastic inventory models is usually very difficult. The cost functions of most stochastic inventory models have been commonly perceived as rather complex and too difficult to be evaluated analytically (Zheng, 1992). In the literature, several stochastic inventory models have been developed.

Surveying the literature, we observe that an important portion of articles on inventory problems suggest mathematical programming. In this section, we mainly focus on linear and Mixed Integer Programming (MIP) modeling approaches. Several approaches have been proposed to solve these mathematical problems. A very commonly used method is the analytical approach for solving small-scale problems, which guarantees exact solutions if the underlying mathematical model is solvable. Besides, general exact solvers, like CPLEX, Lingo, Lindo, Xpress, GAMS, AIMMS, AMPL, can also derive exact solutions, and tailored algorithms (e.g., branch-and-bound/cut, decomposition techniques), can also lead to exact solutions. However, models (Wang et al., 2008, Jain et al., 2011, Amiri-Aref et al., 2018) with exact solutions are often based on a number of assumptions (e.g., constant demand, a small finite period, or no crossover orders for inventory models).

Stochastic multi-echelon inventory optimization is usually intractable due to the inherent combinatorial complexity and the very large number of plausible scenarios necessary to shape supply and demand processes. Therefore in this thesis, the sample average approximation (SAA) technique (Shapiro, 2008) is used in order to approximate the stochastic model by an equivalent deterministic mixed-integer linear program (MILP), namely the SAA model.

To investigate different stochastic inventory models, the role of uncertainty in inventory models is examined.

2.3 Uncertainty

Uncertainties are widely considered in stochastic inventory models. In this section, we classify the inventory models according to two main categories of uncertainties: demand uncertainty, and supply uncertainty. Meanwhile, in order to integrate uncertainties into inventory models, it is usually assumed that parameters (e.g., demand, lead-time) follow certain probability distributions. The majority of models apply this method to capture the impacts of uncertainties. From a generic point of view, based on the previous section, the current inventory optimization models do not provide an adequate level of robustness against supply and demand uncertainty, especially in multi-echelon setting.

2.3.1 Demand Uncertainty

This kind of uncertainty is most commonly taken into account, and can exhibit a very large degree of variability over the course of procurement periods. In multi-supplier inventory models, demand uncertainties are modelled by known parameters, such as certain probability distributions based on the historical demand data. The demand uncertainty could be considered as stationary and non-stationary patterns.

Considering a stationary demand pattern, Yeong-joon et al. (1997) have considered a multi-echelon distribution network. They have proposed an improved DRP approach with single sourcing using the concept of reorder point installation-stock. Based on simulation experiments, they have shown that the proposed system outperforms the classical DRP approach.

Martel (2003) has proposed a stochastic model for multi-echelon inventory optimization systems under the single sourcing strategy which is solved with a DRP-decomposition approach. Using simulation, the results obtained by the proposed approach show a significant improvement in comparison to the classical DRP approach. In Wang (2009), a fuzzy modeling approach has been used for a multi-echelon inventory optimization problem. They have indicated that bythe minimum total cost (including the inventory holding and back-ordering costs) under the continuous review policy could be obtained by applying a DRP method.

Furthermore, Yang et al. (2016) have analyzed two single-product multi-echelon distribution networks with a continuous reorder point replenishment approach in the storage facilities. No storage capacity is assumed and demand is assumed to be stationary and normally distributed. A non-linear simulation-based optimization model has been proposed and solved with a metaheuristic using simulated annealing.

Graves and Willems (2000) have developed a framework for positioning the strategic safety stock in a multi-echelon supply chain considering a base-stock policy with a common review period. They have proposed a guaranteed-service model to evaluate the inventory requirements at each stage as a function of the service times. An optimization algorithm has been applied to find the service times that minimize the holding cost for the safety stock in the supply chain. Another guaranteed-service model with bounded demand has been proposed recently by Graves and Schoenmeyr (2016). They have numerically shown that the modified constant base stock policy is near optimal in a low-capacity condition; however, its performance deteriorates when the constraint (capacity) is relaxed.

Although in this stream of research, most of the work has been developed under the assumption of a stationary demand, few articles have considered the assumption of a non-stationary demand. This latter assumption is important because it fits cases in which the demand is affected by a trend, seasonal factors, or cyclical behaviors.

Note that one of the techniques to model a non-stationary demand is to break the horizon into a set of stationary phases and implement a rolling-horizon approach in which the optimization should be done for each demand phase (Bollapragada and Morton, 1999).

Based on the model presented by Graves and Willems (2000), a supply chain inventory model with a non-stationary demand process has been developed by (Ettl et al., 2000). They have used service-level constraints to calculate safety stocks. Each stage was controlled with a periodic base-stock policy in which the review period is one time unit. This work has been extended by Graves and Willems (2008) in the case of a non-stationary demand.

2.3.2 Supply Uncertainty

According to the literature, the supply uncertainty is categorized into three categories (Qi et al., 2006). The first category is disruptions. When a company's supply is disrupted, its supply process comes to a complete break, or the supply process would be partially operational until the supply disruption process is completely recovered. The second type of supply uncertainty is yield uncertainty. It means that the actual amount of items delivered by the supplier could be a random number dependent on the ordered quantity (Schmitt and Snyder, 2012). For example, among each batch of products delivered, some products might have defects, which make them useless. The number of defective products could be a random variable. The third form of supply uncertainty is lead time uncertainty. When the lead time is stochastic, the delivery of products takes a random amount of time, but at the end of the period, the exact amount of products ordered would arrive (Jokar and Sajadieh, 2008, Hnaien et al., 2010).

In this research, we only consider the first category, supply disruptions. Supply disruption has been increasingly studied in recent years. Due to either internal causes (e.g., machine/equipment breakdowns, or labor strikes) or external causes (e.g., unpredictable natural disasters, political trade intervention or bad weather conditions).

In many studies, supply disruption has been modeled with deterministic demand (Chopra et al., 2007, Jokar and Sajadieh, 2008, Schmitt and Snyder, 2012). Other works (Keskin et al., 2010a, Silbermayr and Minner, 2014) have investigated models with stochastic demand. Measuring the disruption impact is a challenging task.

The most common way that supply disruption has been modeled is that the supplier has two states: normal and disrupted. Supply capacity is infinite in normal and zero in disrupted state. Supply disruptions could have various impacts on the network such as lead time increase, capacity loss, inventory loss, etc. Klibi and Martel (2012) have developed an approach to model the supply disruptions impacting the throughput capacity. In their proposed approach, first, a compound stochastic process has been defined to describe how hazards occur in space and in time, to specify incidentôs impact. Second, the impact of hits on the throughput capacity has been modeled.

It has been extensively discussed how disruption and demand uncertainties affect the topology of different multi-echelon supply chain networks (Keskin et al., 2010a; Schmitt et al., 2015). One of the solutions to overcome the disruptions is to apply a decentralized network design. Schmitt et al. (2015) have used simulation to show the optimal strategy for coping with disruption which is often the exact opposite of the strategy for demand uncertainty. They have concluded that the two forms of uncertainty are mirror image of each other. For example a decentralized design is preferable to reduce the impact of any disruption. In contrast, under demand uncertainty, central design is the optimal one.

Sourcing strategies are considered as a common technique to enhance the robustness of the network against uncertainty. In this subsection we review sourcing strategies in two categories, multi-sourcing using splitting orders and lateral transshipment.

2.4 Distribution strategies

Sourcing strategies are considered as a common technique to enhance the robustness of the network against uncertainty. In this subsection we review sourcing strategies in two categories, multi-sourcing using splitting orders and lateral transshipment.

2.4.1 Multi-Sourcing

An important potential solution to overcome supply uncertainty is multi-sourcing. According to the literature, order splitting can reduce the inventory holding and backordering costs when a network involves random demands and lead-times (Minner, 2003). Despite some clear advantages (e.g., cost and service level) of multi-sourcing, only a limited number of studies present an analytical approach to investigate inventory decision problem in this area.

Splitting order quantity or order allocation between distribution centers depends on supply characteristics, such as capacity, transportation cost, and reliability of the network. Under this order-splitting sourcing mechanism, both inventory level in distribution centers and cycle stock with successive deliveries of smaller split orders can be reduced (Bohner and Minner, 2016).

This multi-sourcing concept can be identified under both dual and multiple sourcing as well as under deterministic or stochastic supply parameters. In scenarios of multi-sourcing with deterministic supply parameters, Glock and Ries (2013) have considered a system in which customers order from multiple suppliers with stochastic demand. They have shown that the shortage can be reduced by splitting the total order quantity among different sources.

Wang et al. (2008) have investigated a fixed demand system where a manufacturer must choose the best suppliers when the on-hand inventory level drops to the reorder point. Based on constraints on supplier capacity and quality, splitting the replenishment order quantity would be considered to improve the performance. Zhou et al. (2011) have proposed a model to examine a finite horizon periodic review system with stochastic customer demands and capacity constraint. The model determines the decisions on the replenishment quantity and order allocation to minimize the total expected cost.

A problem setting with multiple candidate suppliers and multiple warehouses has been presented by (Keskin et al., 2010a, Keskin et al., 2010b). Keskin et al. (2010b) have considered the integrated supplier selection inventory optimization problem under a deterministic demand and proposed an efficient generalized benderôs decomposition algorithm as solution approach.

Extending this deterministic problem by taking into account the stochastic demand and disruptions condition, Keskin et al. (2010a) have proposed a simulation-optimization based solution approach. They have shown that the unit inventory-related costs and fixed allocation cost impact the topology of the network. According to their study, in many cases, the disruptions does not change the topology, however, the inventory decisions and flow decisions may differ.

Moreover, there are recent works considering disruption in order splitting models. (Silbermayr and Minner, 2014) have presented a semi-Markov decision process model with stochastic demands, where lead times and ON and OFF periods of suppliers are identically distributed to minimize a buyer's long run average cost. They find that the percentage of demand allocated to the expensive but reliable supplier is higher with a higher penalty cost, and the long run average cost is less in the backorder model than in the lost sales model.

Song et al. (2014) have considered an inventory system with multiple suppliers subject to stochastic demands and supply disruptions. They present a procedure for determining the total order quantity, reorder point and splitting proportion among multiple suppliers, and assume that the total order quantity is equal among the suppliers with identical lead time distributions. Clemons and Slotnick (2016) investigate the effects of supply chain disruption on a firmôs decisions to invest in quality and on ordering decisions when there is a variable rate of knowledge transfer and a choice between two suppliers with different quality levels. They show that the increasing cost due to disruption can be mitigated by sensibly allocating demand between two suppliers.

Hu and Kostamis (2015) have studied a system where some but not all suppliers face the risks of complete supply disruptions. They have shown that the total order quantity and its allocation between the two suppliers are independent decisions and that unreliable orders are ranked by the ratio between the suppliersô cost advantages over the reliable supplier and their disruption probabilities. They have shown that multi-sourcing is effective since it can reduce the total cost and lead time risks, as well as improve the service level for a high reduction of lead time demand, especially when lead time uncertainty is high or ordering costs are low.

All these works have shown that order splitting among multi-sourcing can reduce the total cost of ordering, procurement, inventory holding, and shortages. Moreover, the optimal strategy of single or multi-sourcing and related optimal sourcing strategies have also been studied (e.g., (Tomlin and Wang, 2005)).

Although multi-sourcing enhances the complexities of the problem, it could reduce and mitigate risks and improve the global performance of the distribution networks. In this thesis we model multi-sourcing by splitting orders between the distribution centers.

2.4.2 Lateral Transshipment

Traditional multi-echelon network only consider product flow from upstream to downstream, while flows of products between different platforms at the same echelon are allowed thanks to a lateral transshipment system.

Lateral transshipment is efficient in some conditions. In the situations when the cost of lateral transshipment is lower than the cost of keeping a higher inventory level, or than the backorder costs, it can be practical. Also, the replenishment time of lateral transshipment is almost always shorter than the one for sourcing from the upstream. Therefore, transshipments could reduce the total cost and increase the service level simultaneously, thereby improving the performance of the whole network.

That is why lateral transshipments have attracted the attention of many researchers as a flexible supply strategy. A comprehensive literature review on inventory models with lateral shipments has been provided by Paterson et al. (2011).

Note that in this work, the models have been classified based on the type of transshipment employed. Based on the timing for replenishment, two streams of transshipment have been identified: 1- proactive (or preventive) lateral transshipment, which occurs before stockouts to minimize the risk of future stockouts, and 2- reactive (or emergency) transshipment, which may happen at any time in response to shortages. In a proactive modeling approach (Agrawal et al., 2004, Lee et al., 2007) transshipments can be affected only at fixed points in time, whereas in a reactive modeling approach, transshipments can occur at any time (Paterson et al., 2012, Zhao et al., 2016, Nakandala et al., 2017).

Grahovac and Chakravarty (2001) have developed an inventory replenishment model based on a base-stock policy that provides a threshold to activate the lateral transshipment flows with the single-sourcing strategy. They have shown that, although the inventory levels may increase in the network (leading to higher inventory holding costs), lateral transshipments reduce the total cost for a stationary demand.

Lee et al. (2007) have proposed a new transshipment strategy for a two-echelon supply chain network, under the single-sourcing strategy, which leads to lower total cost (including the back-ordering, the ordering, and the holding costs) and that deals effectively with stationary demand fluctuations. They have shown that the benefit magnitude of the lateral transshipment depends on the unit transportation cost between the retailers.

Minner et al. (2003) and Zhao et al. (2008) have made an assumption that replenishment lead times are negligible. Based on this assumption they have developed an improved heuristic for deciding on emergency transshipments.

Different inventory policies are used in emergency transshipment scenarios. Previous studies have shown that, in the absence of fixed costs per replenishment order, the base-stock policies are optimal if transshipments are used to compensate for an actual shortage and not for inventory being built up at another stocking location (e.g., (Özdemir et al., 2006, Özdemir et al., 2013)). In other words, transshipment could be applied as an optimal option only to avoid shortage. Gong and Yücesan (2012), and Karsten et al. (2012) have considered multi-location systems with positive replenishment lead times controlled by continuous review base-stock policies to minimize the expected average total cost.

Although the benefits of multi-sourcing and lateral transshipments have been shown in the literature separately to mitigate risks associated with demand uncertainty, applying both strategies in a multi-echelon network has been rarely considered. Considering a specific inventory control policy, (Tiacci and Saetta, 2011) have considered a supply chain system where two retailers who face their final customersô demand have one central depot that supplies them both, and assume that the retailers use a periodic (*s*, *S*) policy to replenishment their inventory.

Firouz et al. (2017) have developed also a mixed integer nonlinear programming for the same setting by considering multi-sourcing and lateral transshipment policies under a stationary stochastic demand following a Poisson distribution. They have studied a two-stage supply chain network in which the warehouses replenish their inventory from multiple suppliers with varying price, capacity, quality, and disruption characteristics. The warehouses are operating under *(R, Q)* policy. They show the benefits of each sourcing strategy under different cost setting.

2.5 Inventory Policies

In this section we review the models and methods proposed in the literature for analyzing inventory policies, especially *(s, S)* policy in multi-echelon inventory systems.

In inventory management, an inventory policy must answer to two questions: when the inventory position must be reviewed and which quantity each order must be placed. Most frequently used inventory policies for multi-echelon inventory systems are base stock policy, (*R*, *Q*) policy and (*s*, *S*) policy. Applying the

There are two control parameters in (*s*, *S*) policy: the reorder point *s* and the order-up-to level *S*. When the inventory position of a platform declines to or below *s*, the platform places an order to bring its inventory position to the maximum level *S*. Compared with (*R*, *Q*) policy, (*s*, *S*) policy no longer orders a multiple of a given order size. Noted that if the reorder point is always reached exactly in case of continuous review and continuous demand, the two policies are equivalent with $s=R$ and $S=R+Q$. In addition, an inventory model operating under (s, S) policy has a complex structure. This leads to the fact that few results exist for the optimization of such a policy in the context of multi-echelon inventory systems. The industry context and the type of product usually determine which policy between continuous (R, Q) and periodic (s, S) policies should be applied. Since (s, S) policy is more generic and poplar in multi-echelon distribution networks, the use of (s, S) policy is more beneficial from a theoretical point of view. For this reason, we only consider (s, S) policy in our research work.

We characterize the literature on multi echelon inventory systems according to the optimal replenishment policy, as well as optimal parameters for given policies. First we show that, according to the literature, (s, S) policy could be optimal for our problem setting, then we review the works in which the (s, S) policy parameters are optimized.

2.5.1 Optimal replenishment policy

The optimality of two policies, base-stock and (s, S), has been proved for controlling inventory with a multi-sourcing setting. (s, S) policy has been proved to be optimal for periodic review systems with fast and slow delivery modes in some specific situations depending on procurement costs. There are several studies that describe the structure of the optimal inventory policy under specific conditions.

One of the first has been done by (Parlar et al., 1995) which considers a finite-horizon problem with random demand, zero lead time, and an unreliable supplier which could be disrupted. They claim that the optimal inventory policy for this problem is order-up-to level (s, S). Another works that determines the structure of the optimal inventory policy under certain conditions is by (Song and Zipkin, 1996). They have indicated that a base-stock policy is optimal if there is no fixed cost and an (s, S) policy is optimal otherwise; both policies are state dependent, with the optimal parameters depending on the state of the supply process. Because of the generality of the proposed model by (Song and Zipkin, 1996) , this policy has been applied to a wide range of inventory problems with disruptions.

Following these original studies by (Parlar et al., 1995) and (Song and Zipkin, 1996) on periodic review inventory systems, multiple researchers have examined the optimality of (*s, S*) policy under different assumptions (Sethi et al., 2003; Fox et al., 2006; Huggins and Olsen, 2010; Zhang et al., 2012).

Fox et al. (2006) have investigated two different inventory systems to prove that (*s*, *S*) policy is optimal, one with a negligible fixed cost but high variable costs, and the other with a positive fixed cost but a low variable cost. They have shown that a reduced form of the generalized (*s*, *S*) policy is optimal for dynamic programming problems with both finite and infinite horizon.

Moreover, Huggins and Olsen (2010) have examined the structure of the optimal expediting policy and indicated that a (*s*, *S*) policy is optimal for regular production. For the special problem settings where the expediting cost function is concave or consists of a fixed and linear per-unit cost, they have shown that the optimal expediting policy is a generalized (*s*, *S*).

Zhang et al. (2012) have considered an inventory control problem with multiple suppliers with different fixed and variable costs under a limited capacity. They have shown that the optimal policy could be considered like (s_{nt}, S_{nt}) for each center and each period. According to their results, a customer has to order from more expensive suppliers if demand exceeds the order quantity from a cheaper supplier, regardless of reliability.

2.5.2 Optimal parameters for given policies

In spite of the fact that it has been known for a long time that there exists an optimal inventory policy under quite general conditions, optimal control parameters of the policy under the stochastic setting are hard to be computed (Feng et al., 2006, Song and Zipkin, 2009). Most of the previous studies on stochastic inventory models were focused on cost evaluation and on determining optimal control parameters for predetermined inventory policies. In contrast, results on optimal policy structures are rare.

Considering (*s*, *S*) as the optimal policy, a lot of researches has been done to approximate (*s*, *S*) policy parameters ((Jain et al., 2011, Fattahi et al., 2015, Amiri-Aref et al., 2018, Cunha et al., 2018). Jain et al. (2011) have presented a conceptual model to approximate the (*s*, *S*) policy parameters to evaluate the cost performance of the network. They have developed a heuristic to provide bounds for the policy parameters under a periodic review system.

In more recent studies, Fattahi et al. (2015) have proposed different mixed-integer linear programming (MILP) models for designing centralized and decentralized supply chains using two-stage stochastic programming. They have studied a multiple period replenishment problem based on (s, S) policy for these supply chain models.

Considering the demand uncertainty, Cunha et al. (2018) have developed mixed integer model to optimize a replenishment policy for single-item single-echelon network with periodic review and variable order quantities.

Zhao et al. (2012) have studied a manufacturer with two transportation modes: a slow mode with low cost and long and stochastic lead time, and a fast mode with high cost and short and deterministic lead time. They assumed a periodically adjusted base stock policy with demand forecast updating.

Amiri-Aref et al. (2018) have extended stationary demand to non-stationary demand, and have investigated a (s, S) policy in a location-allocation problem. They have proposed a two-stage model in a periodic review setting with multi-sourcing suppliers to handle a non-stationary demand pattern.

None of these studies has considered disruption in optimizing the selected inventory policy. Ahiska et al. (2013) have considered a periodic review inventory system for a retailer who has adopted a dual sourcing strategy for coping with potential supply process interruptions. They have derived the optimal parameters for (s, S)-type policies in the presence of fixed ordering costs. Under this policy, they assume that the reorder points for the reliable and the unreliable supplier, which are simply defined as the highest inventory levels below which the respective order quantities, are positive in the optimal policy.

2.6 Discussion on the literature review

We reviewed and discussed a broad range of multi-echelon inventory models, categorizing them according to uncertainty considerations, sourcing strategies, and inventory policies. Various forms of supply and demand uncertainties are extensively investigated and summarized. The modeling and computational complexity may intensely increase with the numbers of uncertain parameters. As indicated in this chapter, many works only consider one kind of uncertainty in their inventory models. Studies addressing inventory models with more types of stochastic parameters are still few.

Furthermore, an important drawback in this field concerns sourcing strategies. Although the benefits of multi-sourcing and lateral transshipments have been shown in the literature separately to mitigate risks associated with demand uncertainty, applying both strategies in a multi-echelon network structure under a capacitated distribution and a non-stationary process setting has not been studied.

In the first part of this research work (chapter 3), the focus is on multi-echelon distribution systems under demand uncertainty. Considering the general drawbacks of single echelon approaches, an alternative approach is commonly considered in inventory optimization models, which is called the Distribution Requirements Planning (DRP) approach.

DRP can handle any number of echelons; it manages lead time efficiently; and it can take economies of scale in transportation into account through the choice of a suitable lot-sizing algorithm (Hnaien and Afsar, 2017).

Despite its advantages, the basic DRP has some weaknesses. Martel (2003) reported that the main drawbacks of this approach in its basic form are that it has been fundamentally designed to support deterministic time-varying demands. Hence, there is very limited research that deals with the DRP approach under demand uncertainty.

Some research has been recently developed to optimize multi-echelons inventory systems under demand uncertainty. This research has considered other inventory control approaches such as the reorder point or the base stock policy (Graves and Willems, 2000, Yang et al., 2017, Graves and Schoenmeyr, 2016).

Most of the work on the multi-echelons inventory optimization has been developed under the assumption of a stationary demand and few articles have considered the assumption of a non-stationary demand. This assumption is important because it fits cases in which the demand is affected by a trend, seasonal factors, or cyclical behaviors (Graves and Willems, 2008) and therefore, it will be considered in this first part of our research.

Moreover, sourcing decisions and strategies are particularly important in tactical planning. Even though multi-echelon inventory optimization problems have been widely discussed in the literature, most of them are restricted to a single-sourcing strategy (Silbermayr and Minner, 2016). This strategy has some advantages such as a stronger long-term relationship with the supplier and the reduction of overheads required for handling multiple suppliers. Single-sourcing strategy has its risks in the form of total dependency of the functioning of the entire supply chain on a single source.

Under demand uncertainty and/or disruptions, several works underlined the necessity to consider more flexible sourcing and distribution strategies, such as dual or multiple sourcing (Silbermayr and Minner, 2014, Snyder et al., 2015). Many companies showed interest in taking the multi-sourcing option in order to increase customer service level and reduce the safety stock level, especially in the presence of demand uncertainties. The benefit of considering such strategies will be evaluated in this thesis. Some other flexible systems also considered lateral transshipments within the network. Although the benefits of lateral transshipments have been shown in the literature to mitigate risks associated with demand uncertainty, applying multi-sourcing and lateral transshipments in a multi-echelon network structure under a capacitated distribution and a non-stationary process setting has not been studied.

Table 1. Literature review analysis for the first part of our research (chapter 3)

Authors	Year	Methodology	MN	Demand Allocation	Capacity	Inventory Deployment Strategy					MP	Demand			LT	MS	Solution Approach
						Base stock	EOQ	(s,S)	(R,Q)	DRP		D	S	NS			
Yeong-joon et al.	1997	MILP	*				*			*			*				Simulation approach
Graves and Williams	2000	Dynamic Programming	*			*							*				Analytical approach
Martel	2003	MILP	*							*			*				Simulation approach
Graves and Williams	2008	Dynamic Programming	*			*							*	*			Analytical approach
Wang et al.	2008	MILP	*	*	*								*			*	Exact Method / Branch and bound
Keskin et al.	2010	MINLP	*		*		*										Benders
Paterson et al.	2012	Dynamic Programming							*				*		*		Analytical approach
Kang and Kim	2012	MINLP	*		*				*							*	Heuristic
Glock and Ries	2013	Conceptual	*						*				*			*	General Solver
Yang et al.	2017	MINLP	*	*					*		*		*		*	*	Metaheuristic
Chapter 3		MILP	*	*	*					*	*	*		*	*	*	CPLEX Solver/ SAA

MN= Multi-echelon network,MP= Multi-period problem, D= Deterministic, S= Stochastic, NS= Non-stationary MS= multiple sourcing, LT = Lateral transshipment, MINLP= Nonlinear mixed integer nonlinear programming, MILP= Linear mixed integer nonlinear programming

Note that the works done by Yoo et al. (1997) and Martel (2003) are the closest to our research work. They have analyzed a multi-echelon distribution network under a DRP approach and facing a stochastic demand. However, both have considered networks without supply capacity, multi-sourcing and lateral transshipment. Table 1 gives a summary of the most relevant works related to chapter 3. Table 1 gives a summary of the most relevant works related to chapter 3.

Following the literature, chapter 4 focuses on computing the optimal policy parameters in multi-echelon distribution networks under supply and demand uncertainties.

In chapter 4, supply uncertainty is taken into account in the form of supply disruption, during which a platform of the supply chain is completely or partially inoperative. Since disruptions are not local and they tend to affect every layer of the supply chain, it is significant to examine disruptions in multi-echelon network settings.

This part of our study aims to contribute to the understanding of multi-echelon systems under the risk of disruptions by proposing a novel model which can optimize the selected inventory policy.

A two-stage stochastic model is proposed to solve a capacitated multi-echelon inventory optimization problem considering a stochastic demand as well as uncertain throughput capacity and a possible inventory loss, due to disruptions. The model minimizes the total cost that is composed of fixed allocation cost, inventory holding cost, transportation and backordering costs by optimizing inventory policy and flow decisions.

The inventory is controlled according to a reorder point order-up-to-level (*s, S*) policy and lateral transshipments in the network are considered. The (*s, S*) policy features two control parameters: reorder point (*s*) and order up-to-level (*S*). According to this policy, the decision maker checks the opening inventory position at the end of each time period: if it drops below the reorder point s, then, replenishment should be placed to reach the order-up-to-level S.

Based on this definition (*s, S*) inventory policy is the most generic policy and it can be the optimal policy for the considered problem setting, However, many works (Firouz et al., 2017, Schmitt and Snyder, 2012) indicate that it is very difficult to find an optimal policy in scenarios where the suppliers could be disrupted.

In order to deal with the uncertainties, several scenario samples are generated by Monte Carlo method and corresponding sample average approximations programs are solved to obtain the adequate response policy to the inventory system under disruptions. For this purpose, extensive numerical experiments are conducted. The results provide insights on the impact of disruptions on the network total cost and service level. Table 2 shows the literature review taken into account for chapter 4.

Table 2. Literature review analysis for the second part of our research (chapter 4)

	Authors	Year	Policy	Policy Optimization	Methodology	LP	Demand Allocation	MS	LT	MN	MP	Uncertainy				Solution Approach
												DIS.	D	CAP	IL	
1	Gurler and Parlar	1997	(s,Q)		Analytic		*	*		*		*				Analytical approach
2	Sethi et al.	2003	(s,S)	*	Dynamic Programming						*		*			Analytical approach
3	Fox et al.	2006	(s,S)	*	Dynamic Programming			*			*		*			Analytical approach
4	Keskin et al.	2010	(Q,R)		MINLP					*		*	*			Simulation and metaheuristic
5	Keskin et al.	2010	EOQ		MINLP					*						Benders
6	Jain et al	2010	(s,Q)	*	Analytic								*			Analytical approach
7	Huggins and Olsen	2010	(s,Q)	App.	Analytic					*			*			Heuristic
8	Jain et al	2011	(s,S)	*	Analytic						*		*			Analytical approach
9	Kang and Kim	2012	(s,Q)	App.	MINLP					*	*		*			Heuristic
10	Kelle et al.	2012	(s,S)	App.	Analytic			*		*			*			Approximation
11	Ahiska et al.	2013	(s,S)	*	Markovian decision process			*		*	*		*			Analytical approach
12	Riezebos and Zhu	2015	(s,S)	*	Dynamic Programming					*	*					Heuristic
13	Fattahi et al.	2016	(s,S)	*	MILP	*				*	*		*			Meta-heuristic/ SAA
14	Firouz et al.	2017	(nQ,R)		MINLP		*	*	*	*		*	*			Decompositon and Simulation
15	Amiri-Aref et al.	2017	(s,S)	App.	MILP	*	*	*		*	*		*			CPLEX Solver/ SAA
16	Cunha et al.	2018	(R,S)	*	MILP	*					*		*			CPLEX Solver/ SAA
17	Chapter 4		(s,S)	*	MILP	*	*	*	*	*	*	*	*	*	*	CPLEX Solver/ SAA

LP= Linear programming, MS= multiple sourcing, LT = Lateral transshipment, MN= Multi-echelon network, MP= Multi-period problem, DIS = Disruption, D= Demand, CAP = Capacity, IL= Inventory Loss

2.7 Conclusion

This chapter has investigated and discussed a wide range of multi-echelon inventory models, categorizing them according to uncertainty considerations, sourcing strategies, and inventory policies.

Different studies considering supply and demand uncertainties are extensively reviewed and summarized. The modeling and computational complexity may intensely increase with the numbers of uncertain parameters. As indicated in the last section, the presented research gap in inventory optimization models considering supply and demand uncertainty would be bridged in the next two chapters. In the next chapter a two-stage stochastic mathematical model would be developed considering multi-sourcing and lateral transshipment to come over the demand uncertainty.

Chapter 3. Distribution Planning for Multi-Echelon Networks Considering Multiple Sourcing and Lateral Transshipments

In this chapter a scenario-based modeling approach is proposed to solve a two-stage multi-echelon inventory optimization problem considering a non-stationary demand. The model is based on a distribution requirements planning (DRP) approach and minimizes the expected total operational and tactical cost. Multi-sourcing and lateral transshipment in a periodic review inventory setting, are considered in this modeling approach. A European retailer case study and managerial insights is provided in the last section of this chapter.

3.1 Introduction

In the inventory management literature ï as discussed in chapter 2- a lot of research papers have been dedicated to optimize inventory decisions considering demand uncertainty. From a supply chain perspective, by an uncertain demand that is accentuated when it has a non-stationary pattern. To deal with this issue, inventory optimization models must be adapted to cover a multi-echelon network structure and to consider alternative distribution strategies such as lateral transshipments and multiple sourcing.

This chapter aims to develop a two-stage multi-echelon inventory optimization modeling approach to deal with a non-stationary demand pattern. The proposed model operates under a periodic DRP-based inventory control policy which is a basic and common policy in multi-echelon distribution systems. The main research objective in this chapter is to measure the benefits of different sourcing options (single sourcing, multi-sourcing and lateral transshipment) in different conditions.

In section 2 of this chapter, the problem context is described and a generic scenario-based inventory optimization model with stochastic demand is first developed and then a SAA-based solution approach is presented. In section 3, numerical experiments are run to show the capability of the proposed model. The solution approach is developed in section 4. Based on a set of problem instances defined in section 5, the solutions produced by the model are compared to their counterpart when multi-sourcing and lateral transshipment features are neglected in section 6. In addition, a real case of a supply chain network of a major French retailer is investigated and managerial insights are provided in section 7. Section 8 concludes the chapter and presents some avenues for further research.

3.2 Problem Definition and Notations

This chapter considers a three-echelon supply chain that includes implicit suppliers, a set of production-distribution centers (PDCs), a set of distribution centers (DCs), and a customer zone (CZ) stage (i.e., consumption points). As illustrated in Figure 4 (chapter 1), each stage is fed from the upper echelon and feeds the ones below. The multiple arrows between PDCs and DCs represent the multi-sourcing opportunities with respect to the throughput capacity per period of each platform. A lateral transshipment (LT) option, which allows replenishment flows in the same echelon, is available between DCs. A tactical planning horizon (e.g., yearly, seasonally) is considered and is partitioned into a set of control periods (e.g., months, weeks, days). For a given period, the demand from customer zones arrives to the DCs and is satisfied from the DCs on-hand inventory. If one or a set of customer zone demands cannot be satisfied, it is back-ordered to the subsequent periods. In the same way, the DC orders are satisfied from the PDCsô on-hand inventory. They are received by the DCs after a fixed lead time. The PDCs are sourced from uncapacitated and reliable suppliers. Consumption point demand is stochastic and follows a non-stationary process over the horizon. At the tactical level of the supply chain, when a make-to-stock policy is considered, a key decision is related to the positioning of inventories in time and space.

To address the previously described problem, a two-stage multi-echelon inventory optimization modeling approach under a periodic DRP-based inventory control policy for a product family (referred hereafter as a single product) is considered. A stochastic scenario-based inventory optimization model is developed to minimize the expected total cost that is composed of the fixed allocation, inventory holding, procurement, transportation, and back-ordering costs. In our proposed two-stage model, the allocation and sourcing decisions are first optimized at the beginning of the planning horizon (i.e., in the first stage). Then, in the second stage, the inventory levels, the transportation flow decisions, and the ordered quantities at all echelons are optimized for all periods of the planning horizon. The proposed model considers a multi-sourcing strategy as well as lateral transshipment opportunities between DCs. According to the periodic review policy employed, the inventory level of each product is inspected at the end of each period and all replenishments are originated based on these inventory reviews. Demand is received from a customer zone and the model decides to assign it totally or partially to a DC or decides to back-order it. It is assumed that inventory levels of the products are maintained in time: they are kept stored before being shipped with respect to the throughput capacity of DCs and PDCs. The flows between DCs and customer zones consist of the demand of the actual period plus back-ordered products in previous and current periods.

When it comes to the replenishment process at DCs, an extra option of lateral transshipment is available in the model. The purpose is to anticipate the day-to-day reaction of the network user to replenish a given DC when global visibility of the on-hand inventory at all DCs is available. In such cases, the DCs could receive their products via lateral transshipments, which could be more expensive; however, the orders would be delivered with a shorter lead time to reduce the back-order costs. As illustrated in Figure 9, the demand allocation and sourcing decisions are fixed at the beginning of the horizon for the entire planning horizon. One should also mention that events occur only at the start or end of a period. The lead time is a pre-planned integer number of periods (i.e., multiple of the review period) covering the transportation time plus the order processing, picking, loading, reception, and inspection lead times.

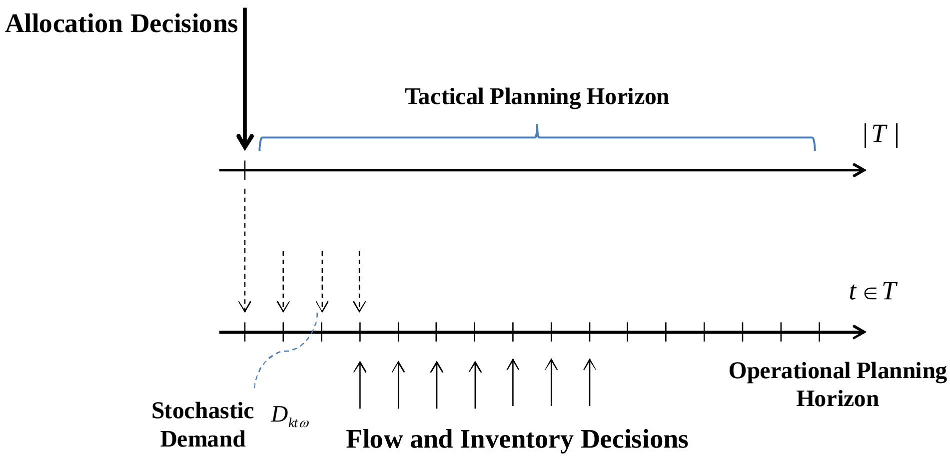


Figure 9. Decision-Time Hierarchy in the Distribution Network

From the network perspective, it is assumed that the platform locations are fixed and that the CZ locations are pre-set (strategic decisions). An order-splitting distribution policy is considered in our model because such a policy can reduce the inventory holding and back-ordering costs when a network involves random demands and lead times (Minner, 2003). Let D_{kt} denote the random variable of the demand of CZ $k \in K$ in period $t \in T$, which follows a given probability distribution, denoted by $F_{kt}(.)$. For a given period, the demand received by an operating DC is the summation of the total or partial demand of the subset of CZs that are assigned to it. We notice that in the multi-sourcing setting considered here, the model controls the number of DC assignments for each CZ, which is fixed in the first stage for the entire horizon, based on cost trade-offs and inventory on-hand availability. Given this situation, the proposed model considers multi-period settings, in which the periods cannot be considered separately due to the state of the inventory constraints. Such optimization models are usually computationally intractable when the number of scenarios and periods is high. They handle a high number of stochastic variables related to flows, demand assignments, inventories, and back orders. In addition, the combinatorial issue of these models is accentuated under multi-sourcing and lateral transshipment features. Therefore a stochastic linear programming (SLP) approach with recourse (Shapiro, 2003, Birge and Louveaux, 2011) is used to cope with this optimization problem under uncertain demand. It builds on the assumption that the probability distribution functions of uncertain parameters are known or can be statistically estimated and that the objective function is estimated by an expected value. Because the demand of CZs along the horizon T is not known when the allocation decisions are made, this information takes the form of the set of demand scenarios, denoted by Ω . A given demand scenario $\omega \in \Omega$ corresponds to a possible realization of the stochastic demand process over the planning horizon T, with a probability of occurrence $p(\omega)$. This leads to the formulation of the model as a two-stage stochastic program (Shapiro, 2008), in which the first stage deals with DC allocation decisions and the second stage deals with the scenario-based daily flows and inventory decisions.

3.3 Mathematical Model Formulation

In this section, a mixed integer stochastic inventory optimization model is presented within four different sourcing strategies,

1- Multi-Sourcing combined with Lateral Transshipment (MSLT)
2- Single Sourcing without lateral transshipment (SS)
3- Multi-Sourcing without lateral transshipment (MS)
4- Single Sourcing combined with Lateral Transshipment (SSLT)

3.3.1 Mathematical Model Formulation for MSLT

Hereafter, are given all the sets, parameters, and decision variables used in the mathematical model.

Table 3. Notation

Sets

S	Set of suppliers $s \in S$
W	Set of PDC platforms $w \in W$
U	Set of DC platforms $u \in U$
K	Set of CZs $k \in K$
T	Set of time periods $t \in T$ (Periodic Review)
Ý	Set of scenarios $\omega \in \Omega$

Parameters

$D_{kt\omega}$	Demand of CZ *k* in the beginning of period *t* under scenario ω
Cap_n	Throughput capacity of platform *n*, $n = \{w,u\}$ available at each period (expressed in flows unit)
$\eta_{nn'}$	Unitary transportation flow cost between site *n* and site *n'*, $n = \{s,w,u\}, \quad n' = \{w,u,k\}$
h_n	Unitary inventory holding cost at platform *n*, $n = \{w,u\}$
π_k	Unitary backorder cost for CZ *k*,
I_s	Inventory level of supplier *s* at the end of period 0 (*t=0*)
$\tau_{nn'}$	Lead-time (expressed in number of periods) from site *n* to site *n'*, $n = \{s,w,u\}, \quad n' = \{w,u,k\}$
a_{uk}	Fixed allocation cost of CZ *u* to DC *k*
$\delta_{nn'}$	Fixed procurement cost for an order from platform *n* to platform *n'*, $n = \{w,u\}, \quad n' = \{u,u'\}$
$Dist_{nn'}$	Distance between site *n* and site *n'*, $n = \{s,w,u\}, \quad n' = \{w,u,k\}$
M	A large positive number

Decision Variables

$I_{nt\omega}$	Inventory level in platform *n* at the end of period *t* under scenario ω, $n = \{w,u\}$
$I^{+}_{nt\omega}$	Inventory on hand in platform *n* at the end of period *t* under scenario ω, $n = \{w,u\}$
$I^{-}_{ukt\omega}$	Backorders level in CZ *k* from DC *u* at the end of period *t* under scenario ω,
$R_{nn't\omega}$	Received products in site *n'* from site *n* in the beginning of period *t* under scenario ω, $n = \{s,w,u\}, \quad n' = \{w,u,k\}$
$x_{ukt\omega}$	Demand level of CZ *k* that is assigned to DC *u* in period *t* under scenario ω
Z_{uk}	Binary variable that takes the value 1 if part of the demand of the CZ *k* is assigned to DC *u* , 0 otherwise
$Y_{nn't\omega}$	Binary variable that takes the value 1 if the replenishment arc (*n*, *n'*) is activated by an order (i.e., $R_{nn't\omega} > 0$) for a given period *t* under scenario ω, 0 otherwise

In this periodic review process, the products shipped from a platform *w* at the end of period *t*ī 1 replenish the inventory of platform *u* at the beginning of period $(t + \tau_{wu})$. The total costs incurred by the network include the fixed allocation cost, the transportation cost, the back-order cost, the fixed procurement cost, and the inventory cost. The back ordering and inventory holding costs for any given platform are linear functions of inventory on hand at the end of the period. The fixed procurement cost is assumed to be independent from the flow levels and it is mostly based on the ordering process fees. It is assumed that the transportation cost per flow unit ($\eta_{nn'}$) is a linear function of the travelled distance on a given network arc. The related formula is $\eta_{nn'} = g\left(Dist_{nn'}\right) = \alpha_{nn'} + \beta_{nn'}.Dist_{nn'}$, where $\alpha_{nn'}$ and $\beta_{nn'}$ are the fixed and variable unitary transportation costs from site n to site n' ($n = \{s, w, u\}, \quad n' = \{w, u, k\}$), respectively. Finally, we recall that the model employs multiple-sourcing and lateral transshipment options with the aim of reducing the amount of back-ordered products, which leads to a better service level. Service level is an implicit performance indicator in the model and will be explicitly evaluated for the solutions produced to evaluate the capabilities of the model. It is considered to be the percentage of satisfied demands from stock on hand without back ordering. According to the given notation, the objective function of the stochastic multi-echelon inventory optimization model is formulated as follow:

$$Min \sum_{u\in U}\sum_{k\in K} a_{uk}.Z_{uk} \tag{1.a}$$

$$+\sum_{\omega\in\Omega} p(\omega)\left[\sum_{t\in T}\left(\sum_{s\in s}\sum_{w\in W}\eta_{sw}.R_{swt\omega} + \sum_{w\in W}\sum_{u\in U}\eta_{wu}.R_{wut\omega} + \sum_{u\in U}\sum_{u'\in U\backslash\{u\}}\eta_{uu'}.R_{uu't\omega} + \sum_{u\in U}\sum_{k\in K}\eta_{uk}.R_{ukt\omega}\right)\right. \tag{1.b}$$

$$+\sum_{t\in T}\left(\sum_{w\in W}\sum_{u\in U}\pi_u.I^-_{wut\omega} + \sum_{u\in U}\sum_{k\in K}\pi_k.I^-_{ukt\omega}\right) \tag{1.c}$$

$$+\sum_{t\in T}\left(\sum_{s\in S}\sum_{w\in W}\delta_{sw}.Y_{swt\omega} + \sum_{w\in W}\sum_{u\in U}\delta_{wu}.Y_{wut\omega} + \sum_{u\in U}\sum_{u'\in U\backslash\{u\}}\delta_{uu'}.Y_{uu't\omega}\right) \tag{1.d}$$

$$\left.+\sum_{t\in T}\left(\sum_{u\in U}h_u.I^+_{ut\omega}\sum_{w\in W}h_w.I^+_{wt\omega}\right)\right] \tag{1.e}$$

The objective function (1) minimises total costs as follows: first, the fixed allocation cost is calculated in the first stage (equation 1.a) independently from the scenarios. Then, the transportation costs among suppliers, PDCs, DCs, and CZs are computed by equation (1.b) based on the flows between these platforms in all periods $t \in T$ and all scenarios $\omega \in \Omega$. Equation (1.c) calculates the total back-order cost based on the level of back-ordered products for all the CZ $k \in K$ in all periods $t \in T$ and all scenarios $\omega \in \Omega$. Next, fixed procurement costs in DCs and PDCs are computed by equation (1.d) based on the number of orders in all periods $t \in T$ and all scenarios $\omega \in \Omega$, and finally equation (1.e) computes the total inventory holding costs in PDCs and DCs, which are considered based on the inventory on hand in all periods $t \in T$ and all scenarios $\omega \in \Omega$. The objective function (1) is subject to the following constraints:

$$\sum_{u \in U} Z_{uk} \geq \rho \qquad \forall k \in K \qquad (2)$$

Constraint (2) defines for each CZ the minimum requirement in terms of the number of assigned DCs. As mentioned, the allocation decisions are made in the first stage, independently from the scenarios, and thus set (2) is considered to be a first-stage constraint set. Herein, the value of ρ is defined in a generic way to underline how the sourcing strategy could be controlled in such a decision model, for instance, when assignments are forced to at least two DCs per CZ $(\rho = 2)$ or more $(\rho > 2)$, or when multiple assignments are only allowed $(\rho = 1)$ (i.e., single sourcing is also feasible for some CZs). We note that in the case of $(\rho = 1)$, this constraint becomes implicit but is kept intentionally to trace the effective DC assignments in comparison to the single-sourcing strategy (see Appendix B).

$$D_{kt\omega} = \sum_{u \in U} x_{ukt\omega} \qquad \forall k \in K, t \in T, \omega \in \Omega \qquad (3)$$

$$x_{ukt\omega} \leq M.Z_{uk} \qquad \forall u \in U, k \in K, t \in T, \omega \in \Omega \qquad (4)$$

Constraint (3) guarantees that the total demand for each CZ is totally allocated among the DCs for each period and each scenario. Constraint (4) checks that a given DC can serve any given CZ only when the DC-CZ assignment decision variable is set to 1.

$$I^{+}_{ut\omega} = I^{+}_{u,t-1,\omega} + \sum_{w\in W} R_{wut\omega} + \sum_{u'\in U'} R_{u'ut\omega} - \sum_{u'\in U'} R_{uu'(t+\tau_{uu'})\omega} - (\sum_{k\in K} x_{ukt\omega} + \sum_{k\in K} I^{-}_{uk,t-1,\omega} - \sum_{k\in K} I^{-}_{ukt\omega}) \qquad \forall u \in U, t \in T, \omega \in \Omega \qquad (5)$$

$$I^{+}_{wt\omega} = I^{+}_{w,t-1,\omega} + \sum_{s\in S} R_{swt\omega} - \sum_{u\in U} R_{wu(t+\tau_{wu})\omega} - \sum_{u\in U} I^{-}_{wu(t-1)\omega} + \sum_{u\in U} I^{-}_{wut\omega} \qquad \forall w \in W, t \in T, \omega \in \Omega \qquad (6)$$

Equations (5) and (6) indicate the inventory on hand in DCs and PDCs, respectively, by balancing the flows in and out of the platform for each period and each scenario. More specifically, the inventory on hand ($I^{+}_{nt\omega}$ $n=\{u,w\}$) in each period $t \in T$ and scenario $\omega \in \Omega$ is the summation of inventory on hand in the last period (*t-1*) and the received products from other platforms minus the products that will be sent out to the subsequent stage (DCs for PDCs and CZs for DCs) and the back-ordered products in period (*t-1*).

$$I_{ut\omega} = I_{u,t-1,\omega} + \sum_{w\in W} R_{wut\omega} + \sum_{u'\in U'} R_{u'ut\omega} - \sum_{u'\in U'} R_{uu'(t+\tau_{uu'})\omega} - \sum_{k\in K} x_{ukt\omega} \qquad \forall u \in U, t \in T, \omega \in \Omega \qquad (7)$$

$$I_{wt\omega} = I_{w,t-1,\omega} + \sum_{s\in S} R_{swt\omega} - \sum_{u\in U} R_{wu(t+\tau_{wu})\omega} \qquad \forall w \in W, t \in T, \omega \in \Omega \qquad (8)$$

$$I_{st\omega} = I_{s,t-1,\omega} - \sum_{w\in W} R_{sw(t+\tau_{sw})\omega} \qquad \forall s \in S, t \in T, \omega \in \Omega \qquad (9)$$

$$\sum_{u\in U} R_{uk(t+\tau_{uk})\omega} = \sum_{u\in U} x_{ukt\omega} + \sum_{u\in U} I^{-}_{uk,t-1,\omega} - \sum_{u\in U} I^{-}_{ukt\omega} \qquad \forall k \in K, t \in T, \omega \in \Omega \qquad (10)$$

Equations (7)ï (9) ensure the adequacy of the inventory level of each platform per period and scenario, based on the inventory levels in the previous periods, outgoing flows, and inflows. Constraint (10) ensures that the outgoing flows of DCs are calculated taking into account the demand of the period and also back orders, and the demand of each CZ is satisfied.

$$\sum_{w\in W} R_{wut\omega} + \sum_{u'\in U'} R_{u'ut\omega} \le Cap_u \qquad \forall u \in U, t \in T, \omega \in \Omega \qquad (11)$$

$$\sum_{s\in S} R_{swt\omega} \le Cap_w \qquad \forall w \in W, t \in T, \omega \in \Omega \qquad (12)$$

$$R_{nut\omega} \le M.Y_{nut\omega} \qquad \forall n = \{w, u'\}\ w \in W, u' \in U / \{u\}, t \in T, \omega \in \Omega \qquad (13)$$

$$I^-_{ukt\omega}, I^+_{nt\omega}, R_{nn't\omega}\ , x_{ukt\omega} \ge 0, \qquad n=\{s,w,u\}, n'=\{w,u,k\}, \ \forall s \in S, w \in W, u \in U, k \in K, t \in T, \omega \in \Omega$$

$$Z_{uk}, Y_{nn't\omega} \in \{0,1\} \qquad n=\{s,w,u\}, n'=\{w,u,k\}, \ \forall w \in W, u \in U, k \in K, t \in T, \omega \in \Omega \qquad (14)$$

$$R_{nn't\omega} = 0 \qquad \forall t \in 1,2,...,\tau_{nn'} \quad n=\{s,w,u\}, n'=\{w,u,k\}, \ \forall s \in S, w \in W, u \in U, \omega \in \Omega$$

Moreover, constraints (11) and (12) restrict the received flows per DC and PDC, respectively, to the throughput capacity limit, defined per period. Constraint (13) guarantees that the fixed procurement cost incurred between two platforms is set to 1 per period and scenario when $R_{nn't\omega} >$ 0. Non-negativity and binary constraints are given by constraints set (14).

3.3.2 Mathematical Model Formulation for SS

According to the above mentioned notations, the SS model is formulated as follow:

$$Min \sum_{u\in U} \sum_{k\in K} a_{uk}.Z_{uk} \qquad (15.a)$$

$$+\sum_{\omega\in\Omega} p(\omega) \left[\sum_{t\in T} \left(\sum_{s\in s} \sum_{w\in W} \eta_{sw}.R_{swt\omega} + \sum_{w\in W} \sum_{u\in U} \eta_{wu}.R_{wut\omega} + \sum_{u\in U} \sum_{u'\in U\setminus\{u\}} \eta_{uu'}.R_{uu't\omega} + \sum_{u\in U} \sum_{k\in K} \eta_{uk}.R_{ukt\omega} \right) \right. \qquad (15.b)$$

$$+\sum_{t\in T} \left(\sum_{w\in W} \sum_{u\in U} \pi_u.I^-_{wut\omega} + \sum_{u\in U} \sum_{k\in K} \pi_k.I^-_{ukt\omega} \right) \qquad (15.c)$$

$$+\sum_{t\in T} \left(\sum_{s\in S} \sum_{w\in W} \delta_{sw}.Y_{swt\omega} + \sum_{w\in W} \sum_{u\in U} \delta_{wu}.Y_{wut\omega} \right) \qquad (15.d)$$

$$+\sum_{t\in T}\left(\sum_{u\in U} h_u . I^+_{ut\omega} \sum_{w\in W} h_w . I^+_{wt\omega}\right)\Bigg] \tag{15.e}$$

$$I^+_{ut\omega} = I^+_{u,t-1,\omega} + \sum_{w\in W} R_{wut\omega}$$
$$-(\sum_{k\in K} x_{ukt\omega} + \sum_{k\in K} I^-_{uk,t-1,\omega} - \sum_{k\in K} I^-_{ukt\omega}) \qquad \forall u \in U, t \in T, \omega \in \Omega \tag{16}$$

$$I_{ut\omega} = I_{u,t-1,\omega} + \sum_{w\in W} R_{wut\omega} - \sum_{k\in K} x_{ukt\omega} \qquad \forall u \in U, t \in T, \omega \in \Omega^N \tag{17}$$

$$\sum_{u\in U} Z_{uk} = 1 \qquad \forall k \in K \tag{18}$$

$$\sum_{w\in W} R_{wut\omega} \le Cap_u \qquad u \in U, t \in T, \omega \in \Omega \tag{19}$$

$$R_{wut\omega} \le M . Y_{wut\omega} \qquad u \in U, w \in W, t \in T, \omega \in \Omega \tag{20}$$

and constraints (3), (5)-(9), (12) and (14).

The objective function in (15) minimizes the expected total cost in the network. The main difference between (15) and (1) is the removal of the lateral transshipment cost term. Constraints (16) and (17) indicate respectively the inventory on hand and the inventory level in the DCs by balancing the flows-in and flows-out of in each center, period and scenario. Constraint (18) sets the single sourcing requirements. It enforces the model to assign a unique source for each customer. Constraints (19) and (20) replace respectively constraints (11) and (13) to guarantees the respect of the capacity and the procurement decisions for each platform.

3.3.3 Mathematical Model Formulation for MS

This model allows only the multi-sourcing options (i.e. without LT) and thus considers the removal of the lateral transshipment term in the objective function as in (15). Accordingly, MS minimizes the objective function (15) subject to constraints (3), (5)-(10), (12), (14), (16), (17), (19) and (20).

3.3.4 Mathematical Model Formulation for SSLT

This model differs from the MSLT model (1-14) by the sourcing constraint (10), which must be replaced by (18). Accordingly SSLT minimizes the objective function (1) subject to the constraints (2)-(9), (11)-(14) and (18).

3.4 Solution Approach

3.4.1 Scenario Generation

As reported in Section 2.3, the sample average approximation (SAA) technique (Shapiro, 2008) is used in order to approximate the stochastic model by an equivalent deterministic mixed-integer linear program (MILP), namely the SAA model. This latter relies on a simulation-based optimization approach using a Monte-Carlo scenario sampling method. Recall that the non-stationary demand process is characterized by the formula in (Zhao and Xie, 2002) proposed in page 16 of the chapter. In fact, for CZ k in period t the mean demand is generated using the formula: $\mu_{kt} = b + sl.\sin\left(2\pi t / sc\right) + no.snormal()$,

which enables to generate the demands for scenario ω (i.e. $\left[D_{kt\omega}\right]_{k\in K, t\in T}$).

3.4.2 Solution Methodology SAA

As mentioned, the stochastic Multi-echelon Inventory Optimization Model (1)ï (14) is intractable due to the inherent combinatorial complexity and the very large number of plausible scenarios necessary to shape entirely the demand process. The sample average approximation (SAA) technique (Shapiro, 2008) is used in order to approximate the stochastic model by an equivalent deterministic mixed-integer linear program (MILP), namely the SAA model. This latter relies on a simulation-based optimization approach using a Monte-Carlo scenario sampling method. The SAA method has been widely used in the recent years to find near-optimal solutions for stochastic problems in the supply chain (Klibi et al., 2010, Benyoucef et al., 2013, Brandimarte, 2006). The Monte-Carlo sampling method is a common technique that uses statistical information on uncertain parameters to generate possible future scenarios occurring during a given planning horizon. When a sample of scenarios is generated using this method, all the scenarios in the sample are equiprobable, which simplifies the estimation of the optimal solution using an average of the scenarios. Running the Monte-Carlo procedure N times gives a sample of independent demand scenarios $\left\{\omega^1, \omega^2, ..., \omega^N\right\} = \Omega^N \subset \Omega$. Since N equiprobable scenarios are produced, then $p(\omega) = 1/N$, and the presented model (1)-(14) could be rewritten as the following SAA model:

$$Min \sum_{u \in U} \sum_{k \in K} a_{uk}.Z_{uk} + \frac{1}{N} \left(\sum_{\omega \in \Omega^N} \sum_{t \in T} \left[\sum_{s \in s} \sum_{w \in W} (\eta_{sw}.R_{swt\omega} + \delta_{sw}.Y_{swt\omega}) + \sum_{w \in W} h_w.I^+_{wt\omega} + \sum_{u \in U} h_u.I^+_{ut\omega} \right. \right.$$
$$+ \sum_{w \in W} \sum_{u \in U} (\delta_{wu}.Y_{wut\omega} + \eta_{wu}.R_{wut\omega} + \pi_u.I^-_{wut\omega}) \qquad (21)$$
$$\left. \left. + \sum_{u \in U} \sum_{u' \in U \setminus \{u\}} (\delta_{uu'}.Y_{uu't\omega} + \eta_{uu'}.R_{uu't\omega}) + \sum_{u \in U} \sum_{k \in K} (\eta_{uk}.R_{ukt\omega} + \pi_k.I^-_{ukt\omega}) \right] \right)$$

Subject to constraints (2) ï (14) $\forall \omega \in \Omega^N$

The solvability of the SAA model is very dependent on *N*, which makes the problem intractable for large-scale instances, even when powerful optimization software is used. The sample size calibration could be done by computing a statistical gap based on the framework of Shapiro, et al. (2009). The SAA approach and the computation of the related statistical gap are detailed below:

***Step1* .** Generate *M* independent sample each of size *ns*, i.e. $(\Omega_1^N, \Omega_2^N, \ldots, \Omega_M^N)$ using Monte Carlo and Hierarchical sampling procedure.

***Step2* .** For *m* =1 to *M*, Solve the SAA model (28). Let TC_m^N be the objective function value for sample *m*, and let (ς_m^N, ξ_m^N) be the solution vector of the SAA model obtained with a scenario sample *m* of size *N*, where ς_m^N and ξ_m^N correspond to the first stage design decisions and to the second stage decisions, respectively.

Step 3. Compute the average and the variance of *M* SAA models by equations (22) and (23).

$$\overline{TC}_{N,M} = \frac{1}{M} \sum_{m=1}^{M} TC_m^N \qquad (22)$$

$$\tilde{\sigma}^2_{N,M} = \frac{1}{M(M-1)} \sum_{m=1}^{M} \left(TC_m^N - \overline{TC}_{N,M} \right)^2 \qquad (23)$$

Step 4. Considering the average and the variance of *M* SAA programs, Calculate an approximate 100(1- α)% confidence upper bound with equation (24), where θ is the α -critical value of the *t*-distribution with *M-1* degrees of freedom.

$$U_{N,M} = \overline{TC}_{N,M} + \theta_{\alpha,M-1}\hat{\sigma}_{N,M} \tag{24}$$

Step 5. Obtain $\bar{\varsigma}^{N}$, The average of first stage solution among the *M* samples.

Step 6. Generate the sample $\omega \in \Omega^{N'}$, N' >> *nc*, using Monte Carlo procedure.

Step 7. Solve SAA model (26) for all scenario $\omega \in \Omega^{N'}$ considering $\bar{\varsigma}^{N}$ as the determined first-stage solutions.

Step 8. Get the optimal objective function $T\hat{C}_{m}^{N}$ and solution vector $\left(\bar{\varsigma}^{N}, \hat{\xi}_{m}^{N'}\right)$.

Step 9. Considering the variance of samples by equation (25), calculate the 100(1-α)% confidence lower bound for the expectation of optimal TC^{*} with equation (26).

$$\hat{\sigma}_{N'}^{2} = \frac{1}{N'\left(N'-1\right)}\sum_{\omega=1}^{N'}\left(T\hat{C}_{\omega}(\bar{\varsigma}_{N}, \hat{\xi}_{\omega}) - T\hat{C}_{N'}(\bar{\mathbf{X}}_{N}, \bar{\varsigma}_{N}, \hat{\xi}_{N'})\right)^{2} \tag{25}$$

$$L_{N'} = T\hat{C}_{N'}(\bar{\varsigma}_{nc}, \hat{\xi}_{N'}) - z_{\alpha}\hat{\sigma}_{N'} \tag{26}$$

Step 10. Calculate the statistical optimality gap with equation (27) by $gap_{nc,M,N'} = U_{nc,M} - L_{N'}$

$$gap_{N,M,N'}\% = \frac{gap_{nc,M,N'}}{U_{nc,M}} \times 100\% \tag{27}$$

If this gap is acceptable, stop. Otherwise, increase *nc* and/or *M* and return to step 1.

In the experimental plan section, we discuss how to select an acceptable value of N to solve the SAA model and to produce good quality.

3.5 Experimental Plan

In order to cover several business contexts, several problem instances are generated considering four dimensions: network size (Small (*SN*), Medium (*MN*), Large (*LN*)), PDC and DC capacity levels (Cap^{low}, Cap^{high}), back-order cost levels (π^{low}, π^{medium}, π^{high}), and inventory holding cost levels (h^{low}, h^{high}). The combination of these four dimensions yields 36 problem instances. Each instance is denoted by the quadruplet:

$$(w, x, y, z,) \big| w \in \{SN, MN, LN\}, x \in \{\text{Cap}^{low}, \text{Cap}^{high}\}, y \in \{\pi^{low}, \pi^{medium}, \pi^{high}\}, z \in \{h^{low}, h^{high}\}$$

A planning horizon covering a season is used, which includes 90 working days composed of several demand cycles.

Table 4 provides the network parameters used to generate the various instances. The DC and PDC capacity levels are expressed in throughput units per period.

Table 4. Network Parameters

Network Size	*SN*		*MN*		*LN*	
Customer zones (|K|)	10		60		200	
Distribution centers (|U|)	2		8		12	
Production distribution centers (|W|)	1		2		4	
	Cap^{low}	Cap^{high}	Cap^{low}	Cap^{high}	Cap^{low}	Cap^{high}
Capacity in DCs (Cap_u)	2 800	4 000	4000	9 000	8 500	20 000
Capacity in PDCs (Cap_w)	4 000	9 000	7 500	17 000	12 000	30 000

Next, the initial stock at each echelon is fixed to the average lead-time demand (demand of upper echelon). It's worth mentioning that these experimental settings and parameters are consistent with the ones in the literature (see (Martel, 2003, Fattahi et al., 2015, Hnaien and Afsar, 2017)). The unit back-order cost (per day) in the three tested levels ($\pi^{low}, \pi^{medium}, \pi^{high}$) are set to (1ú, 4ú, 8ú), and the unit inventory holding cost (per day) in the two levels tested (h^{low}, h^{high}) are set to (0.01ú, 0.1ú), respectively. For all the network sizes studied, the replenishment lead times between the locations (i.e., $\tau_{sw'}, \tau_{wu}$, and $\tau_{uu'}$) were fixed to 3, 2, and 1 in days, respectively. The unit flow costs ($\eta_{nn'}$) were computed with the distance-based transportation cost function with the values $\alpha_{nn'}$ = 0.0432 and $\beta_{nn'}$ = 0.0035 for the fixed and variable cost components, respectively. The maximum distance between the network nodes doesnôt exceed the 800 km in all the network sizes. The unit flow cost ranges [0.04ú, 2.4ú] for the sourcing flows, [0.07ú, 2.1ú] for the transshipment flows, and [0.04ú, 2.4ú] for the outbound flows. The fixed procurement cost ranges [20ú, 50ú]. The fixed allocation costs ranges [200ú, 1000ú] per DC-CZ pair. The values for all these parameters presented in Table 4 are based on realistic parameter value ranges obtained from a case in the retail industry. Recall that the unit flow costs ($\eta_{nn'}$) is computed by taking into account the transportation cost function parameters and the distances between the network sites. We notice that the unit flow costs dedicated to lateral transshipment flows are augmented by factor ɔ=1.5 compared to the sourcing flows for the same distance (i.e., using the expression $\eta_{nn'} = \gamma g(Dist_{nn'})$). This is set initially superior to 1 in order to characterize the effect of such unplanned transportation decisions on the availability of transportation resources. Next we provide a sensitivity analysis on this factor.

Furthermore, we consider a network including three market segments that are reflected by their demand level ï large-size CZs (L), medium-size CZs (M), and small-size CZs (S) ï with proportions of 20%, 60%, and 20%, respectively. Because a stochastic non-stationary demand process with seasonal trends is applied, the following demand function is used to generate demand realizations. This function, proposed by Zhao and Xie (2002), considers a demand distribution whose parameters follow an additive seasonal pattern over the planning horizon. For each CZ *k* and period *t* the mean demand is given by:

$$\mu_{kt} = b^n + sl.\sin\left(2\pi t / sc\right) + no.snormal() \qquad n \in \{S, M, L\}.$$

In this function, the fixed parameter b^n represents the *base* for the network size *n* and takes the values of $b^S = 80$, $b^M = 150$, and $b^L = 240$. The *Slope (sl)* and *Noise (no)* values are fixed to 40 and 50, respectively. Note that *sc* is considered to be a monthly cycle (i.e., *sc*=30) and that $snormal()$ is a standard normal random number.

We recall that it is very difficult to solve to optimality the presented model for the entire set of scenarios. Therefore, a number of sample sizes are tested and their related statistical optimality gap values are computed. For each problem size, three different sample sizes are tested (30, 50, and 100) and for each of them the obtained gap values for *SN, MN,* and *LN* are (1.5%, 1.76%, 1.95%), (1.63%, 1.8%, 2.2%), and (1.8%, 2.15%, 2.24%), respectively. The largest SAA problems that could be solved optimally, without truncating the solution optimality gap, is *N* = 100 for small and medium instances; however, for some large instances, the SAA is solvable only when it does not exceed *N*=70. The validation analysis shows that, with this latter sample size, the SAA method provides satisfactory statistical optimality gaps (always less than 2.25%), which argues in favor of good-quality solutions. It is worth mentioning that, because the planning horizon includes 90 periods, when *N* scenarios are used in the SAA model, 90*N* instances are sampled from the probability distribution (i.e., 9,000). Thus, the multiplicity of scenarios and periods explains the low statistical gaps obtained, which is congruent with the findings of other stochastic problems proposed in the literature (Klibi et al., 2016). The Monte Carlo procedure and the statistical gap computation details are given in Appendix C. Finally, we notice that the SAA models are generated with OPL Studio 12.1 and solved with CPLEX-12.6 using a *MIP relative tolerance* of 0.005. All the experiments are run on a 64-bit operating system server with a 2.7 GHz CPU on Intel(R) processor and 72 GB of RAM.

3.6 Numerical Experiments

Given the four SAA models to inspect (MSLT, MS, SSLT, and SS) and the 36 problem instances previously proposed, 144 instances were run and their results are provided hereafter. Noticeably, the MSLT distribution strategy has shown its superiority in terms of expected total costs and service level compared to the alternative tested strategies. The service level considered in this work is the fill rate that is measured empirically by averaging the number of satisfied demands without back orders over the total number of product flows

To start with, Table 5 summarizes the comparative results in percentage of cost reduction of each distribution strategy compared to the baseline SS strategy. These results report the expected total costs $\mathbb{E}(C)$, which are aggregated per capacity levels and per back-order cost levels. The first row of Table 5 represents the distribution strategy employed in the model and the instanceċs labels, in which the dot denotes a specific capacity level and unit back-order cost and the dash (-) represents the average of all instances for the related attribute.

Table 5. Expected Total Costs Gap Compared to SS Strategy

		SSLT (-, . , ., -) %	MS (- , . , ., -) %	MSLT (- , . , ., -) %
Cap ^{high}	ˊ ***low***	6.8	8.43	10
	ˊ ***medium***	9.6	10.6	11.3
	ˊ ***high***	9.7	13.4	13.5
Cap ^{low}	ˊ ***low***	9.4	13.1	14.4
	ˊ ***medium***	13.1	16.7	18.1
	ˊ ***high***	15.8	18.1	18.2

When looking at the expected total cost gaps, it is clear that MSLT has the lowest total cost because it provides the largest gaps for all the considered cases. Also, when inspecting all the instances, these gaps are always positive, which means that SS is a dominated strategy. As illustrated in Table 5, MSLT leads to a cost reduction that can reach 18.2% when the DC capacity is low and the back-order cost is high. Another observation from Table 5 is that SSLT is always more expensive than both MS-based strategies, which are in general close in terms of the average relative gap (margin between 0.1% and 1.6%) with a small advantage for the strategies allowing LT. In general our results show a higher benefit for lateral transshipments in instances with fewer distribution capabilities and high back-order costs, which is congruent with the arguments towards promoting the flexibility of this strategy.

Table 6. Expected Total Costs Gap per Cost Component for MSLT Strategy

		MSLT (., ., -, -)					
		$\mathbb{E}(C)\%$	TC%	BC%	PC%	HC%	FC%
Cap ***high***	SN	6.50	9.49	30.19	15.70	50.13	-15.00
	MN	7.01	2.22	8.96	49.96	75.59	-46.94
	LN	9.11	3.32	15.34	32.02	74.90	-32.78
Cap ***low***	SN	8.00	9.72	41.00	12.40	44.32	-31.00
	MN	9.70	4.44	20.17	23.64	61.92	-54.72
	LN	12.73	6.73	28.20	31.67	59.81	-44.58

Moreover, Table 6 supports these results by reporting the relative gaps of the expected total costs for the MSLT strategy from the view of the network and capacity sizes. Table 6 also reports, for these instance attributes, the detailed cost partitions among the components of the objective function, that is, the back-order cost (BC), the holding cost (HC), the procurement cost (PC), the fixed allocation cost (FC), and the transportation cost (TC). Table 6 mainly reveals that the MSLT strategy benefits increase when the network size increases. The lowest gap with MSLT remains higher than 6.8% compared to the SS strategy. These results confirm the importance of the network resource dispositions on the distribution strategy effectiveness. One can also underline that, in most of cases, the inventory holding cost (HC) component shows the highest improvement in average gap percentages. Even though this behavior clearly confirms the sensitivity of these costs to the distribution strategy, we should notice that a close look at the detailed numerical results (given in Appendix A) shows that the impact of this cost component on the total cost remains, however, relatively minor because the unit holding costs in the context of the retail industry, considered in this research work, are low. By looking at the allocation cost in the small instances (SN), we notice that the MSLT strategyï based model does not activate the multiple-sourcing option when it is possible to use lateral transshipments, which happens for a high-capacity and low-unit back-order cost.

More generally, we notice that the results are not very sensitive to the variation of the unit-holding cost as opposed to the back-ordering cost. Therefore, hereafter we focus more on the variation of the unit back-ordering cost in order to gain more insights. We find that the lateral transshipment strategy reduces the expected total cost. These cost reductions are sometimes achieved through increasing overall inventory levels in the supply chain. This observation is also evidenced in the literature (Grahovac and Chakravarty, 2001). Due to the context of retailing industry, the lateral transshipment cost is dominant over the holding cost; thus, when lateral transshipment flows are used, the expected total cost and back-order cost are reduced, whereas the holding costs and transportation costs are augmented (Table 6). Because the MSLT strategyï based model uses more lateral transshipment flows, fewer orders would be made and consequently the procurement cost is decreased. It appears that the SS strategy forces the DCs to order more in each period to deal with the demand uncertainty, which explains why the SS strategy always has the highest procurement cost among the multiple-sourcing strategies. Consequently, MS strategyï based model produces considerably lower inventory levels in DCs, which leads to lower inventory holding costs.

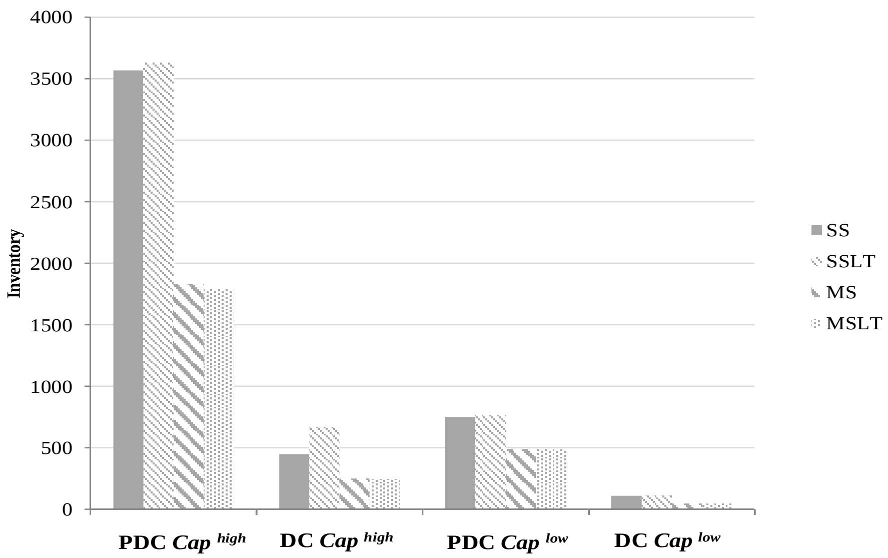


Figure 10. Inventory Level in DCs and PDCs

Moreover, the results show that in cases of low capacity, the performance enhancement is much higher than for high capacity (the difference in the expected total cost can go up to 20% for some instances). Because the MSLT strategy could be more practical due to the flow capacity constraint, the proposed model performance is more interesting. This shows the important role of the throughput capacity in inventory optimization. Figure 10 provides the average inventory on hand at PDCs and DCs, which may explain how these performances are obtained. It shows that the SS-based strategies (SS and SSLT) tend to maintain higher inventory on hand than the MS-based strategies (MS and MSLT), which provides a key insight when investments in capabilities are made at the strategic level.

Because higher expected total costs are often associated with higher achieved service levels, efficiency curves are needed to allow a fair comparison of the cost-service performance of the four distribution strategies. Such curves are obtained by varying the unit back-order cost (π_k from low to high) as shown in Figure 11.

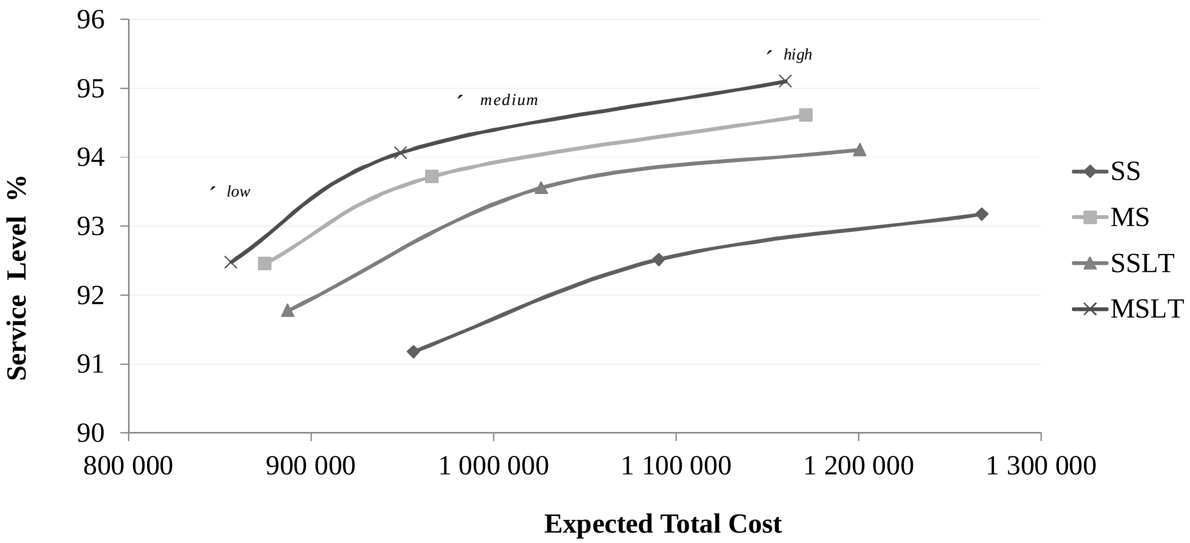


Figure 11. Cost-Service Efficiency Curves of the Four Strategies

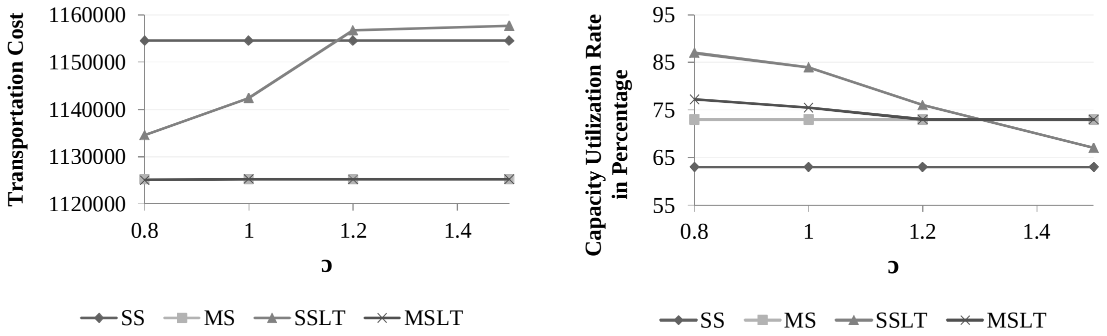
The efficiency curves confirm again that the MSLT is the most efficient strategy by providing the highest achieved service level (with a gap that exceeds 2% compared to that of SS) for the same expected total cost. When the unit back-order cost increases, the total back-order cost increases as expected but leads to improved service levels. This means that, although the back-order unit cost increases, the model uses all the available features to reduce the number of back-ordered products, which results in achieving higher service levels.

Table 7 reports the average percentage of the number of effective DC allocations per CZ. Each row presents the instance label and the selected sourcing strategy. The results show that the number of effective DC allocations is sensitive to the unit back-order cost and the capacity. More specifically, even when the minimum number of DCs to be allocated per CZ is set to 1, the stochastic model finds the best trade-offs between a high number of allocations and network cost minimization thanks to the recourse variables. In fact, the model tends to allocate more DCs to CZs when the unit back-order cost increases and when the throughput capacity is limited. This is confirmed by considering four instances with different unit back-order costs and different capacities in each network size. Note also that MSLT performs better than MS in these instances, which is due to the lateral transshipment flows. The expected number of lateral transshipment orders (average from the scenarios) and the expected total cost gap between the two strategies and the SS strategy (average from the scenarios and instances) are reported in the last two columns. The results show that the number of lateral transshipment orders increases when the network size increases, which means that it is more profitable to use lateral transshipments instead of allocating more DCs to CZs.

Table 7. The Impact of Capacity and Unit Back-Order Cost on the Number of Allocations per CZ

Strategy	**Instance**	**Number of allocations per CZ (j) %**					**Expected total cost gap with SS %**	**Expected number of LT orders per DC**
		1	**2**	**3**	**4**	**5**		
MS	(SN, Cap^{high}, ^{low}, -)	100	0	0	0	0	6	0
MS	(SN, Cap^{low}, ^{high}, -)	0	100	0	0	0	6	0
MSLT	(SN, Cap^{high}, ^{low}, -)	100	0	0	0	0	7	2.25
MSLT	(SN , Cap^{low}, ^{high}, -)	20	80	0	0	0	12	2.25
MS	(MN, Cap^{high}, ^{low}, -)	46.7	41.7	8.3	3.3	0	1	0
MS	(MN, Cap^{low}, ^{high}, -)	10	46.7	30	10	3.3	6	0
MSLT	(MN, Cap^{high}, ^{low}, -)	78.33	21.77	0	0	0	3	2
MSLT	(MN, Cap^{low}, ^{high}, -)	26.7	73.3	0	0	0	8	4.38
MS	(LN, Cap^{high}, ^{low}, -)	70.5	22.5	5.5	1	0.5	4	0
MS	(LN, Cap^{low}, ^{high}, -)	50.5	39	8	2	0.5	13	0
MSLT	(LN, Cap^{high}, ^{low}, -)	92	8	0	0	0	6	2
MSLT	(LN, Cap^{low}, ^{high}, -)	48	49	3	0	0	15	7.42

Given these results, we conducted a sensitivity analysis of problem instances with tight capacity and high holding cost attributes in order to investigate distribution strategy behaviors under different unit lateral transshipment costs. In linkage with the transportation costs estimation function, the cost of LT arcs (*n*, *nô*) is perturbed with the expression $\gamma.\eta_{nn'}$ where three new situations (in addition to $\gamma = 1.5$) are tested as follows : (1) The unitary LT flow cost is cheaper than the unitary sourcing flow cost $(\gamma = 0.8)$, (2) the unitary LT flow cost is as costly as the unitary sourcing flow cost $(\gamma = 1)$, and (3) the unitary LT flow cost is slightly more expensive than the unitary sourcing flow cost $(\gamma = 1.2)$. The plots in Figure 12 show the performance of the alternative strategies in terms of transportation cost (a) and capacity utilization level (b) for the different ɔ values. We calculate the capacity utilization in percentage per DC as the ratio of the average product flow during the period divided by the total throughput capacity available at the DC for the period.



a) **b)**

Figure 12**.** Backorder Cost, Transportation Cost, and Capacity Utilization Rates for Different ɔ Values

We observe in Figure 12b that when ɔ decreases, capacity utilization increases especially in the SSLT model. This means that in SSLT, fewer numbers of orders are made by the DCs to the PDCs. After each flow between PDC and DC, the products would be redistributed in the same echelon; consequently, the procurement cost is decreased (by about 26%). Note that when the lateral transshipment unit cost in SSLT decreases, the transportation cost decreases, making it even lower than that of SS. According to the results in Figure 12a, when the transportation cost between DCs is low, SSLT could be a very beneficial strategy, which confirms the findings of Lee, Jung, and Jeon (2007). Also the results in Figure 12b show that the dispersion of the inventory through the network using the MSLT strategy produces a lower utilization rate of capacity compared to the other strategies. Because here the capacities are assumed to be fixed a priori, this result appeals to a strategic insight, which is the reduction of unused capacities and thus the reduction of the fixed costs of the network structure. Finally, it is noticeable that the MSLT-based solution is not sensitive to the variation in operational costs, which clearly underlines the robustness of these solutions produced by the stochastic optimization.

3.7 Case study

In order to test the empirical validity of the obtained findings, we consider in this section the case of the distribution network of a major retailer in France that consists of 206 stores, eight distribution centers, and two production distribution centers located all over France. The production distribution centers are fed, for the product category considered in this section, by a European supplier.

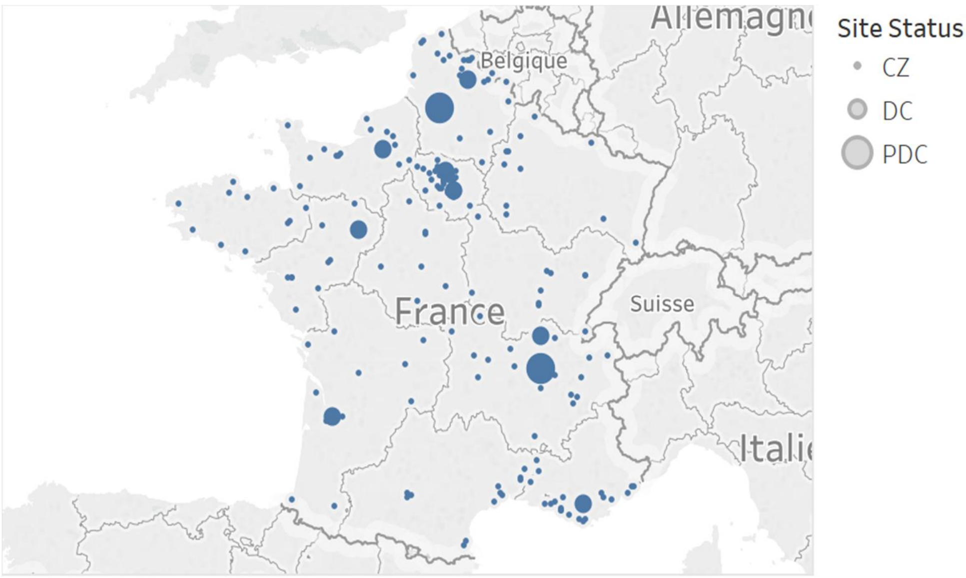


Figure 13. The Distribution Network of the French Retailer

The planning horizon considered in this case study covers three business months partitioned into 90 working periods (days). A non-stationary demand is estimated based on the equation presented in Section 3.3, and a sample of 100 scenarios is generated with the Monte Carlo procedure (Appendix C). The lead time between PDCs and DCs is two days. The unit cost parameters are presented in Table 8.

Table 8. Cost Parameters

Unit holding cost	Euro/palette/year	ú81
Procurement cost	Order	ú20
Unit back-order cost	Euro/item/day	ú1
Unit price	Euro/item	ú32

The inventory holding unit cost and the transportation unit cost could be easily estimated based on the number of products per pallet (28 products). In order to do so, an average for the category of the considered products is considered, which allows determining the holding cost per day. In addition, the unit flow cost ranges [0.04ú, 0.47ú] for the sourcing flows, [0.1ú, 2.1ú] for the transshipment flows, and [0.04ú, 0.60ú] for the outbound flows. Regarding the network capacities, the throughput capacity for each operated DC is set to 4,200 flow units per day and the throughput capacity for each operated PDC is set to 17,000 flow units per day. Fixed allocation cost for each CZ-DC pair is 500ú.

Table 9 presents the comparative results in percentage gap of each distribution strategy compared to the baseline SS strategy in terms of expected costs and service level. As expected, the proposed model with multiple sourcing and lateral transshipment (MSLT) has the best expected results (with 9.1% cost reduction and 4.1% service level improvement) compared to the current state of the distribution strategy in the network, which is the single sourcing without lateral transshipment (i.e., SS strategy). Recall that to show the impact of these features, we compare the SS model to MS, SSLT, and MSLT. It should be noted that if we consider only the lateral transshipment feature (SSLT), the transportation cost increases, whereas this leads to a back-order cost reduction and a better service level. Because the SS model is not flexible and does not have any option to deal with the non-stationary demand process, it results in higher numbers of orders and larger inventories in the DCs; therefore, the procurement and holding costs are higher than MSLT.

Table 9. Results of the Case Study

Models	$\mathbb{E}(C)$	Transportation Cost %	Back-Order Cost %	Procurement Cost %	Holding Cost %	Allocation Cost %	Service Level %
MS	8.21	6.04	54.42	44.94	61.49	-73	-3.4
SSLT	6.79	1.97	50.24	-4.09	47.15	0	-1.84
MSLT	9.10	6.07	56.74	43.99	67.74	-51	-4.1

In Figure 14, we plot the efficiency curves of the cost-service performance of each distribution strategy for different unit back-order costs (i.e., low, medium, and high). Figure 14 shows that, regardless of the unit back-order cost, the MS-based strategies (i.e., MS and MSLT) result in about 4% increase of the service level when compared to the baseline SS strategy, whereas SSLT leads to 2% service level increase. Figure 14 clearly confirms that MSLT is the most efficient strategy among the four considered ones. Note, though, that, in this case, there is a very small difference between the performances of the MS and the MSLT strategies, which means that the MSLT strategy uses very rarely the lateral transshipment option. This is expected because the holding cost and the back-order cost in this case are low, which implies less need to use lateral transshipment in the MSLT strategy.

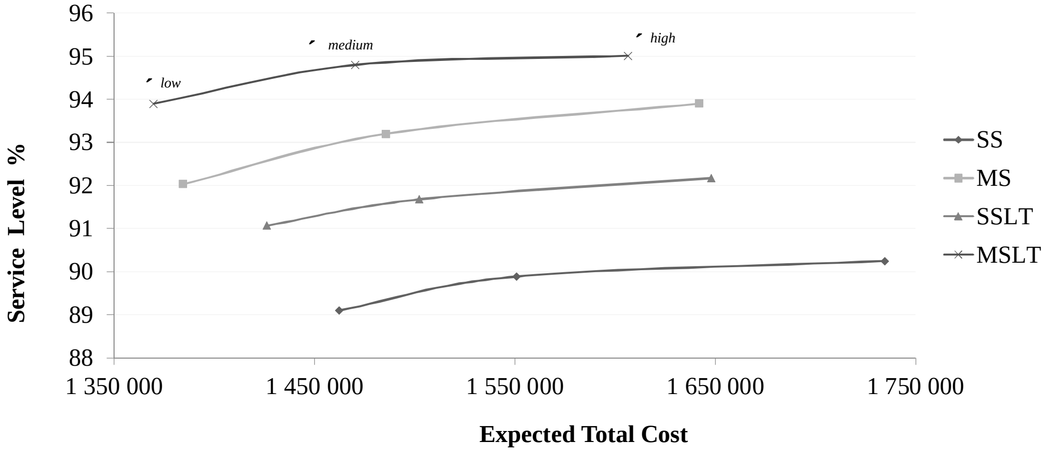


Figure 14. Efficiency Curves

Furthermore, the results show that there is a significant difference between the inventory levels resulting from the four strategies. Figure 15 indicates the average inventory level in the DCs and the PDCs under the four strategies. The multiple sourcingï based strategies lead to a significant reduction of the inventory levels in the DCs. In fact, the demand in each period could be satisfied from various DCs, so there is no need to keep large inventories to hedge the demand variability.

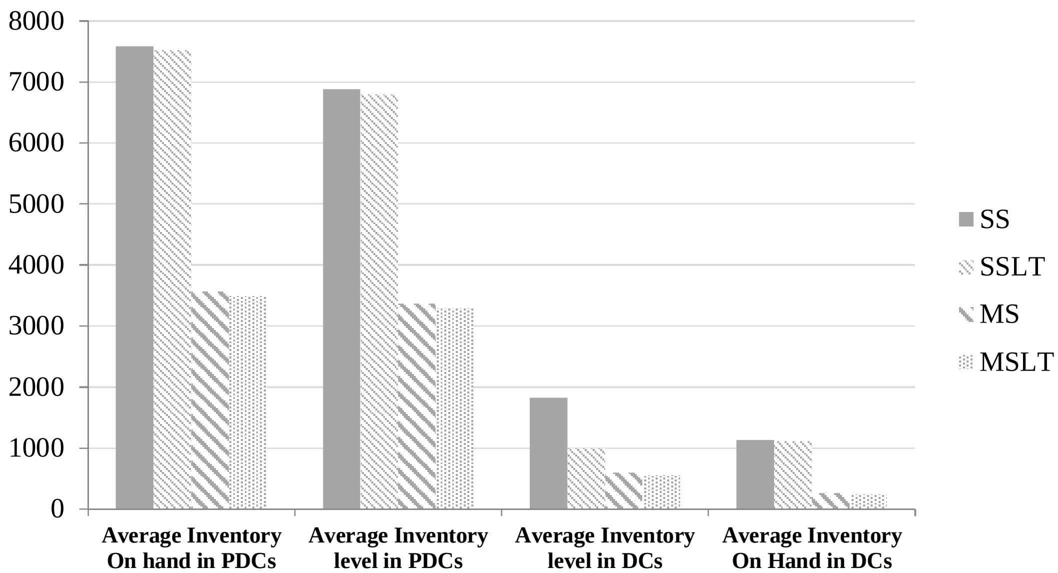


Figure 15. Inventory Level in DCs and PDCs

By looking closely at the results by DC, an interesting insight is gained, for instance, with the DC located in the south of Ile de France (Greater Paris area), which is a very strategic region for the retailer. The inventory level in this DC under the SS strategy is 8,620, whereas in the MSLT strategy it is about 3,240, which is explained by the decentralization effect offered by the multi-sourcing strategy in the latter. Hence, the current state of the network (i.e., the SS strategy) tends to carry more inventories in the periods when the demand fluctuates, but at the same time, the number of back-ordered products increased in the network.

In the same way as in the previous section, the sensitivity of the distribution strategies to the lateral transshipment unit cost is investigated. It is useful to analyze the variation of the total cost by applying different lateral transshipment costs that depend on the regular transportation unit cost. Figure 16 shows the results of the sensitivity analysis with respect to ɔ (ɔ = 1, 1.2, 1.4, and 2). This figure underlines that when the lateral transshipment unit cost is low (i.e., ɔ tending towards 1), SSLT leads to a considerable cost reduction, which renders its total cost close to the one of MS-based strategies.

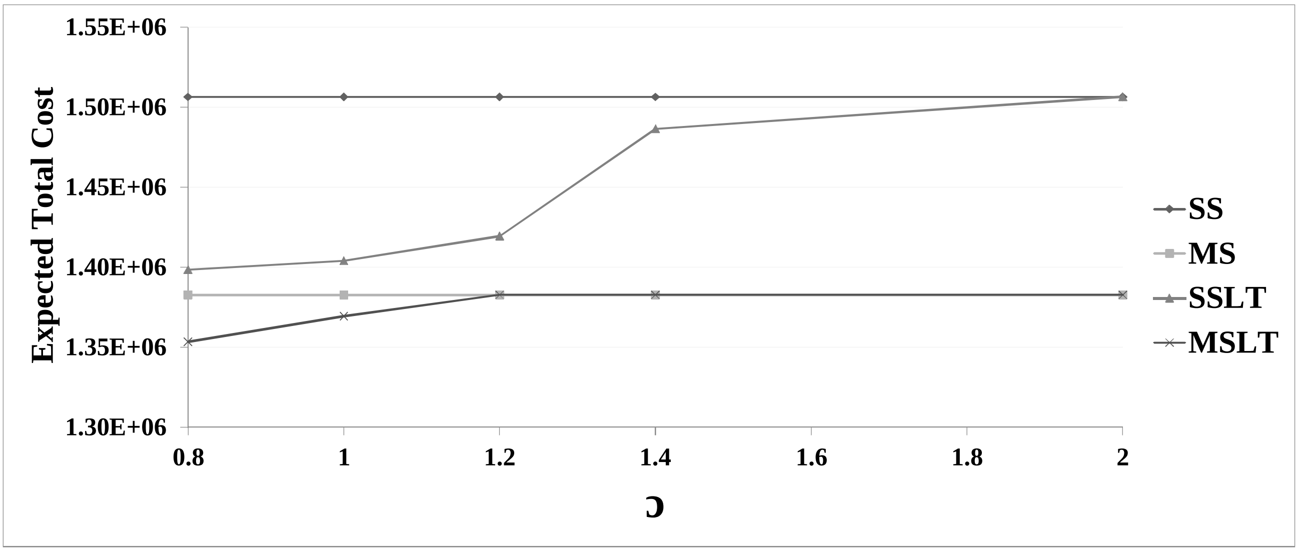


Figure 16. Expected Total Cost with Different Unit Lateral Transshipment Costs

In fact, for low ɔ values, the model uses all the potential lateral transshipment flows to reduce the total cost, and for the same unit transportation and unit lateral transshipment costs (i.e., ɔ = 1) the flows in the lateral transshipment strategy tend towards the case of multiple sourcing. However, if ɔ is very high, the model does not tend to use the lateral transshipment flows, which consequently leads to the same results as in the SS model.

3.8 Conclusions

This chapter proposed a modeling and a solution approach for a multi-echelon inventory optimization problem under non-stationary demand. Lateral transshipment and multiple sourcing have been considered to improve the performance of the distribution network. A two-stage stochastic multi-echelon inventory optimization model is developed and run on different numerical instances and also on real data coming from a major French retailer. We have examined the tactical implications of the multiple sourcing and the lateral transshipment strategies on the distribution network. We have shown substantial savings obtained using the MSLT and the SSLT strategies, which can go up to 23.6% and 21%, respectively.

The results also show that a combination of the lateral transshipment and the multiple-sourcing strategies leads to a considerable improvement of the service level, which can reach 6% when compared to that of the baseline SS strategy. However, the magnitude of the benefits of lateral transshipments and multiple-sourcing depends on the network flow capacity and the unit costs. Proactive lateral transshipments can help managers to reduce the expected total cost especially when the unit back-order cost is high. Another important finding in this contribution is that the expected total cost of the proposed MSLT strategy is not very sensitive to the lateral transshipment and inventory holding unit costs.

In this chapter, supply uncertainty has not been considered in the modeling approach. Chapter 4 presents a two-stage stochastic modeling approach considering disruption and the related impact on the network. A specific inventory control policy (s, S) would be applied and optimized in each platform. In the next chapter, the impacts of disruption in all inventory management decision would be examined.

Chapter 4. A Scenario-based Inventory Optimization Approach for a Multi-Echelon Network Operating under Disruptions

In this chapter, a two-stage stochastic model is proposed to solve a capacitated multi-echelon inventory optimization problem considering a stochastic demand as well as uncertain throughput capacity and possible inventory loss, due to disruptions. The model minimizes the expected total operational and tactical cost. The inventory is controlled according to a reorder point order-up-to-level (s, S) policy and lateral transshipments in the network are considered. In order to deal with the uncertainties, several scenario samples are generated by Monte Carlo and corresponding sample average approximations programs are solved to obtain the adequate response policy to the inventory system under disruptions. Extensive numerical experiments are conducted and the results enable insights to be gained into the impact of disruptions on the network total cost and service level.

4.1 Introduction

In chapter 3 we developed a modeling and a solution approach for a multi-echelon inventory optimization problem under demand uncertainty. As discussed in chapters 1 and 2, disruptions could be caused for example by discontinuities in supply, political instability, natural disasters and labor strikes, and could have a severe effect on the supply chain performance.

To deal with such disruptions, inventory optimization models must be adapted to cover a multi-echelon network structure and consider alternative sourcing strategies such as lateral transshipment and multi-sourcing. In this chapter, a two-stage stochastic model is proposed to solve a capacitated multi-echelon inventory optimization problem considering a stochastic demand as well as uncertain throughput capacity and possible inventory loss, due to disruptions. The model minimizes the total cost that is composed of fixed allocation cost, inventory holding, transportation and backordering costs by optimizing inventory policy and flow decisions.

The inventory is controlled according to a reorder point order-up-to-level (*s*, *S*) policy and lateral transshipments in the network are considered. In order to deal with the uncertainties, several scenario samples are generated by Monte Carlo and corresponding sample average approximations programs are solved to obtain the adequate response policy to the inventory system under disruptions. Extensive numerical experiments are conducted and the results enable insights to be gained into the impact of disruptions on the network total cost and service level.

Following the literature presented in chapter 2, this chapter focuses on computing the optimal (*s*, *S*) policy parameters in multi-echelon distribution networks under supply and demand uncertainty. We consider two types of strategies to overcome supply and demand uncertainties: inventory decisions and sourcing strategies.

Inventory decisions include the ordering and stocking decisions and could be considered as mitigating, proactive techniques. Sourcing strategies could be reactive to an actual delay in the network or used proactively in planning for a potential shortage within lateral transshipment and multi-sourcing.

The main challenge in this chapter is the anticipation of the future demand variability and disruption occurrences in order to improve the quality of the inventory decisions. At the operational level, this issue could be handled by a set of scenarios. In order to efficiently mitigate supply and demand uncertainty, sourcing and inventory decisions should proactively take the risk exposure into account. In the third chapter we have investigated the effect of flexible sourcing strategies on the performance of the network under the demand uncertainty.

In this chapter, we take into account supply uncertainty. We propose a two-stage stochastic mixed integer linear programming formulation (MILP) for a multi-echelon distribution network under a stationary demand behavior and disruption risks.

As discussed before in chapter 2, when making the first-stage decisions two sources of uncertainties are considered. The first regards the demand for final products that is not known with certainty when the planning of the inventory has to be made. This reflects the real-world setting in which demand is highly variable and the second one is the occurrence of different disruption. When a disruption occurs, some depots may lose part of their capacity and their inventory, which is difficult to predict when inventory has to be deployed.

Section 2 presents the problem definition by explaining the preliminaries and the main assumptions. The uncertainty modeling approach is also described in this section. In section 3, the stochastic two-stage mathematical model is developed. Solution approach and scenario generation are presented in section 4. Computational experiments and the insights are presented in section 5 and section 6 concludes the chapter.

4.2 Problem Definition

This chapter considers a three-echelon supply chain that includes a set of suppliers $v = 1, 2, 3, \ldots$, a set of Production-Distribution Centers (PDC) p = 1, 2, 3, ..., a set of Distribution Centers (DC) w= 1, 2, 3, é , and a set of Customer Zones (CZ) z = 1, 2, 3, ...Each stage is fed from the upper echelon and feeds the below ones itself. The platforms $l, l \in L, l = \{p, w\}$ are defined as a set of network locations $L = P \bigcup W$.

There are multi-sourcing opportunities between PDCs and DCs with respect to the throughput capacity per period of each platform. Each platform operates under (s, S) policy. The (s, S) policy features two control parameters: reorder point (s) and order up-to-level (S). According to this policy, the decision maker checks the inventory position at the end of each time period: if it drops below the reorder point s, then, replenishment should be placed to reach the order-up-to-level S. Unfortunately, computing the optimal (s, S) policy parameters remain a computationally intensive task.

A lateral transshipment (LT) option, which allows replenishment flows in the same echelon, is available between DCs. In our proposed two-stage stochastic model, the decisions concerning the inventory policy and the demand allocation are considered as the first-stage decisions, taken based on the available data. The inventory policy parameters are anticipated at the beginning of the planning horizon as decision rules, then in the second stage, inventory levels, and transportation flow decisions and order quantity at all echelons are optimized.

4.2.1 Preliminaries and Assumptions

In our modeling approach, we assume that each CZ $z \in Z$ in the supply network faces a stationary demand for a product family (referred hereafter as a single product). The demand at each period τ, $\tau \in T$, is assumed to be independently distributed, consistent with the assumptions of most of the studies in the literature of multi-period inventory problems (Jain et al., 2011; Kang and Kim, 2012; Cunha et al., 2018). Let $d_{z\tau}$ denotes the random variable of the demand of zone z on the period τ $\tau \in T$ with mean and standard deviations $\mu_{z\tau}$ and $\sigma_{z\tau}$, respectively. Each DC $w \in W$ faces an independent stationary, stochastic demand. In the multi-echelon distribution inventory models, the demand of a DC $w \in W$ is the summation of the customer demands , which are allocated to that DC. The subset of CZs $z \in Z$ allocated to a DC is determined by the demand fraction decision variables X_{wzt} and demand allocation variables Y_{wz} .These variable decisions are determined at the tactical level. The multi-sourcing is allowed to satisfy the demand of CZs, $z \in Z$ either partially or totally. Therefore, the assigned demand to a given DC, $w \in W$ becomes a random variable and it would be determined by the demand allocation decision variables. In other words, demand allocation decisions answer how much product should be transported from which DC to which CZ.

As mentioned before, in order to optimize the inventory decisions in multi-echelon distribution networks, the inventory policy parameters should be integrated in the mathematical model, while these parameters are often considered in the literature as a constant estimated value or included in the cost objective function (Yao et al., 2010; Berman et al., 2012). The inventory policy parameters are integrated in the model and optimized in our study. To find the periodic review inventory policy parameters, the reorder point level *s* and the order-up-to level *S*, are considered as decision variables and optimal (*s, S*) policy parameters are computed. In order to do so, a two-stage stochastic model is developed to optimize inventory decisions; in fact, the model optimizes two problems with different time granularities. As illustrated by Figure 17 , Inventory policy parameters and demand allocation decisions are determined in each planning period (season, month,..) (*t*) and the flow decisions are determined on operational periods (weeks) (τ). Note that the allocation decisions would be fixed in the first period (*t=0*) for the whole planning horizon. In the proposed model the first-stage variables are the demand allocation, demand fraction, reorder point and order-up-to level for each platform (DC, $w \in W$ and PDC, $p \in P$) in a centralized multi-echelon distribution network setting. The second-stage variables are the quantity of flows to be carried between echelons, binary variables to set ordering, on-hand inventories, inventory position and stock out amounts.

Moreover, the stochastic parameters in this problem are demand and disruption parameters which are considered in a scenario-based framework. When platform l, $l=\{p,w\}$ is disrupted, the platform would be partially operational for a stochastic number of $\theta_l,\ l=\{p,w\}$ periods and a stochastic percentage of inventory on-hand ($\zeta_l, l=\{p,w\}$) would be considered as the inventory loss.

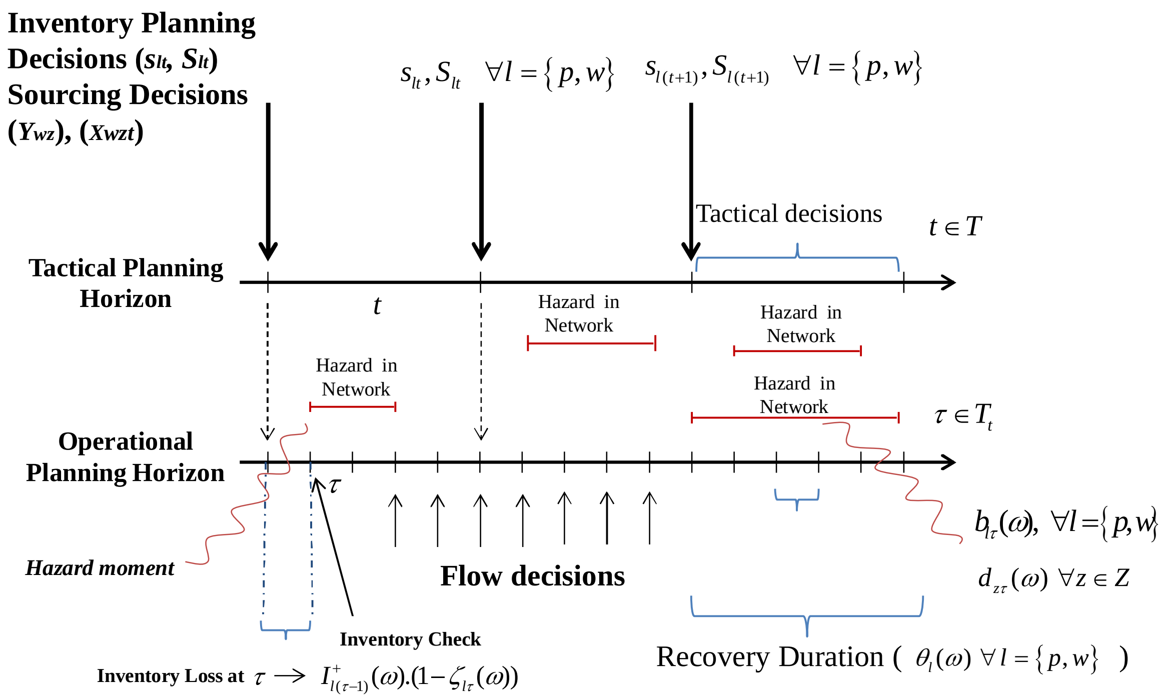


Figure 17. Network decisions and planning horizon

At operational period (Ǿ), the demand which arrives from customer zones $d_{z\tau}$ to the DCs should be satisfied from the DCs on-hand inventory $I^{+}_{w\tau}$ but if the demand cannot be satisfied, it is backordered to the subsequent periods. According to the considering periodic review policy (s, S)

for all DCs and PDCs during each season (t) (s_{lt}, S_{lt}) $l = \{p, w\}$, the inventory position $I_{p\tau}, I_{w\tau}$ of each product is inspected at the end of each period and all replenishments are originated based on these reviews. Noted that operational periods (τ) are defined as a subset of planning periods $\tau \in T_t$. After receiving demands from all market zones at the end of the period τ, the inventory is reviewed: if the inventory position is less or equal to the reorder point s, an order $Q_{wp\tau}, Q_{pv\tau}$ is placed to raise the inventory position up to the order-up-to-level S. The quantity that is ordered at the end of period τ will then be received ($R_{pw\tau}, R_{vp\tau}$) at the beginning of period $\tau + LT + 1$ where LT is the length of lead-time period.

The DCs orders are satisfied from the PDCs on-hand inventory. Each platform's flow is restricted to a throughput capacity ($b_{p\tau}, b_{w\tau}$). When a platform is facing a disruption, the throughput capacity and the inventory on-hand will be affected. With regards to the matters enumerated, a multi-echelon inventory optimization modeling approach under a periodic (s_{lt}, S_{lt}) control policy is considered. The proposed model considers multi-period settings, where the periods can't be considered separately, due to the inventory state constraints. A two-stage stochastic programming approach (Shapiro et al., 2009) is used to cope with this optimization problem under uncertain demand and capacity. It builds on the assumption that the probability distribution functions of uncertain parameters are known or can be statistically estimated, and that the objective function is estimated by an expected value. Thus, a possible realization of the stochastic demand process, throughput capacity and inventory loss for each platform over the planning horizon T can be generated. Such a realization constitutes a scenario ω, the set of all demands, capacity and inventory losses are denoted by Ω. The probability of occurrence of a given scenario $\omega \in \Omega$ is denoted by $p(\omega)$.

4.2.2 Uncertainty Modeling Approach

A multi-echelon distribution network planning must consider the operational and natural risks. However, at the point of tactical planning the future events are not known with certainty. Generally two types of events influencing the business environment can be distinguished by, namely business-as-usual random events and low-probability high-impact disruptions. The business-as-usual and low-probability high-impact events mentioned are the main sources of uncertainty, but the available information to characterize them may be lacking. In order to form these events, it is essential to characterize the occurrence, severity, intensity, and duration of each event depending on the availability of data. In this chapter, customer demands and network disruptions are modeled as compound stochastic processes. The proposed disruption modeling approach is based on the framework developed in Klibi and Martel (2012). First, a compound stochastic process is defined to describe how disruption events occur in space and in time, to specify incidentôs impact. Second, the impact of hits on the throughput capacity and the inventory on-hand is modeled. The occurrence, severity, intensity, and duration of hazards are characterized depending on the location of each platform $l, l=\{p,w\}$. When the platform l is hit at the beginning of period ***Ű*** this leaves perturbed capacity and the inventory loss. More specifically, when a hazard hits a network location at the beginning of a period, its intensity is felt as follow:

1) In terms of capacity reduction, for a number of subsequent periods θ_l.

2) In terms of inventory loss, for a percentage of the inventory on-hand in platform l in (***Ű****1*)

Figure 17 also indicates the order of the events in one chosen platform $w, w \in W$. If the disruption occurs at the beginning of the period ***Ű*** the inventory loss would be calculated based on the inventory on-hand at the end of the period (***Ű****1*).

Recall that in the present framework, the platforms $l, l=\{p,w\}$ define a set of network locations $L = P \bigcup W$. The platforms l have different disruption profiles in terms of impact and time to recovery. It is assumed that the hazards occur independently in different zones $l \in L$, and the time between their occurrences is a random variable λ_l characterized by a stochastic arrival process with cumulative disruption function $F_l^{\lambda}(.)$. When platform $l \in L$ is hit by a hazard, the severity of the disruption is formed by correlated random variables β_l, depending to the location with cumulative distribution function $F_l^{\beta}(.)$. As mentioned before, each hazard could have two different impacts on the network. The intensity of the disruption which impacts the throughput capacity is shaped by a random variable χ_l and time to recovery θ_l.

The time to recovery and the intensity are directly related to the severity of the hazard through the functions $\theta_l = f^{\theta}(\beta_l) + \varepsilon$ and $\chi_l = f^{\chi}(\beta_l) + \varepsilon$ where ε is a random error term with probability distribution function $F^{\varepsilon}(.)$. We also need another random variable ζ_l to model the inventory loss in each platform. The intensity which impacts the inventory loss ζ_l is likewise related to the severity of the hazard through the function $\zeta_l = f(\beta_l) + \varepsilon$.

When the platform l is hit at the beginning of period $\tau' \in T_t$, this leaves perturbed capacity $b_{l_\tau}(\omega)$ and inventory loss $I^{+}_{l(\tau-1)}(\omega).(1-\zeta_{l\tau}(\omega))$. The impact of the hit on the throughput capacity is not necessarily uniform during the recovery time. After arriving a hit on a platform, the throughput capacity drops and there may be a stagnation phase for η number of periods while recovery measures are organized. The impact could be characterized by a discrete recovery function $(r(\chi, \tau), \tau = \tau', ..., \tau' + \theta - 1)$ providing capacity amplification percentages for the θperiods affected by the hazards (Figure 18). Figure 18 shows the impacts of the hazard at a given DC *w*, a scenario ω in two different views. In the throughput capacity view, the regular throughput capacity $b_{w\tau}$ would decrease based on the hazard severity$\beta_w, w \in W$. The intensity $\chi_w$and the duration $\theta_w$would be calculated and it would partially be operational for θ_w period. During these periods the DCȏs throughput capacity would be gradually recovered which is shaped by a discrete recovery function. The difference between the usual capacity and the assigned demand is considered as the capacity buffer. Capacity buffer is the available capacity that could be used for lateral transshipment flows and the backorders of other DCs.

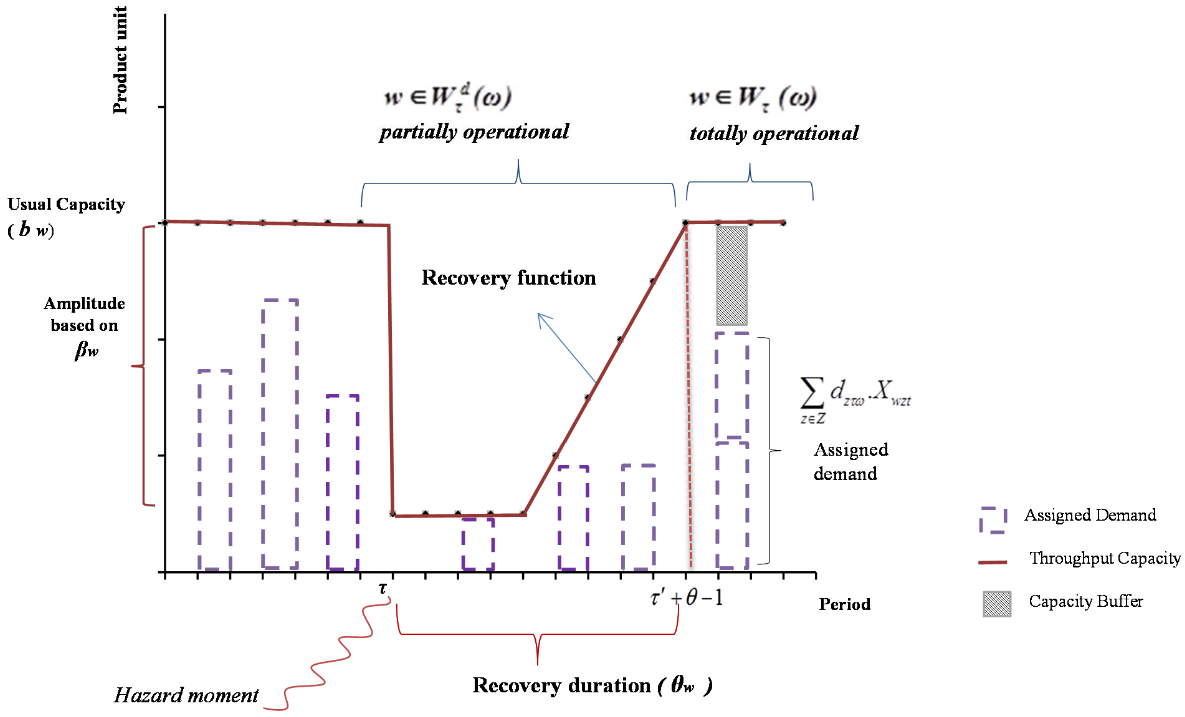


Figure 18. Capacity view of impact of hazard at a given DC *w* and scenario ω

Klibi and Martel (2012) have considered this impact as the only effect of the disruption; however, a hazard could result in addition to an inventory loss. If a hazard occurs in period ***Ű*** the inventory loss $I^{+}_{l(\tau-1)}(\omega).(1-\zeta_{l\tau}(\omega))$ would be a percentage of the inventory on-hand in the period before the disruption happens. Figure 19 indicates the possible inventory loss caused by a disruption. The ζ_w amount of inventory on-hand at the period (***Ű****1*) can be unavailable due to the severity of the event at the period ***Ű***

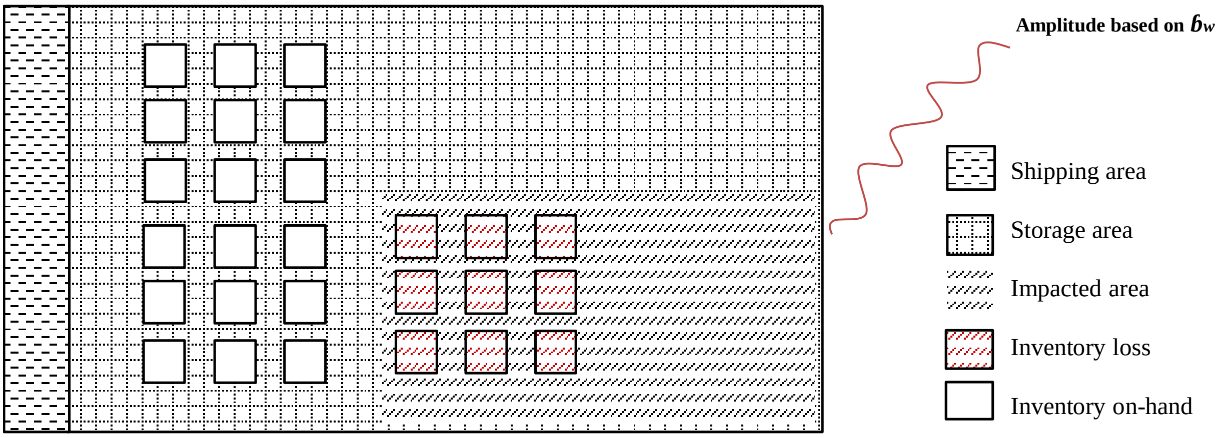


Figure 19. Inventory view of impact of hazard at a given DC w and scenario ω

Itôs worth mentioning the intensity on the throughput capacity and the inventory loss is modeled using a function $F^i(\beta)\ i=\{1,2\}$ in which i=1 corresponds to $F^{\chi}(\beta)$ and i=2 gives $F^{\varsigma}(\beta)$. Here is an example of the intensity function for a disruption with three different levels of severity in which A^i, B^i, and C^i are parameters related to the type of the disruption. Note that ψ_{Low}, ψ_{High} are defined as risk tolerance parameters for low risk level and high risk level respectively.

$$F^i(\beta)=\begin{cases} A^i.\beta & \beta<\psi_{Low} \\ B^i.\beta & \psi_{Low}\le\beta\le\psi_{High} \\ C^i.\beta & \psi_{High}<\beta \end{cases}$$

Furthermore, we assume that the demand of CZ, $z\in Z$ follows a stochastic process with a random demand size. The cumulative distribution functions of the random variable is denoted by $F^z(.)$

The instantiation of demand and disruption processes over all the possible values of the involved random variables yields a set Ω of plausible future scenarios with associated probabilities $\pi(\omega)$. Monte Carlo procedure is used to generate a scenario instance ω including vectors of daily demands, inventory loss, and throughput capacities.

4.3 Mathematical Model Formulation

As mentioned before, modeling (s_{lt}, S_{lt}) policy is very complex especially in the presence of demand uncertainty and disruption. In this study, a two-stage stochastic model is proposed to optimize inventory decisions with different time granularities. Inventory policy parameters and demand allocation decisions are determined in te first-stage and the flow decisions are determined at the second-stage. We also introduce cuts to improve the efficiency of the proposed model for optimizing (s_{lt}, S_{lt}) parameters.

We have three challenges in modeling this problem:

- Approximate the multi-echelon setting by a two-stage setting with all (s_{lt}, S_{lt}) fixed at the first-stage.
- Approximate the non-linearity implied by the modelling the ordering process by a linear set of equations.
- Introduce cuts for (s_{lt}, S_{lt}) in the first-stage.

This section presents a mixed integer stochastic inventory optimization model. Hereafter, are given all the sets, parameters, and decision variables used in the mathematical model.

Table 10. Notation

Sets	
V	Set of suppliers $v \in V$
P	Set of PDC platforms $p \in P$
W	Set of DC platforms $w \in W$
Z	Set of CZs $z \in Z$
T	Set of time planning periods $t \in T$
T_t	Set of time operational periods in planning period t $\tau \in T_t$ (Periodic Review)
Ω	Set of scenarios $\omega \in \Omega$

Parameters	
$d_{z\tau\omega}$	Demand of CZ *z* at the beginning of period τ under scenario ω
$\mu_{z\tau}$	The average of CZ *z* demand at period τ
$b_{l\tau\omega}$	Available throughput capacity of platform *l*, $l=\{p,w\}$ at the beginning of period τ under the scenario ω, (expressed in flows unit)
$tc_{ll'}$	Unitary transportation flow cost between site *l* and site *l'*, $l=\{v,p,w\}$, $l'=\{p,w,z\}$
h_l	Unitary inventory holding cost at platform *l*, $l=\{p,w\}$
c_l	Unitary backorder cost for site *l*, $l=\{p,w\}$
I^{+}_{v}	Inventory on-hand of supplier *v* at the end of period 0 ($\tau=0$)
$LT_{ll'}$	Lead-time (expressed in the number of operational periods) from site *l* to site *l'*, $l=\{v,p,w\}$, $l'=\{p,w,z\}$
$\delta_{ll'}$	Fixed ordering cost from platform *l* to platform *l'*, $l=\{p,w\}$, $l'=\{v,p\}$
a_{wz}	Fixed allocation cost of CZ *z* to DC *w*,
π_{ω}	The probability of occurrence of scenario ω
$Dist_{ll'}$	Distance between site *l* and site *l'*, $l=\{v,p,w\}$, $l'=\{p,w,z\}$
M	A large positive number
Decision Variables	
$Q_{ll'\tau\omega}$	Ordered quantity from site *l* to site *l'* at period τ under scenario ω, $l=\{v,p,w\}$, $l'=\{p,w,z\}$ (expressed in flows unit)
$\breve{Q}_{ll'\tau}$	Expected order quantity (anticipated at first-stage) at period *t* from platform *l* to platform *l'*, $l=\{p,w\}$, $l'=\{v,p\}$ (expressed in flows unit)
s_{lt}	re-order point at platform *l* at period *t*, $l=\{p,w\}$
S_{lt}	order-up-to level at platform *l* at period *t*, $l=\{p,w\}$
$I_{l\tau\omega}$	Inventory position (inventory on hand – backorder + orders in transit) at platform *l* at the end of period τ under scenario ω, $l=\{p,w\}$
$I^{+}_{l\tau\omega}$	Inventory on-hand at platform *l* at the end of period τ under scenario ω, $l=\{v,p,w\}$
$I^{-}_{ll'\tau\omega}$	Backorders of site *l'* from site *l* at the end of period τ under scenario ω, $l=\{z,w\}$, $l'=\{w,p\}$
$I^{-}_{z\tau\omega}$	Backorders level in CZ *z* at the end of period τ under scenario ω,
$F_{wz\tau\omega}$	Product flow from DC *w* to CZ *z* at the beginning of period τ under scenario ω
$R_{ll'\tau\omega}$	Received products from site *l* to site *l'* at the beginning of period τ under scenario ω, $l=\{p,w\}$, $l'=\{v,p,w\}$ (expressed in flows unit)
Y_{wz}	Binary variable that takes the value 1 if part of the demand of the CZ *z* is assigned to DC *w*, 0 otherwise
X_{wzt}	The fraction of demand of CZ to DC *w* assigned a priori for period *t*
$O_{ll'\tau\omega}$	Binary variable that takes the value 1 if the sourcing arc (*l*, *l'*) is used (platform *l* placed an order to platform *l'* i.e., $Q_{ll'\tau\omega}>0$) for a given period τ under the scenario

ω, 0 otherwise $l = \{p, w\}, \quad l' = \{v, p, w\}$

According to the above notations, the objective function (1) of the stochastic multi-echelon inventory optimization model is formulated as follow:

$$Min \sum_{w\in W}\sum_{z\in Z} a_{wz}.Y_{wz} \tag{1.a}$$

$$+\sum_{\omega\in\Omega}\pi(\omega)\left[\sum_{\tau\in T}\left(\sum_{v\in V}\sum_{p\in P} tc_{vp}.R_{vp\tau\omega} + \sum_{p\in P}\sum_{w\in W} tc_{pw}.R_{pw\tau\omega} + \sum_{w\in W}\sum_{w'\in W\setminus\{w\}} tc_{ww'}.R_{ww'\tau\omega} + \sum_{w\in W}\sum_{z\in Z} tc_{wz}.R_{wz\tau\omega}\right)\right. \tag{1.b}$$

$$+\sum_{\tau\in T}\left(\sum_{p\in P}\sum_{w\in W} c_w.I^-_{pw\tau\omega} + \sum_{w\in W}\sum_{z\in Z} c_z.I^-_{wz\tau\omega}\right) \tag{1.c}$$

$$+\sum_{\tau\in T}\left(\sum_{v\in V}\sum_{p\in P} \delta_{vp}.O_{vp\tau\omega} + \sum_{p\in P}\sum_{w\in W} \delta_{pw}.O_{pw\tau\omega}\right) \tag{1.d}$$

$$\left.+\sum_{\tau\in T}\left(\sum_{w\in W} h_w.I^+_{w\tau\omega}\sum_{p\in P} h_p.I^+_{p\tau\omega}\right)\right] \tag{1.e}$$

The objective function (1) minimizes the total cost as follows: First, the fixed allocation cost is calculated in the first stage (equation, 1.a) independently from the scenarios. Then, the transportation costs between suppliers, PDCs, DCs, and CZs are computed by equation (1.b) based on the flows between these platforms at all periods $t\in T$ and all scenarios $\omega\in\Omega$. Equation (1.c) calculates the total backorder cost based on the level of backordered products for all the CZ $z \in Z$ in all periods $t \in T$ and all scenarios $\omega\in\Omega$. Next, fixed procurement cost in DCs and PDCs, is computed by equation (1.d) based on the number of orders in all periods $\tau\in T$ and all scenarios $\omega\in\Omega$ and finally the equation (1.e) computes the total inventory holding costs in PDCs and DCs which are considered based on the inventory on-hand in all periods $\tau\in T$ and all scenarios $\omega\in\Omega$. The objective function (1) is subject to the following constraints:

First-stage constraints:

$$\sum_{w\in W}\sum_{\tau\in T_t} \mu_{z\tau} X_{wzt} = \sum_{\tau\in T_t} \mu_{z\tau} \qquad \forall z \in Z, t \in T \tag{2}$$

$$\sum_{p \in P} \sum_{\tau \in T_t} \breve{Q}_{wp\tau} = \sum_{z} \sum_{\tau \in T_t} \mu_{z\tau} X_{wzt} \qquad \forall w \in W, t \in T \tag{3}$$

$$X_{wzt} \leq Y_{wz} \qquad \forall w \in W, z \in Z, t \in T \tag{4}$$

Constraints (2-4) are related to the first-stage decisions. Since sourcing decisions are the first-stage variables and could not vary on daily basis, they should be determined based on the approximations. Constraints (2) and (3) determine the fraction variable of the demand of CZs and the orders of DCs. Constraint (4) guarantees that each CZ *z* is served from the DC *w* which is allocated in the first stage.

Based on the method proposed by Porteus (1985), in (*s*, *S*) policy the order point of each DC and PDC is a function of the expected assigned demand and the lead time.

$$(s, S) \simeq \breve{F}(\mu_{z\tau}, LT_{wz})$$

One of the techniques to strengthen the mathematical formulation is to add valid inequalities to the model. For that purpose, the mathematical formulation (5) and (6) are just presented below to reinforce the model with the following valid inequality. Recent studies in inventory management also show that in (*s, S*) policy, the order point of each DC and PDC is interrelated by the expected assigned demand and the lead time (Snyder and Shen, 2011). This inequality imposes a boundary on the reorder points (*s*) in each platform *l*, in any period *t*. Constraints (5) and (6) present a lower bound for reorder point in each DC and each PDC. However we optimize the reorder point (*s*) and order up to level (*S*) in this model, these constraints would make the feasible solutions area bounded according to the allocated demand and could be considered as the valid inequalities. These formulations are valuable in assisting CPLEX to generate some new cuts. We show the advantage of applying these constraints in section 4.

The approximate lower bound for reorder point level in each platform *l*, is expressed as follows:

$$s_{wt} \geq \left(LT_{wz} + 1\right).(\sum_{z} \mu_{z\tau} X_{wzt}) \qquad \forall w \in W, \tau \in T_t, t \in T \tag{5}$$

$$s_{pt} \geq \left(LT_{pw} + 1\right).(\sum_{w \in W} \breve{Q}_{wp\tau}) \qquad \forall p \in P, \tau \in T_t, t \in T \tag{6}$$

Demand satisfaction constraint:

$$\sum_{w\in W} F_{wz\tau\omega} + I^-_{z\tau\omega} - I^-_{z(\tau-1)\omega} = d_{z\tau\omega} \qquad \forall z\in Z, \tau\in T, \omega\in\Omega \qquad (7)$$

Constraint (7) ensures that the demand of each CZ z , in each period $\tau \in T$ and each scenario $\omega\in\Omega$ is satisfied through the outgoing flow of DCs *w*, taking into account the demand of the period as well as backorders.

sourcing constraint:

$$F_{wz\tau\omega} \le MY_{wz} \qquad \forall w\in W, z\in Z, \tau\in T, \omega\in\Omega \qquad (8)$$

Equation (8) assures that the product flow to each CZ *z* could be sent from a given DC *w* only when the latter is assigned to the CZ *z*.

Backorder splitting between the DCs:

$$I^-_{z\tau\omega} = \sum_{w\in W} I^-_{wz\tau\omega} \qquad \forall z\in Z, \tau\in T, \omega\in\Omega \qquad (9)$$

Constraint (9) divides the backorder of CZs between DCs. At the end of each period $\tau\in T$, the unmet demand of each CZ would be distributed between the DCs which are able to send them products satisfy from next period (τ+1) regarding their available stock and capacity.

Throughput capacity constraint at DCs:

$$\sum_{z\in Z} F_{wz\tau\omega} + \sum_{w'\in W\setminus\{w\}} Q_{w'w\tau\omega} \le b_{w\tau\omega} \qquad \forall w\in W, \tau\in T, \omega\in\Omega \qquad (10.1)$$

$$\sum_{p\in P} R_{pw\tau\omega} + \sum_{w'\in W\setminus\{w\}} R_{ww'\tau\omega} \le b_{w\tau\omega} \qquad \forall w\in W, \tau\in T, \omega\in\Omega \qquad (10.2)$$

$$\sum_{w\in W} Q_{wp\tau\omega} \le b_{p\tau\omega} \qquad \forall p\in P, \tau\in T, \omega\in\Omega \qquad (11.1)$$

$$\sum_{v\in V} R_{vpt\omega} \le b_{p\tau\omega} \qquad \forall p\in P, \tau\in T, \omega\in\Omega \qquad (11.2)$$

$$\sum_{p\in P} Q_{pvt\omega} \le b_v \qquad \forall v\in V, \tau\in T, \omega\in\Omega \qquad (12)$$

We have considered the same inflow and outflow capacity for each platform *l*, ($b_{l\tau\omega}$). Constraints (10-12) restrict the outflows per platform $l \quad l=\{v,p,w\}$ to the throughput capacity limit of the period. Let b_l denotes the usual capacity of platform $l \quad l=\{v,p,w\}$. If a platform $l \quad l=\{p,w\}$ would be partially operational as shown in Figure 18, then it performs to ($L_\tau^d(\omega)=\{l \mid b_{l\tau\omega}<b_l\}$, $l=\{p,w\}, L=\{W,P\}$), would be considered. Since we assume to have reliable suppliers $v \in V$, the outflows of the supplier do not depend on the scenarios (b_v).

Flow - Information equilibrium constraints (Supplier-PDCs – DCs):

$$R_{pw\tau\omega} = Q_{wp(\tau-LT_{pw}-1)\omega} - I^-_{pw(\tau-LT_{pw}-1)\omega} + I^-_{pw(\tau-LT_{pw}-2)\omega} \qquad \forall p \in P, w \in W, \tau \in \{LT_{pw}+2,..,T\}, \omega \in \Omega \tag{13}$$

$$R_{w'w\tau\omega} = Q_{ww'(\tau-LT_{w'w}-1)\omega} \qquad \forall w' \in W \setminus \{w\}, w \in W, \tau \in \{LT_{w'w}+2,..,T\}, \omega \in \Omega \tag{14}$$

$$R_{vp\tau\omega} = Q_{pv(\tau-LT_{vp}-1)\omega} \qquad \forall v \in V, p \in P, \tau \in \{LT_{vp}+2,..,T\}, \omega \in \Omega \tag{15}$$

Constraints (13-15) show the flow equilibrium between supplier, PDCs, and DCs. Since the orders are made at the end of period τ they will then be received ($R_{pw\tau\omega}, R_{w'w\tau\omega}, R_{vp\tau\omega}$) at the beginning of period $\tau + LT + 1$ where LT is the length of the lead-time period. Recall that there is no backorder between supplier and PDCs. Constraint (13)shows the received orders at period τ $\tau \in T$ which is equal to the orders that DC *w* has sent at the end of period $(\tau - LT_{pw} - 1)$ considering the related backorders. Recall that if DC *w* sends an order to PDC *w* ($Q_{wp\tau\omega}$) or to a DC *w'* (via lateral transshipment, $Q_{ww'\tau\omega}$) in period τ the order will be sent at the end of the same period τ

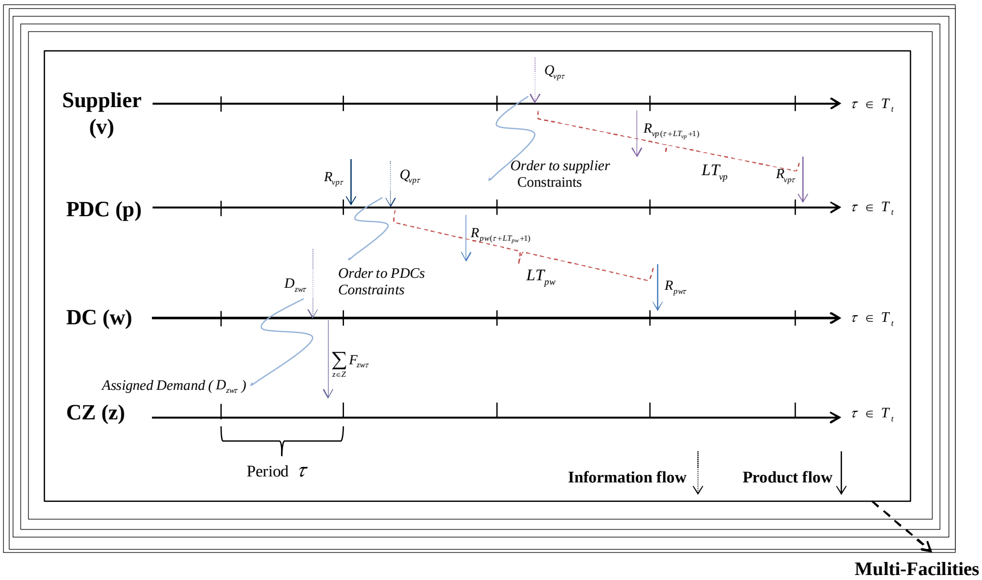


Figure 20. Dynamics of a multi-echelon, multi-period distribution network flows

Figure 20 indicates the order of events in the whole multi-echelon distribution network. Product and information flows are distinguished to have a better understanding of the flow-information equilibrium constraints. The presented model could be applied to multi-echelon distribution networks with several DCs and PDCs. In Figure 20, we take a very simple case of one supplier, one PDC, one DC and a set of CZs to show the flow management and order of events. The flows below and above each line represent the out-flows and in-flows respectively.

Inventory on-hand constraints at DCs and PDCs

$$I^{+}_{w\tau\omega} = (1-\zeta_{w\tau\omega}).I^{+}_{w(\tau-1)\omega} + \sum_{p\in P} R_{wp\tau\omega} - \sum_{z\in Z} F_{wz\tau\omega} + \sum_{w'\in W\backslash\{w\}} R_{w'w\tau\omega} - \sum_{w'\in W\backslash\{w\}} Q_{w'w\tau\omega} \qquad \forall w\in W, \tau\in T, \omega\in\Omega \qquad (16)$$

$$I^{+}_{p\tau\omega} = (1-\zeta_{p\tau\omega})I^{+}_{p(\tau-1)\omega} + \sum_{v\in V} R_{vp\tau\omega} - \left(\sum_{w\in W} Q_{wp\tau\omega} - \sum_{w\in W} I^{-}_{pw\tau\omega} + \sum_{w\in W} I^{-}_{pw(\tau-1)\omega}\right) \qquad \forall p\in P, \tau\in T, \omega\in\Omega \qquad (17)$$

$$I^{+}_{v\tau\omega} = (1-\zeta_{v\tau\omega})I^{+}_{v(\tau-1)\omega} - \sum_{p\in P} Q_{pv\tau\omega} \qquad \forall v\in V, \tau\in T, \omega\in\Omega \qquad (18)$$

Equations (16-18) indicate the inventory on-hand in DCs, PDCs, and suppliers by balancing the flows-in and flows-out of the platform per period and scenario. More specifically, the inventory on-hand ($I^{+}_{l\tau\omega}$, $l = \{w, p\}$) at each period $\tau \in T$ and scenario $\omega \in \Omega$ is the summation of inventory on-hand at the last period (**Ű**1) and the received products from other platforms minus the products that will be sent out to the subsequent stage (PDCs for supplier, DCs for PDCs and CZs for DCs) and the backordered products at period (**Ű**-1). The inventory on-hand of the previous period $(1-\zeta_{w\tau\omega}).I^{+}_{w(\tau-1)\omega}$ could be decreased due to the disruption.

Inventory Position at DCs and PDCs:

In general distribution policy, inventory position is considered as a decent indicator for inventory management. The inventory position is the inventory on hand at the same period plus the orders which have been made but are not yet received minus the backordered products. In common, an ordering decision should not be based only on the inventory on-hand level. The ordering decision should also consider the replenishment orders which have been placed earlier and not yet been delivered. The overall state of the system can be then characterized by the inventory position, denoted $I_{l\tau}$.

$$I_{w\tau\omega} = I^{+}_{w\tau\omega} + \sum_{p\in P}\sum_{l\in\{1,\dots,LT_{pw}\}} Q_{wp(\tau-l)\omega} - \sum_{z\in Z} I^{-}_{wz\tau\omega} \qquad \forall w\in W, \tau\in T, \omega\in\Omega \qquad (19)$$

$$I_{p\tau\omega} = I^{+}_{p\tau\omega} + \sum_{v\in V}\sum_{l\in\{1,\dots,LT_{vp}\}} Q_{pv(\tau-l)\omega} - \sum_{w\in W} I^{-}_{pw\tau\omega} \qquad \forall p\in P, \tau\in T, \omega\in\Omega \qquad (20)$$

Depending on the information system availability, an inventory position may be controlled at periodic times through a periodic review policy. Then, the control policy determines when and how much to order. The (s_{lt}, S_{lt}) policy has been shown to be optimal for the problems having fixed ordering cost and stationary stochastic demand in a single stage system and could be an appropriate policy for a multi-echelon distribution network

Based on the (s_{lt}, S_{lt}) policy, if the inventory position ($I_{l\tau\omega}$) in each DC and PDC falls below the re-order point (s_{lt}) in period $\tau \in T_t$, an order up to level would be sent to the upper echelon at the end of period $\tau \in T_t$. As it is mentioned before, the order quantity in the (s_{lt}, S_{lt}) policy is the difference of maximum possible inventory level and current inventory position. In all the constraints below, we define $L = \{W, P\}$ and $L' = \{P, V\}$ .In other words,

if $I_{l\tau\omega} \le s_{lt}$ $\quad\quad \forall l \in L, \tau \in T, t \in T, \omega \in \Omega$

Then,

$$Q_{ll'\tau\omega} = S_{lt} - I_{l\tau\omega} \quad\quad \forall l \in L, l' \in L'_{\tau}(\omega), \tau \in T_t, t \in T, \omega \in \Omega$$

Else,

$$Q_{ll'\tau\omega} = 0 \quad\quad \forall l \in L, l' \in L'_{\tau}(\omega), \tau \in T_t, t \in T, \omega \in \Omega$$

These two constraints clearly determine the order quantity base on the selected policy. However, they are non-linear ($Q_{ll'\tau\omega} = (S_{lt} - I_{l\tau\omega}).O_{ll'\tau\omega} \quad \forall l \in L, l' \in L'_{\tau}(\omega), \tau \in T_t, t \in T, \omega \in \Omega,\ l = \{w, p\},\ l' = \{p, v\}$). So, a binary variable ($O_{ll'\tau\omega}$) is defined to end up with a MIP. Constraints (21-25) are the linear mode of above equations.

$$I_{l\tau\omega} - M(1 - O_{ll'\tau\omega}) \le s_{lt} \quad\quad \forall l \in L, l' \in L', \tau \in T_t, t \in T, \omega \in \Omega \quad (21)$$

$$I_{l\tau\omega} + M.O_{ll'\tau\omega} > s_{lt} \quad\quad \forall l \in L, l' \in L', \tau \in T_t, t \in T, \omega \in \Omega \quad (22)$$

$$Q_{ll'\tau\omega} - M.(1 - O_{ll'\tau\omega}) \le S_{lt} - I_{l\tau\omega} \quad\quad \forall l \in L, l' \in L', \tau \in T_t, t \in T, \omega \in \Omega \quad (23)$$

$$Q_{ll'\tau\omega} + M.(1 - O_{ll'\tau\omega}) \ge S_{lt} - I_{l\tau\omega} \quad\quad \forall l \in L, l' \in L', \tau \in T_t, t \in T, \omega \in \Omega \quad (24)$$

$$Q_{ll'\tau\omega} \le M.O_{ll'\tau\omega} \quad\quad \forall l \in L, l' \in L', \tau \in T_t, t \in T, \omega \in \Omega \quad (25)$$

This order will be delivered at the beginning of the period $\tau + LT_{ll'} + 1 \in T_t$. Based on the different time granularities, there would be same reorder points and order-up-to levels for a set of daily periods $\tau \in T_t$. Constraints (25) guarantees that fixed ordering cost incurred between two platforms is set to *1* per period and scenario when $Q_{ll'\tau\omega} > 0$.

Initialization constraints:

$$
\begin{array}{ll}
R_{ll'\tau\omega} = 0 & \forall\ \tau \in 1,2,...,LT_{ll'}+1 \quad l=\{v,p,w\}, l'=\{p,w\} \\
 & \forall v \in V, p \in P, w \in W, w' \in W \setminus \{w\}, \omega \in \Omega \\
Q_{ll'0\omega} = 0 & \forall\ l=\{p,w\}, l'=\{v,p,w\}, \\
 & \forall v \in V, p \in P, w \in W, w' \in W \setminus \{w\}, \omega \in \Omega \\
I^{-}_{ll'0\omega} = 0 & l=\{p,w\}, l'=\{w,z\} \\
 & \forall p \in P, w \in W, z \in Z, \omega \in \Omega \\
I^{+}_{w0\omega} = ((\sum_{p \in P} LT_{pw} / |P|)+1).\sum_{z \in Z} \breve{\mu}_{z\tau} / |W| & \forall\ w \in W, \omega \in \Omega \\
I^{+}_{p0\omega} = ((\sum_{v \in V} LT_{vp} / |V|\ )+1).\sum_{z \in Z} \breve{\mu}_{z\tau} / |P| & \forall\ p \in P, \omega \in \Omega \\
I^{+}_{v0\omega} = I_v & \forall\ v \in V, \omega \in \Omega
\end{array} \tag{26}
$$

Non-negativity constraints:

$$
\begin{array}{l}
s_{pt}, s_{wt}, \widehat{Q}_{wp\tau}, X_{wzt} \geq 0 \quad \forall p \in P, w \in W, z \in Z, t \in T, \tau \in T_t \\
X_{wzt} \subset [0,1] \quad , Y_{wz} \in \{0,1\} \\
I_{l\tau\omega} \in \mathbb{R} \qquad \forall\ \tau \in T, \omega \in \Omega,\ l=\{p,w\} \\
I^{-}_{ll'\tau\omega}, I^{+}_{l\tau\omega}, Q_{ll'\tau\omega}, R_{ll'\tau\omega}, F_{wz\tau\omega} \geq 0, \quad l=\{v,p,w\}, l'=\{p,w,z\} \ \forall v \in V, p \in P, w \in W, z \in Z, \tau \in T, \omega \in \Omega \\
O_{ll'\tau\omega} \in \{0,1\}, \quad l=\{p,w\}, l'=\{v,p\} \ \forall v \in V, p \in P, w \in W, \tau \in T_t, \omega \in \Omega
\end{array} \tag{27}
$$

The initialization of the proposed model, the non-negativity, and binary constraints are given by constraints (26) and (27). In the initialization inventory on on-hand in each platform, we consider the average of demand $\breve{\mu}_{z\tau}$ for each CZ to estimate the initial stock.

It is important to remark that the mathematical formulation (1) to (27) can only be solved to optimality by commercial integer linear programming solvers for very small instance sizes. For instances of realistic size, the problem cannot be solved to optimality. The solution approach will be developed and presented in the following subsection.

4.4 Solution approach

4.4.1 Monte-Carlo

The Monte-Carlo sampling method is a common technique that uses statistical information on uncertain parameters to generate possible future scenarios occurring during a given planning horizon. When a sample of scenarios is generated using this method, all the scenarios in the sample are equiprobable, which simplifies the estimation of the optimal solution using an average of the scenarios. Running the Mont-Carlo procedure M times gives a sample of independent scenarios $\left\{\omega^1, \omega^2, ..., \omega^M\right\} = \Omega^M \subseteq \Omega$. Note that the functions used in the ñ*demand and disruption modeling*ò section to generate the scenarios, are described in Figure 21 and pseudorandom number **ɜ**, uniformly distributed on the interval $[0,1]$ is used to generate random variable realizations based on the inverse of probability functions and **F** denotes the set of all the previously defined probability distributions. $\boldsymbol{d}(\omega), \boldsymbol{b}(\omega), \chi(\omega)$ and $\zeta(\omega)$ denote the vectors of demand $\left[d_{z\tau}(\omega)_{z\in Z,\tau\in T}\right]$, capacity $\left[b_{l\tau}(\omega)_{l\in L,\,\tau\in T}\right]$,impact of disruption on capacity $\left[\chi_{l\tau}(\omega)_{l\in L,\,\tau\in T}\right]$ and impact on inventory loss $\left[\zeta_{l\tau}(\omega)_{l\in L,\tau\in T}\right]$. Number of hits on the network in each scenario is presented by $N(\omega)$.

The Monte-Carlo procedure used to generate stochastic demands, capacities and inventory loss for the scenario ω is given in a generic format in Figure 21.

Monte Carlo $\left((L, l \in L), T, \lambda^l, \boldsymbol{F}, r(\chi, \theta); \chi(\omega); N(\omega), \boldsymbol{d}(\omega), \boldsymbol{b}(\omega), \zeta(\omega)\right)$

For all platforms, set the normal capacity and initialize the intensities

set $b_{l\tau}(\omega) = b_l, \beta_{l\tau}(\omega) = 0, \; \zeta_{l\tau}(\omega) = 0, \; \chi_{l\tau}(\omega) = 0, \; l \in L, p \in P, w \in W, \tau \in T_t$

Hazard Moments

For all $l \in L, do$:

Using $F_l^{\lambda}(.)$, the distributin of λ_l, generate a list of hazard moments $T_l \subseteq T_t$

Disruption Characteristics

For all $\tau' \in T_l$, do :

Generate υ and compute hazard severity $\beta_{l\omega} = F^{\beta^{-1}}(\nu)$

Compute the intensity on throughput capacity and duration

$\chi_{l\tau\omega} = F^{\chi}(\beta_{l\omega})$ and $\theta_{l\omega}$=(0.8.$\beta_{l\omega}$^2+4.$\beta_{l\omega}$)

Compute η , $\eta = \lfloor 0.25 * \theta_{l\omega} \rfloor$

Compute recovery function

For $\tau = \tau' : \tau' + \eta$

$r_\tau(\chi_{l\omega}, \theta_{l\omega}) = 1 - \chi_{l\omega}$ #Recovery function on stagnation phase

End For

For $\tau = \tau' + \eta + 1 : \tau' + \theta_l - 1$

$r_\tau(\chi_{l\omega}, \theta_{l\omega}) = 1 - \chi_{l\tau\omega}((\theta_{l\omega} + 1 - \tau) / (\theta_{l\omega} + 1 - \eta))$

End For

Update Throughput Capacity

$b_{l\tau}(\omega) = r_{\tau - \tau' + 1}(\chi_{l\omega}, \theta_{l\omega}).b_{l\tau}(\omega), \tau = \tau', ..., \tau' + \theta_{l\omega} - 1$

Compute the intensity on inventory loss:

$\zeta_{l\tau\omega} = F^{\zeta}(\beta_{l\omega})$

Update the number of hits on the network ($N(\omega)$)

End For

For all $z \in Z$ and $\tau \in T_t$, do : #*CZ Demand*

$d_{z\tau\omega} = 0, z \in Z, \tau \in T_t$

calculate $F_\tau^z(.)$ distributions parameters

compute $F_\tau^{z^{-1}}(i)$

calculate the demand of CZ, $d_{z\tau\omega} = F_\tau^{z^{-1}}(i)$

End For

Figure 21. Monte-Carlo procedure.

In order to consider the attitude of decision makers towards serious events, we need to distinguish between the scenarios that the decision maker would consider as *acceptable*, in terms of the involved perturbation, and those that would raise a serious concern. Typical measures to assess the perturbation level associated to a scenario $\omega \in \Omega$ are the number of hits over the planning horizon or the cumulative capacity loss during the planning horizon (Klibi and Martel, 2012). Figure 22 shows the histograms obtained with these two measures by investigating a large sample of scenarios (1000 scenarios). In order to differentiate the acceptable and the serious concerned scenarios, a hazard tolerance level is defined. This level is defined as the number of maximum hits κ or the maximum cumulative capacity loss κ' that the decision maker can tolerate without any serious concern. Using this tolerance level, the set of scenarios Ω is partitioned into two subsets: Ω^A the set of acceptable-risk scenarios and Ω^S the set of serious-risk scenarios. In Figure 22, the tolerance level is a maximum of 3 hits on the network and a maximum tolerated capacity loss of 70000 flow units (about 10% of total capacity). In this study, a hazard tolerance level κ is defined based on the number of hits.

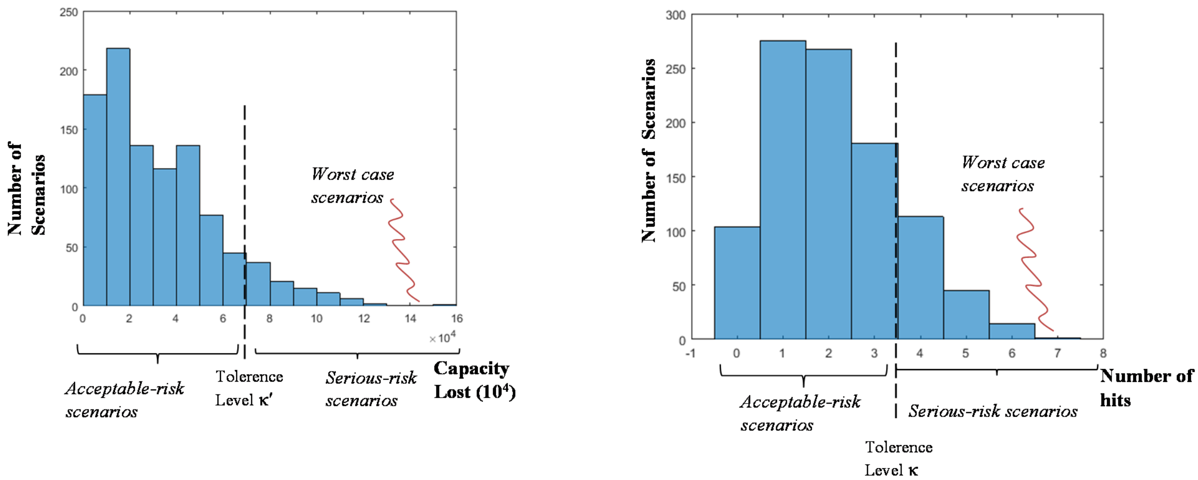


Figure 22. Distribution of hits and capacity loss for a large scenario sample

4.4.2 Hierarchical Sampling

The number of plausible future scenarios $|\Omega^M|$ to shape the uncertain parameters remains a big number in real cases and the probabilities $\pi(\omega), \omega \in \Omega^M$, cannot be estimated explicitly. In order to solve our stochastic program, one therefore needs to limit the number of scenarios considered and to find a way to calculate their probability. This can be done by replacing the scenario set in Ω^M in SAA model considering M scenarios by representative Monte Carlo samples Ω^{m_A} and Ω^{m_S} of m_A equiprobable acceptable-risk scenario and m_S equiprobable serious-risk scenario, respectively. Obviously the quality of the solutions depends on the number $m = m_A + m_S$ of considered scenarios.

Monte-Carlo procedure can be used to generate all the scenarios, however, to generate the scenario probabilities we need another procedure called *Hierarchical Sampling*. As explained in Figure 23, a large sample of M scenarios Ω^M is generated and partitioned into acceptable and serious-risk subsets Ω^{M_A} and Ω^{M_S}, using the tolerance level κ. From these samples, the estimates of the probabilities $\pi_A(\omega)$ and $\pi_S(\omega)$ are calculated with $\hat{\pi}_A = |\Omega^{M_A}|/M$ and $\hat{\pi}_S = |\Omega^{M_S}|/M$. Then, the small scenario samples Ω^{m_A} and Ω^{m_S} are randomly selected among Ω^{M_A} and Ω^{M_S}, respectively. Through this hierarchical sampling, we consider that all scenarios in Ω^{m_A} and Ω^{m_S} are equiprobable, with the probability $\hat{\pi}_{m_A}(\omega)$ and $\hat{\pi}_{m_S}(\omega)$.

Hierarchical Sampling $(m, M, \kappa; \pi_A(\omega), \pi_S(\omega), \omega \in \Omega^m)$

$M_A = 0, M_S = 0,\ \Omega^M = \varnothing,\ \Omega^m = \varnothing$

1) *Forming the scenario clusters*

for all $i = 1 : M, do:$

  Monte Carlo $\big((L, l \in L), T, \lambda^l, \boldsymbol{F}, r(\chi, \theta); N(\omega), \boldsymbol{d}(\omega), \boldsymbol{b}(\omega), \chi(\omega), \zeta(\omega)\big), \{\omega\} \subset \Omega^M$

Classify the scenarios based on the total number of hits given the threshold

 If $N(\omega) < \kappa$

  $\{\omega\} \subset \Omega^{M_A}, M_A = M_A + 1$

  Else

  $\{\omega\} \subset \Omega^{M_S}, M_S = M_S + 1$

 End If

End for

Compute the probability associated to each *risk level*

$\pi_A(\Omega) = M_A / M,\ \pi_S(\Omega) = M_S / M$

2) *Select the samples*

Fix $m_A = m/2$ and $m_S = m/2$

 Choose randomly m_A scenarios from subset Ω^{M_A} and m_S scenarios, Ω^{M_S}

 Compute the probability associated to each scenario $\omega \in \Omega^m$

 $\pi_{m_A}(\omega) = \pi_A / m_A, \pi_{m_S}(\omega) = \pi_S / m_S$

Figure 23. Hierarchical sampling procedure

The presented objective function of equation (1) is rewritten as the following model:

$$
\begin{aligned}
& Min \sum_{w \in W} \sum_{z \in Z} a_{wz}.Y_{wz} \\
& + \sum_{g=A,S} \breve{\pi}_{m_g}(\omega) \sum_{\omega \in \Omega^{m_g}} \left[\sum_{\tau \in T} \left(\sum_{v \in V} \sum_{p \in P} tc_{vp}.R_{vp\tau\omega} + \sum_{p \in P} \sum_{w \in W} tc_{pw}.R_{pw\tau\omega} + \sum_{w \in W} \sum_{w' \in W \setminus \{w\}} tc_{ww'}.R_{ww'\tau\omega} + \sum_{w \in W} \sum_{z \in Z} tc_{wz}.R_{wz\tau\omega} \right) \right. \\
& + \sum_{\tau \in T} \left(\sum_{p \in P} \sum_{w \in W} c_w.I^{-}_{pw\tau\omega} + \sum_{w \in W} \sum_{z \in Z} c_z.I^{-}_{wz\tau\omega} \right) \\
& + \sum_{\tau \in T} \left(\sum_{v \in V} \sum_{p \in P} \delta_{vp}.O_{vp\tau\omega} + \sum_{p \in P} \sum_{w \in W} \delta_{pw}.O_{pw\tau\omega} \right) \\
& \left. + \sum_{\tau \in T} \left(\sum_{w \in W} h_w.I^{+}_{w\tau\omega} \sum_{p \in P} h_p.I^{+}_{p\tau\omega} \right) \right]
\end{aligned}
\tag{29}
$$

Subject to:

Constraints (2) ï (6)

Constraints (7) ï (27) $\quad \forall \omega \in \Omega^m$

4.4.3 Statistical Gap

Let g_M^* be the optimal objective function value and let $\left(A_M^*, B_M^*\right)$ be the optimal solution vector of the proposed model with a scenarios sample of size *M*, where A_M^* and B_M^* correspond to the first stage design decisions and to the second stage decisions, respectively. The value of g_M^* converges to optimality as *M* tends towards infinity. Since finding the true optimal value g^* of the optimal solution $\left(A^*, B^*\right)$ is impossible due the extremely large required number of scenarios, in this section, we estimate statistical lower and upper bounds and compute the statistical gap for each size of scenario samples *m* from *M* scenario. This is done in order to qualify solutions produced depending on the scenario sample sizes used in proposed hierarchical sampling method. First, a valid statistical upper bound for the expectation of the optimal solution g^* of the actual stochastic problem can be estimated by averaging. This is obtained with solving m individual scenario samples based on independently *M* generated scenarios. Let g_M^m be the computed optimal objective function values of m scenario samples from M scenarios and $\left(A_M^m, B_M^m\right), m=1,\ldots,m$ be the corresponding. Therefore the objective average of these m scenario samples, denoted by $\bar{g}_{M,m}$, is an unbiased solutions vectors estimator of the upper bound for the expectation of optimal g^* which is given by:

$$\bar{g}_{M,m} = \frac{1}{m}\sum_{m=1}^{m} g_M^m \,. \tag{30}$$

Since the generated samples are independent and they have identical distributions, we can estimate the variance of $\bar{g}_{M,m}$ by :

$$\hat{\sigma}_{M,m}^2 = \frac{1}{m(m-1)}\sum_{m=1}^{m}\left(g_M^m - \bar{g}_{M,m}\right)^2 \tag{31}$$

Considering the average and variance of m scenario samples, we can apply an approximate 100(1-α)% confidence upper bound for the expectation of optimal g^* using

$$U_{M,m} = \bar{g}_{M,m} + \theta_{\alpha,m-1}\hat{\sigma}_{M,m}, \tag{32}$$

Where □ is the α -critical value of the *t*-distribution with m-1 degrees of freedom. Then, for estimating a statistical lower bound for the expected optimal g^*, the second-stage model will be solved based on a larger scenario sample of size m , denoted by m' ($\omega \in \Omega^{m'} \subset \Omega$) selected independently. In this case, the first-stage solution of the initial model with m scenario sample from M scenarios, denoted by $\bar{A}_M$ is an input. We denote by $\hat{g}_{m'}(\bar{A}_M, \hat{B}_{m'})$ the optimal objective function value of the second stage program. Obviously solving the second-stage model by a given first-stage solution would be easier. In addition, let $\hat{\sigma}_{m'}^2$ be an estimate of the variance of solutions $\hat{g}_{m'}(\bar{A}_M, \hat{B}_{m'})$ where the samples are selected independently. Then the sample m' variance is defined as follows,

$$\tilde{\sigma}^2_{m'} = \frac{1}{m'(m'-1)} \sum_{\omega=1}^{m'} \left(\tilde{g}_{\omega}(\bar{A}_M, \tilde{B}_{\omega}) - \tilde{g}_M(\bar{A}_M, \tilde{B}_{m'}) \right)^2 \qquad (33)$$

Having the mean and the variance of the second-stage model with a large scenario sample *mm*, we can compute an approximate 100(1-α)% confidence lower bound for the expectation of optimal g^* using

$$L_{m'} = \tilde{g}_{m'}(\bar{A}_M, \tilde{B}_{m'}) - z_{\alpha} \tilde{\sigma}_{m'} \qquad (34)$$

The statistical optimality gap is then computed by $gap_{M,m,m'} = U_{M,m} - L_{m'}$. The statistical optimality gap percentage is then calculated by:

$$gap_{M,m,m'}\% = \frac{gap_{M,m,m'}}{U_{M,m}} \times 100\%, \qquad (35)$$

4.5 Computational Experiments

In this section, we show the results of the proposed two-stage model through an extensive experimental investigation. The model is tested with several problem instances corresponding to various business contexts and different disruption characteristics. The purpose of these experiments is to identify the parameters that have significant impact on the model performance and on the *(*s_{lt}, S_{lt}*)* policy parameters. Another goal of this section is to inspect the implications of using the multiple-sourcing and lateral transshipment strategies in a multi-echelon distribution network within different disruption profiles. Moreover, we show the impact of different disruption types on the tactical and operational decisions by comparing the obtained solutions with the instances without disruptions. We also show the performance of our proposed solution methodology via the statistical gap.

The two-stage stochastic programming model and the hierarchical sampling technique are implemented using OPL Studio 12.8 and solved with CPLEX-12.8 using a MIP relative tolerance of 0.005. All the experiments are run on a 64-bit operating system server with a 2.7 GHz CPU on Intel(R) processor and 72 GB of RAM.

Furthermore, in order to discuss the added value of the proposed sourcing strategies (multi-sourcing and lateral transshipment) within the network (hereafter as MSLT), the solutions are compared to those obtained by a model with alternative distribution strategies where neither multi-sourcing nor lateral transshipment are used. This case is referred to a single sourcing (SS) case. Accordingly, each instance is also run for the inventory optimization model with a single-sourcing (SS) strategy. The SS model is presented in appendix (B).

4.5.1 Experimental Plan

In order to verify and validate the presented model, several problem instances are generated considering four dimensions: Capacity level $\{b^{low}, b^{high}\}$, Backorder cost level $\{c^{low}, c^{high}\}$, holding cost level $\{h^{low}, h^{high}\}$ and exposure level $\{E_{ignorance}, E_{low}, E_{high}\}$. We test different exposure levels including: low risk, high risk, risk ignorance level (referred hereafter as type 1, type 2 and base case, respectively). Actually by presenting the base case, we aim to show the potential benefits of disruption risk considering in planning. These four dimensions are solved with two different sourcing strategies, multi-sourcing with lateral transshipment (MSLT) and single sourcing (SS). Note that these experimental settings and parameters are consistent with the ones in the literature (see (Firouz et al., 2017, Cunha et al., 2018, Amiri-Aref et al., 2018)).

$$(i, j, f, k,) \middle| w \in \{b^{low}, b^{high}\}, x \in \{h^{low}, h^{high}\}, y \in \{c^{low}, c^{high}\}, z \in \{\mathrm{E}_{ignorance}, \mathrm{E}_{low}, \mathrm{E}_{high}\}$$

Since we are proposing an optimization approach for inventory management, it is mandatory to define initial inventory levels. In chapter 3, it has been shown that the initial stock could impact the behavior of the network.

The unit back-order cost (per day) in the three tested levels (c^{low}, c^{high}) are set to (1ú, 10ú), and the unit inventory holding cost (per day) in the two levels tested (h^{low}, h^{high}) are set to (0.01ú, 0.1ú), respectively. For all the instances, the replenishment lead times between the locations were fixed to one period. The unit flow costs ($tc_{ll'}$) were computed with the distance-based transportation cost function with the values $a_{ll'}^{tr}$ = 0.0432 and $\beta_{ll'}^{tr}$ = 0.0035 for the fixed and variable cost components, respectively. The parameter $Dis_{ll'}$ defines the distance between the site *l* and *l*X

$$tc_{ll'} = a^{tr}_{ll'}.Dis_{ll'} + \beta^{tr}_{ll'}$$

The unit flow cost ranges [0.04ú, 1.4ú] for the sourcing flows, [0.07ú, 1ú] for the transshipment flows, and [0.04ú, 2.4ú] for the outbound flows. The fixed procurement cost ranges [20ú, 50ú]. The fixed allocation costs ranges [300ú, 1000ú] per DC-CZ pair. The values for all these parameters are based on realistic parameter value ranges obtained from a case in the retail industry. Recall that the unit flow costs ($tc_{ll'}$) is computed by taking into account the transportation cost function parameters and the distances between the network sites. Demand of each CZ follows a normal distribution, with an average of 300 and variance of 50, in each period.

By testing two exposure levels, we aim to investigate the effect of various disruption scenarios on each problem setting. Based on available historical data for natural disasters, relevant information for these two event type can be derived. Inter-arrival times in a platform l are considered exponential and the severity of the disruption β_l is obtained from a function on a scale from 1 to 10. The arrival process parameter λ_l for the disruption types 1 and 2 is considered 19 and 35 (operational periods, two weeks), respectively. The severity of disruption types 1 and 2 would be uniformly generated in the intervals U (1, 7) and U (6, 10) respectively. Recall that each disruption type could have two different impacts on the network, capacity reduction and inventory loss. Inventory loss usually does not occur by disruption type 1.

Figure 24 shows an example of the impact of two different disruption types for one scenario and one DC. There is no inventory loss caused by disruption type 1; however there are two disruptions in 3rd and 15th operational periods. Since the severity is not serious, the capacity is recovered fast in both events.

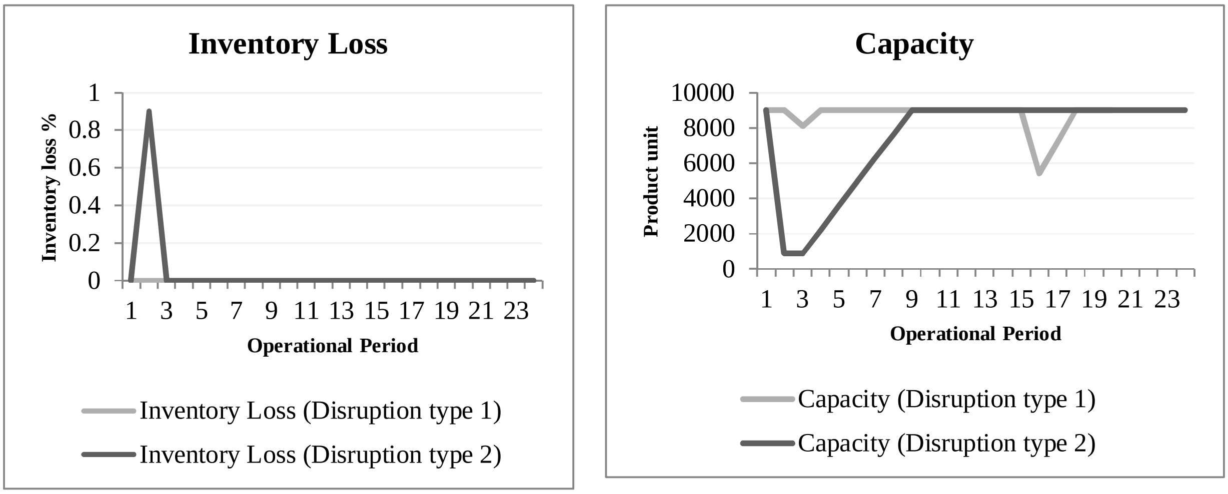


Figure 24. Capacity and inventory loss for one scenario and one DC

The combination of these four dimensions by two sourcing strategies yields 48 problem instances. A network including 10 CZ, 3 DCs and 1 PDC and 1 supplier is considered. A planning horizon covering a year was used, which includes 4 planning periods and 24 operational periods.

As we mentioned before, it is very difficult to solve to optimality the presented model for the entire set of scenarios. Therefore a number of sample sizes in hierarchical sampling are tested and their related statistical optimality gap values are computed. Two different sample sizes are chosen (10 and 20) through a 1000 generated scenarios and the related gaps are (2.76% and 1.95%), respectively computed by equation (35). The largest SAA problems that could be solved optimally, without truncating the solution optimality gap, is M = 1000 and m=20. The validation analysis shows that with this latter sample size, the SAA method provides satisfactory statistical optimality gaps (always less than 3%), which argues in favor of good-quality solutions. It is worth mentioning that because the planning horizon includes 24 periods, when m scenarios are used in the SAA model, 24*m instances are sampled from the probability distribution (i.e., 480). Thus, the multiplicity of scenarios and periods explains the low statistical gaps obtained, which is congruent with the findings of other stochastic problems proposed in the literature (Klibi, Martel, and Guitouni 2016). We have also tested larger scenario samples which provide models with more than 5 million decision variables and 500,000 constraints which cannot be solved using

CPLEX. The model with presented problem setting provides 272872 decision variables (including 6080 binary variables) and 114,160 constraints which could be solved in 12 hours on average with a MIP Relative Tolerance of 0.005.

4.5.2 Numerical Experiments

The given 48 problem instances are run and their results are provided hereafter. The expected total costs are computed with the objective function (29) for each instance using the same scenario samples.

An important aspect of a distribution network planning is its robustness towards the changes in the parameters of the network. In our parameter sensitivity experiments in this section, we investigated the effect of possible parameter changes on the total cost savings across the different sourcing options : MSLT and SS. The robustness of each one of these sourcing options towards the changes in the network parameters is investigated.

The numbers are calculated as: ($\frac{Expected\ Total\ Cost^{MSLT} - Expected\ Total\ Cost^{Base\ case}}{Expected\ Total\ Cost^{Base\ case}} \times 100$

and $\frac{Expected\ Total\ Cost^{SS} - Expected\ Total\ Cost^{Base\ case}}{Expected\ Total\ Cost^{Base\ case}} \times 100$

As expected the performance of the proposed model in large capacity instances with MSLT strategy is better against different types of disruption. When looking at the expected total cost, it is clear that the capacity is the most important parameter in this model. We notice that the model is sensitive to the variation of any factor in the instances with limited capacity. We observe that the MSLT setting has the least increase in total cost in all instances.

Table 11. Expected total costs gap (%) compared to the base case instances

Capacity Level	Backorder Level	Holding Level	Disruption Type/ Sourcing Strategy 1 MSLT	1 SS	2 MSLT	2 SS
Large	low	low	10.1	10.1	10.8	14.5
		high	8.5	9.4	8.8	10.5
	high	low	15.0	30.4	24.8	109.8
		high	19.3	22.7	19.2	85.3
Tight	low	low	21.1	58.9	59.6	71.3
		high	17.3	45.5	65.5	87.1
	high	low	22.0	60.2	66.4	114.8
		high	20.0	51.9	67.7	307.4

Essentially, as observed in theory and practice, the cost benefits follow the increasing of flexibility in the network. Additionally, we note that in all the tests, a large capacity results in a better overall total cost of the system, regardless of the disruption type.

However, the more interesting results are related to the relative difference between the sourcing strategies. In the base case instances, there is almost no difference between the MSLT and SS with large capacity (less than 1%). It can be seen that applying lateral transshipment and multi-sourcing significantly decreases the total cost, especially in the high backorder cost instances (31% and 240% in tight capacity instances). When a DC faces a disruption, the assigned customers have no alternatives, therefore the backorder cost is increased and consequently the expected total cost increases.

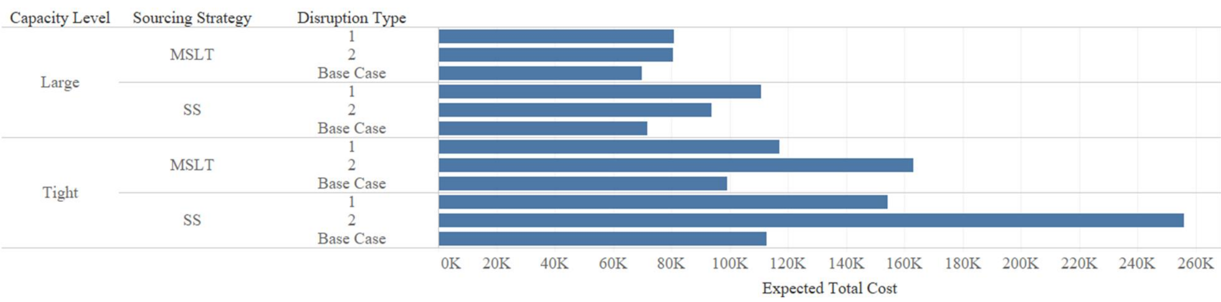


Figure 25. Expected total cost

Figure 25 demonstrates the total costs of all instances. The results are aggregated per backorder cost level and per holding cost level. In tight capacity instances, sourcing strategies plays an important role. Applying MSLT sourcing strategy could reduce the total cost up to 72%.

Table 12. Expected operational costs in detail

Capacity Level	Sourcing Strategy	Disruption Type	Transportation Cost	Holding Cost	Ordering Cost	Backorder Cost	Fixed Allocation Cost	Lateral Transshipment Cost
Large	MSLT	1	40 609	6 098	10 632	5 237	18 000	405
		2	44 218	6 117	10 595	4 194	15 000	396
		Base Case	40 422	6 350	10 314	48	12 000	820
	SS	1	42 695	6 855	10 867	38 535	12 000	
		2	43 540	7 376	10 560	20 308	12 000	
		Base Case	41 850	6 729	10 355	748	12 000	
Tight	MSLT	1	43 174	5 680	12 234	31 568	22 800	1 501
		2	42 936	11 262	11 902	71 756	24 000	1 110
		Base Case	41 654	3 931	11 378	28 434	12 600	1 132
	SS	1	43 563	7 971	12 803	77 816	12 000	
		2	42 842	11 016	9 462	180 684	12 000	
		Base Case	42 396	4 558	11 348	42 476	12 000	

Table 12 reports the expected operational costs in detail. The results are distinguished by the sourcing strategy, capacity level and disruption type. Note that the results are aggregated per backorder cost level and per holding cost level.

Transportation cost does not change significantly in this problem setting so we focus on the other operational costs. One of the mitigation technics in inventory management to deal with the disruption is to increase the stock level in the network (Snyder et al., 2015). Our experiments confirm this for rare and long disruptions (type 2). Itôs worth mentioning that in multi-echelon networks, the positioning of the stock depends on the vulnerability of the platforms. For example if a hazard hit a PDC, the inventory on-hand level in DCs would increase. Moreover, in the MSLT instances, the holding cost is less than in the related SS instances. This shows that an increasing of the network flexibility could decrease the stock all over the network, which confirms our results in chapter 3.

Furthermore, by looking at the fixed allocation cost, we clearly see that the model tends to use more DCs when the disruptions happen. In other words, when a platform l faces a disruption with a high severity $\boldsymbol{\beta}$, the whole network tries to activate the other potential DCs. This confirms the finding of Keskin et al. (2010).

Moreover, Table 12 shows that the lateral transshipment flows are activated more in tight capacity instances to avoid the potential backorder cost. The ordering cost is also more sensitive to the disruption in tight capacity instances.

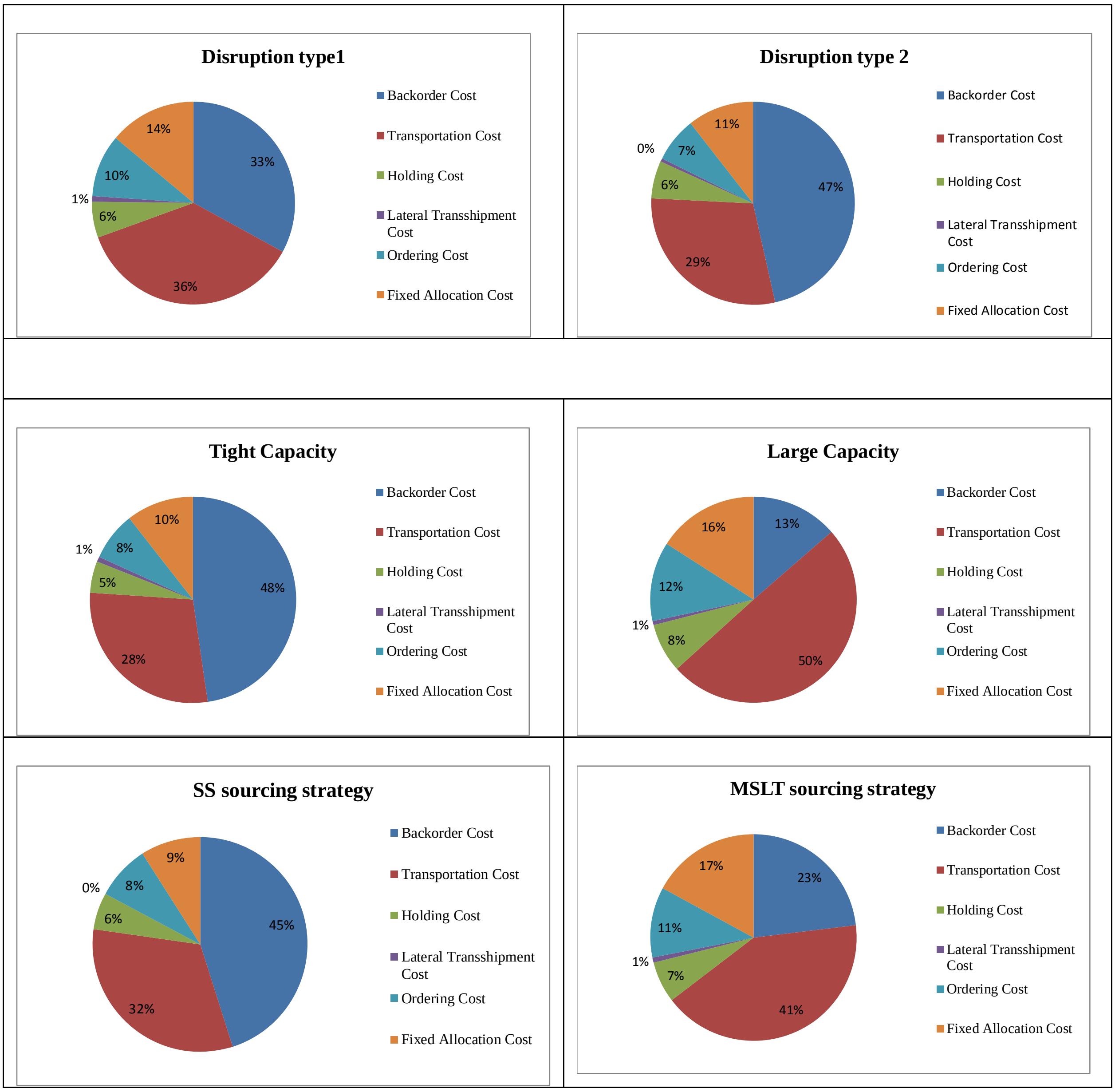


Figure 26. The contribution percentage of each cost factor in different instances

Figure 26 shows the contribution percentage of each cost factor in the expected total cost within each disruption type, capacity level and sourcing strategy for all scenarios.
As expected, when comparing disruption type 1 to 2, one observes a decrease in the contribution of the ordering cost in the expected total cost while giving a higher contribution to backorder costs. In both SS sourcing strategy and tight capacity instances, the contribution of backorder cost increases pointedly.

Note that the model applies multi-sourcing and lateral transshipment options with the aim to reduce the amount of backordered products, which leads to a better service level. Service level is an implicit performance indicator in the model and will be explicitly evaluated for the solutions produced to evaluate the capabilities of the model. It is considered as the percentage of satisfied demands from stock on-hand without backordering.

Table 13 represents the average percentage of the number of effective DC allocations per CZ. Each row presents the instance label, in which dash (-) denotes the average of all instances for the related attribute.

The results show that the number of effective DC allocations is sensitive to the disruption type and the capacity. More specifically, the stochastic model finds the best trade-offs between a high number of allocations and network cost minimization. In fact, the model tends to allocate more DCs to CZs within the disruption and when the throughput capacity is limited. This is confirmed by considering four instances with different disruption types and different capacities. The expected number of lateral transshipment orders (average from the scenarios) is reported in the last column. The results show that the number of lateral transshipment orders increases when in the tight capacity instances, which means that it is more profitable to use lateral transshipments instead of allocating more DCs to CZs.

Table 13. The Impact of capacity and disruption type on the Number of Allocations per CZ

Instance	Number of allocations per CZ %			Expected number of LT orders per DC
	1	2	3	
(b^{low}, - , - , *base case*)	90	10	0	7.46
(b^{high}, - , - , *base case*)	100	0	0	5.46
(b^{low}, - , - , *hazard 1*)	40	50	10	10.01
(b^{high}, - , - , *hazard 1*)	50	50	0	2.7
(b^{low}, - , - , *hazard 2*)	20	60	20	7.55
(b^{high}, - , - , *hazard 2*)	60	40	0	2.64

As we discussed before, two independent impacts on the network are considered in this study. Figure 27 indicates the expected inventory loss all over the network at all platforms. The number of stocks (inventory on-hand) under disruption type 1 is less than disruption type 2. Even though, the risk of inventory loss is more important under disruption type 2, the model tends to stock more, especially with SS sourcing policy. Since there is no flexibility in SS sourcing strategy, the only way to avoid the shortage is to increase the inventory on-hand level in the network.

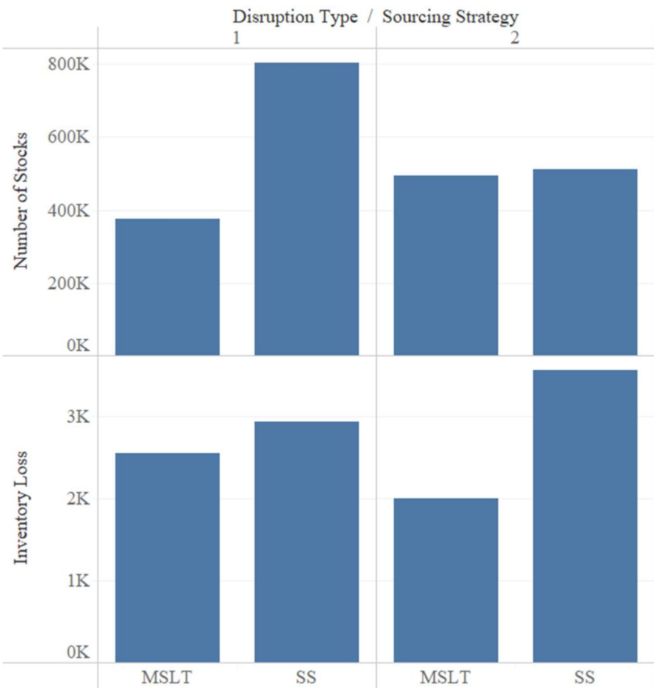
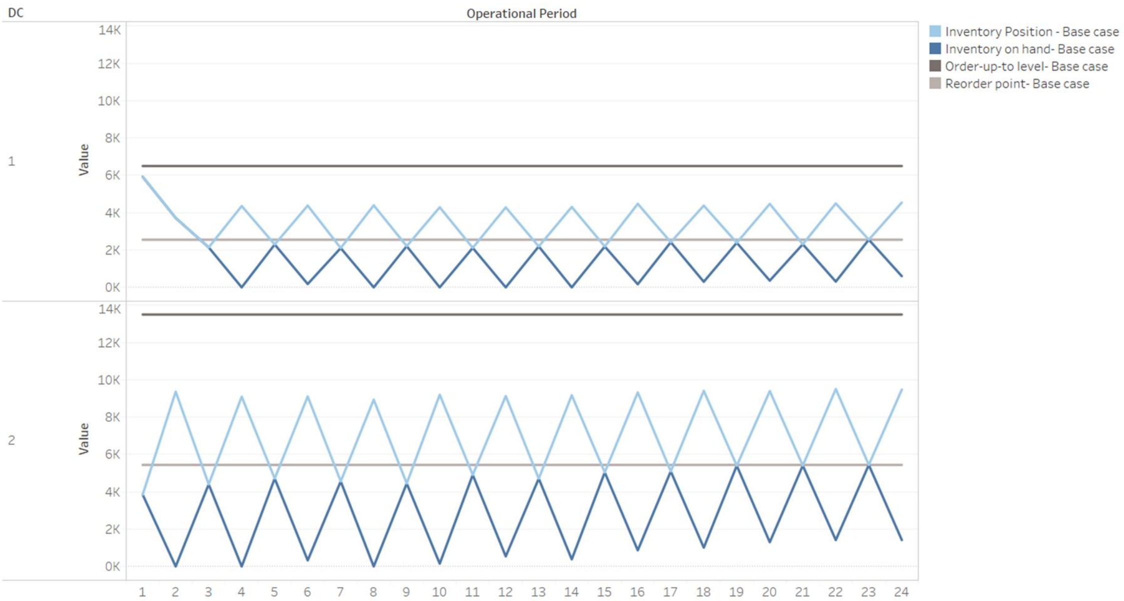


Figure 27. Inventory loss

In order to isolate the effect of disruption, we take one scenario from the experiments and analyze the effect of each disruption types on the (s_{lt}, S_{lt}) policy parameters. Figure 28 illustrates (s_{lt}, S_{lt}) policy behavior under disruption. Inventory position, inventory on-hand, reorders point and order up-to-level are shown for the base case and disrupted network for all DCs. We can see four different figures. In this instance only DC 1 is hit by disruption. The hazard arrives at period 7 and the duration of hit is 8 periods with an impact on the capacity (90%).

The Figure 28 (a) indicate the behavior of (s_{lt}, S_{lt}) policy for DC1 and DC2 respectively under a scenario without disruption. Since the number CZs allocated to DC2 is more than DC1, DC2 has the higher level of stock. The ordering process for both DCs is constant during the planning horizon. The reason for this constant behavior is the absence of uncertainty and disruption.

(a)



(b)

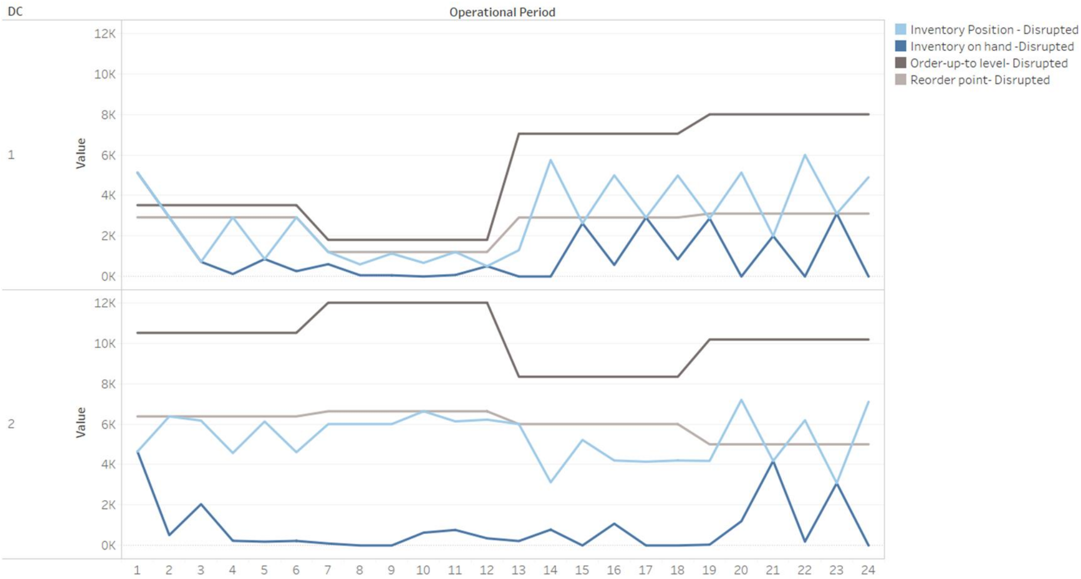


Figure 28. The impact of disruption on (*s*, *S*) parameters

As we can see in the Figure 28 (b), when disruption occurs in DC1, the optimized (s_{lt}, S_{lt}) would be different. Unlike the base case instance, the model will give different reorder point and order-up-to-level for each planning period (*t*). To have a better understanding of the impact of varying (s_{lt}, S_{lt}) policy parameter, we investigate separately each variable in Figure 29.

In Figure 29 (a), the inventory position of both DCs is shown within the throughput capacity in the planning horizon. The solution of each instance (base case and disrupted) is indicated with different colours. The inventory position is decreased due to the disruptions. Decreasing the inventory position could increase the number of orders by DCs. The inventory position increases at operational period 14, when the DC 1 has recovered its lost capacity.

In Figure 29 (b), the inventory on-hand of both DCs is illustrated. We can see that DC1 has lost 400 products by arriving the hazard. The model tends to stock less in both DCs, however, we will show that in the PDC, the stock level would be increased.

Figure 29.The impact of disruption on inventory decisions

It is obvious that stocking in DC1 is not a decent option due the major hit. This result approves the finding of (Schmitt and Snyder, 2012) which shows that the DCs with high exposure levels stock less than the other platforms.

In of Figure 29 (c), reorder point and order-up-to level of each DC is indicated. We observe in DC1, by happening the disruption, the reorder point increases and the order-up-level decreases. It allows DC1 to limit its order size so that DC1 can use the maximum available capacity. This is a very important point as a managerial point view. In other words, one the effective inventory strategy to deal with disruption is to decrease the order size and to order more frequently.

The PDC inventory decisions are shown in Figure 30. PDC also augments the order-up-to-level to be able to replenish the extra order by DC2. That means that in these instances the model increases its order size in the PDC level which leads to a higher stock level.

As the result, the inventory on-hand level would be decreased in DCs which do not necessarily decrease the holding cost because in the same time the PDC stocks more.

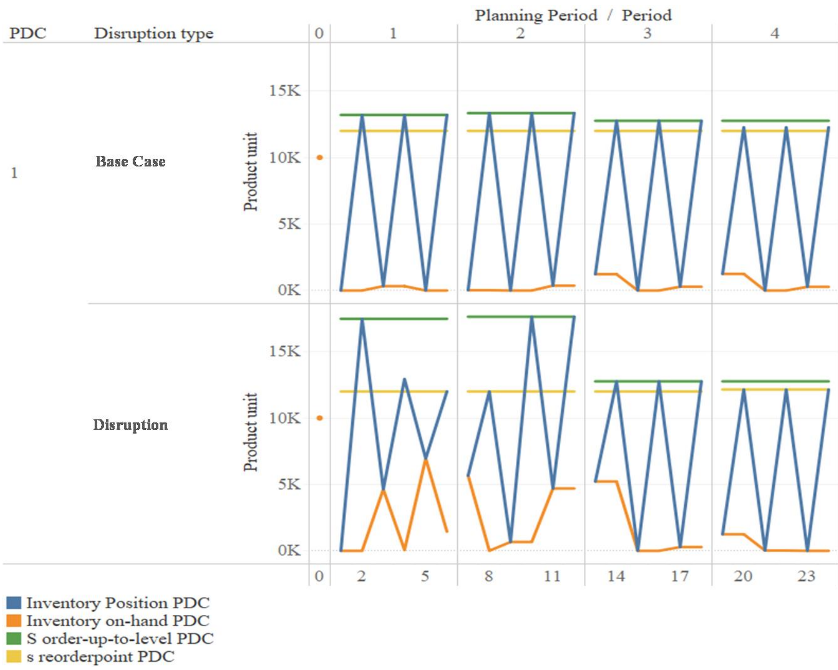


Figure 30. The impact of disruption on (s, S) parameters (PDC)

4.6 Conclusion

In this chapter we proposed a scenario-based stochastic two-stage model to solve a capacitated multi-echelon inventory optimization problem considering both demand uncertainty and disruption. When a platform is facing a disruption, the throughput capacity and the inventory on-hand will be affected and through put capacity decrease and/or inventory loss would be possible. The inventory is controlled according to a reorder point order-up-to-level (*s*, *S*) policy and lateral transshipments in the network were considered. The main challenge in this chapter was considering disruption occurrences and the related impacts in order to improve the quality of the inventory decisions.

A numerical experiment has been run for MSLT and SS model to show the effectiveness of the proposed approach. According to the results of the experiments, lateral transshipment and multi-sourcing significantly decrease the total cost, especially in the high backorder cost instances (up to 72%). The interesting insight was about the stock positioning in multi-echelon networks considering disruption. In multi-echelon networks, the positioning of the stock depends on the vulnerability of the platforms. For example if a hazard hit a PDC, the inventory on-hand level in DCs would increase.

Furthermore, the model tended to use more DCs when the disruptions happen. In other words, when a platform *l* faces a disruption with a high severity ***б***, the whole network tries to activate the other potential DCs. Also the lateral transshipment flows were activated more in tight capacity instances to avoid the potential backorder cost.

The results indicated that the stochastic model tried to find the best trade-offs between a high number of allocations and network cost minimization. In fact, the model allocated more DCs to CZs within the disruption and when the throughput capacity is limited.

The most important limitation of the proposed model is the solvability. Considering the complexity of the defined problem the proposed modeling approach could not solve the big size problems with exact solutions.

Chapter 5. Conclusions and Perspectives

Conclusions and Research Perspectives

Multi-echelon inventory systems are challenging systems to analyze when compared to single echelon systems. The optimal inventory policy, and the allocation policy if required, is still unknown for many different types of multi-echelon inventory systems. Our work contributes to the research on optimizing multi-echelon supply chain planning under supply and demand uncertainties.

The thesis has two major research topics:

(i) Optimizing multi-echelon distribution networks operating under DRP policy considering demand uncertainty

(ii) Optimizing multi-echelon distribution networks operating under (s, S) policy considering disruptions

In chapter 3 we proposed a modeling and a solution approach for a multi-echelon inventory optimization problem under non-stationary demand. Lateral transshipment and multiple sourcing have been considered to improve the performance of the distribution network. A two-stage stochastic multi-echelon inventory optimization model is developed and run on different numerical instances and also on real data coming from a major French retailer. We have examined the tactical implications of the multiple sourcing and the lateral transshipment strategies on the distribution network. We have shown substantial savings obtained using the MSLT and the SSLT strategies, which can go up to 23.6% and 21%, respectively. The results also show that a combination of the lateral transshipment and the multiple-sourcing strategies leads to a considerable improvement of the service level, which can reach 6% when compared to that of the baseline SS strategy. However, the magnitude of the benefits of lateral transshipments and multiple-sourcing depends on the network flow capacity and the unit costs. Proactive lateral transshipments can help managers to reduce the expected total cost especially when the unit back-order cost is high. Another important finding in this contribution is that the expected total cost of the proposed MSLT strategy is not very sensitive to the lateral transshipment and inventory holding unit costs.

In chapter 4 we developed a modeling and a solution approach for a multi-echelon inventory optimization problem under demand uncertainty and disruption. Inventory and sourcing decisions have been considered as the mitigation strategies to deal with such uncertainty. In order to efficiently handle the disruption and demand uncertainty, a two-stage stochastic multi-echelon inventory optimization model is developed to optimize (s, S) policy parameters and run on different numerical instances. Two different impacts of the disruption (capacity and inventory loss) are modeled. The results of this chapter demonstrate that disruption could have significant negative impact if it has not been considered in the planning. Our results show that proactive sourcing planning and large capacity can reduce the expected total cost up to 72% in case of disruptions.

Based on the above declarations, the opportunities and limitations described in the previous chapter, it has become clear that there is still some room left for further research.

The proposed approach currently only focuses on measuring operational and tactical performances. Further research might be aimed at figuring out a method to consider the strategic level. This would be very interesting since considering different types of uncertainties could affect significantly the strategic decisions such as the locations, supplier selection, etc. Sourcing options also could influence the strategic decisions like the topology of the network. In fact, in this thesis we have developed our approach by considering only the tactical level of the multi-echelon distribution network, so the DC locations are fixed. Obviously, if a proactive lateral transshipment strategy is considered in the multi-echelon distribution network design, higher benefits could be reached.

Finally, the formulated models become more difficult to solve with larger samples and larger number of periods, which raises the need for the elaboration of decomposition methods or heuristic approaches.

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Appendix A

Table 14. Results of MSLT model

Instance's Label	Total Expected Cost	Transportation Cost	Backorder Cost	Procurement Cost	Holding Cost
$(SN, Cap^{high, low}, h^{low})$	77710	54 858	15412	7440	498
$(SN, Cap^{high, medium}, h^{low})$	90727	58 071	25216	7440	1 400
$(SN, Cap^{high, high}, h^{low})$	196965	54925	61680	80360	898
$(SN, Cap^{high, low}, h^{high})$	76439	54 891	13888	7660	4343
$(SN, Cap^{high, medium}, h^{high})$	90327	54 891	27 776	7660	11976
$(SN, Cap^{high, high}, h^{high})$	205301	54 891	69 440	80970	82
$(SN, Cap^{low, low}, h^{low})$	87539	56257.3	23391.2	7890	84.421
$(SN, Cap^{low, medium}, h^{low})$	111370	56257.3	46782.4	8330	97
$(SN, Cap^{low, high}, h^{low})$	252094	56296	109018	86780	120
$(SN, Cap^{low, low}, h^{high})$	89482	57505	24008	7970	795
$(SN, Cap^{low, medium}, h^{high})$	111884	56551	46833	8500	664
$(SN, Cap^{low, high}, h^{high})$	260936	56590	114956	89390	685
$(MN, Cap^{high, low}, h^{low})$	474823	418802	38581	17440	1331
$(MN, Cap^{high, medium}, h^{low})$	612547	425802	169305	17440	1631
$(MN, Cap^{high, high}, h^{low})$	729795	418802	291793	19200	1672
$(MN, Cap^{high, low}, h^{high})$	458823	405879	34454	18490	6921
$(MN, Cap^{high, medium}, h^{high})$	602507	412881	171136	18490	6921
$(MN, Cap^{high, high}, h^{high})$	737625	418879	298396	20350	8921
$(MN, Cap^{low, low}, h^{low})$	526461	422806	85165	18490	9001.23
$(MN, Cap^{low, medium}, h^{low})$	869176	432902	417615	18660	9701
$(MN, Cap^{low, high}, h^{low})$	1163295	422806	719749	20740	9701
$(MN, Cap^{low, low}, h^{high})$	517803	410884	87319	19600	31206
$(MN, Cap^{low, medium}, h^{high})$	605494	415869	169845	19780	26066
$(MN, Cap^{low, high}, h^{high})$	1221877	425799	774098	21980	26066
$(LN, Cap^{high, low}, h^{low})$	1086398	898400	161328	26670	1659.61
$(LN, Cap^{high, medium}, h^{low})$	1277726	898400	352656	26670	1659.61
$(LN, Cap^{high, high}, h^{low})$	1807710	898400	881640	27670	1659.61
$(LN, Cap^{high, low}, h^{high})$	1102894	898566	176328	28000	15520
$(LN, Cap^{high, medium}, h^{high})$	1306162	908566	370656	26940	15520
$(LN, Cap^{high, high}, h^{high})$	1808156	898566	881640	27950	15520

Instanceôs Label	Total Expected Cost	Transportation Cost	Backorder Cost	Procurement Cost	Holding Cost
$(LN, Cap^{low, low}, h^{low})$	1362410	941670	392470	28270	13363.2
$(LN, Cap^{low, medium}, h^{low})$	2409160	941670	1438950	28540	13363.2
$(LN, Cap^{low, high}, h^{low})$	3818920	941670	2847370	29880	13363.2
$(LN, Cap^{low, low}, h^{high})$	1402644	959379	413295	29970	96359
$(LN, Cap^{low, medium}, h^{high})$	2491737	945905	1515302	30530	107945
$(LN, Cap^{low, high}, h^{high})$	3976523	945798	2998455	32270	111405

Appendix B

SS model

According to the mentioned notations in chapter 4, the SS model is formulated as follow:

$$
\begin{aligned}
& Min \sum_{w\in W}\sum_{z\in Z} a_{wz}.Y_{wz} \\
& +\sum_{\omega\in\Omega} p(\omega)\left[\sum_{\tau\in T}\left(\sum_{v\in V}\sum_{p\in P} tc_{vp}.R_{vp\tau\omega} + \sum_{p\in P}\sum_{w\in W} tc_{pw}.R_{pw\tau\omega} + \sum_{w\in W}\sum_{z\in Z} tc_{wz}.R_{wz\tau\omega}\right)\right. \\
& +\sum_{\tau\in T}\left(\sum_{p\in P}\sum_{w\in W} c_w.I^-_{pw\tau\omega} + \sum_{w\in W}\sum_{z\in Z} c_z.I^-_{wz\tau\omega}\right) \\
& +\sum_{\tau\in T}\left(\sum_{v\in V}\sum_{p\in P}\delta_{vp}.O_{vp\tau\omega} + \sum_{p\in P}\sum_{w\in W}\delta_{pw}.O_{pw\tau\omega}\right) \\
& \left.+\sum_{\tau\in T}\left(\sum_{w\in W} h_w.I^+_{w\tau\omega}\sum_{p\in P} h_p.I^+_{p\tau\omega}\right)\right]
\end{aligned} \tag{36}
$$

$$\sum_{w\in W} Y_{wz} = 1 \qquad z\in Z \tag{37}$$

$$\sum_{z\in Z} F_{wz\tau\omega} \leq b_{w\tau\omega} \qquad \forall w\in W, \tau\in T, \omega\in\Omega \tag{38.1}$$

$$\sum_{p\in P} R_{pw\tau\omega} \leq b_{w\tau\omega} \qquad \forall w\in W, \tau\in T, \omega\in\Omega \tag{38.2}$$

$$I^+_{w\tau\omega} = (1-\zeta_{w\tau\omega}).I^+_{w,(\tau-1),\omega} + \sum_{p\in P} R_{wp\tau\omega} - \sum_{z\in Z} F_{wz\tau\omega} \qquad \forall w\in W, \tau\in T, \omega\in\Omega \tag{39}$$

And constraints:

(2-7), (9), (11-12), (14-16), (18-25).

The objective function in (36) minimizes the expected total in the network. The main difference between (36) and (1) is the removal of the lateral transshipment cost term. Constraint (37) sets the single sourcing requirements. It enforces the model to assign a unique source for each customer. Constraints (38.1) and (38.2) guarantee the respect of the capacity and the procurement decisions for each platform. Constraint (39) indicates the inventory on hand in the DCs by balancing the flows-in and flows-out of in each center, period and scenario.