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Comments on "Effects of Variable Viscosity and Thermal Conductivity on Unsteady MHD Flow of Non-Newtonian Fluid over a Stretching Porous Sheet" by Gamal M. Abdel-Rahman

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by

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Short paper
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Several fundamental errors listed below were found in the paper of G. M. Abdel-Rahman.

(1) Large parts of the Introduction and list of References were copied from the published paper by Prasad and Vajravelu [1].
(2) Due to the coping from the published paper by Prasad and Vajravelu [1], the author mentioned in the Introduction that "the resulting equations are solved numerically by the Keller-Box method." While the equations solved numerically by using the sixth order Runge-Kutta integration accompanied with the shooting iteration scheme (see Numerical computations section).
(3) The physical meaning of ε should be the thermal conductivity parameter, not small parameter as defined in Nomenclature .
(4) Although the main aim of the above paper is to study the effects of variable viscosity (in spite of the author mentioned the effect of the variable viscosity parameter in Conclusion section, no viscosity parameter defined in the paper) and thermal conductivity (the thermal conductivity parameter $\varepsilon = 0$ in all figs. (2)-(12)), no discussions for these effects have been presented in this work.
(5) The first term in the right hand side of eq. (2) "$-(1/\rho)(\partial/\partial y)[-\mu(x, t)(\partial u/\partial y)]^n$" is wrong but should be "$-(1/\rho)(\partial/\partial y)[\mu(x, t)(-\partial u/\partial y)]^n$".
(6) The fourth term in the right hand side of eq. (2) "$-(\nu/K)u$" is wrong but should be "$-(\mu\phi/\rho K)u^n$" where ϕ is the medium porosity (see [2]).
(7) The fourth term in the right hand side of eq. (3) "$1/\rho c_p[\mu(x, t)(\partial u/\partial y)]^{n+1}$" is wrong but should be "$[\mu(x, t)/\rho c_p](\partial u/\partial y)^{n+1}$".
(8) The similarity variable η is wrong (because η has units [ms^{-2}] but should be dimensionless).

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(9) The dimensionless stream function $f(\eta)$ is not in dimensionless form as the author mentioned but has units [s^2m^{-1}].Therefore $f(\eta)$ is also wrong.
(10) The mass transfer parameter R is wrong (because R has units [ms^2]) .
(11) $\theta(\eta)$, Nu, β, J, G, Ec, S, Pr, and k_0 are not correct because T_0 is not defined in the paper.
(12) v is not defined in the paper.
(13) $\Delta T = T_w - T_\infty$ defined in Nomenclature as sheet temperature. This definition is wrong and should be the difference between the temperature at the sheet and the temperature at infinity.
(14) From eqs. (5) and (9) the boundary condition $\theta(0) = 1$ is wrong and should be:

$$\text{“}\theta(0) = \frac{T_w - T_\infty}{T_0 bx\left(\dfrac{1}{1-\gamma t}\right)} = \frac{A_l\left(\dfrac{x^2}{l^2}\right)\dfrac{1}{1-\gamma t}}{T_0 bx\dfrac{1}{1-\gamma t}} = k_0 \neq 1\text{”}$$

Note that $k_0 = 0.4$ in figs. (2)-(12).
(15) μ is the consistency coefficient not the kinematic viscosity of the power law as defined in Nomenclature.
(16) K is the permeability of the porous medium not consistency coefficient as defined in Nomenclature .
(17) $B(x, t)$, $\beta(x, t)$, $Q(x, t)$, and $\Im(x, t)$ are independent on x not as the author mentioned that they are depends on x.
(18) The wall skin-friction defined in the above paper as [1]: "$-(-\mu(x, t)\partial u/\partial y)^n_{y=0}$" is wrong and should be "$-\mu(x, t)(-\partial u/\partial y)^n_{y=0}$.
(19) Even the fundamental equations (2) and (3) , η, $f(\eta)$, and $\theta(\eta)$ in eq. (9) are correct, eqs. (10)-(12) are wrong and should be:

$$\frac{(-\mu_0)^n}{\rho} nf'''(f'')^{n-1} - (M+S)f' + ff'' - f'^2 - A(f' + 2\eta f'') + G\theta = 0$$

$$(k_0 + \delta + \varepsilon\theta)\theta' + \varepsilon\theta'^2 + \beta\theta - \Pr A(\theta + 2\eta\theta') + \Pr(f\theta' - \theta f') + Jf'^2 + \mathrm{Ec}(f'')^{n+1} = 0$$

with the boundary conditions:

$$f' = 1, \quad f = R, \quad \theta = k_0, \quad \text{at} \quad \eta = 0$$

$$f' \to 0, \quad \theta \to 0, \quad \text{as} \quad \eta \to \infty$$

where

$$k_0 = \frac{A_1 x}{bl^2 T_0}, \quad \Pr = \frac{A_1 x\rho c_p}{b^4 l^2 T_0 \alpha_\infty}, \quad A = \frac{\gamma}{b}, \quad S = \frac{S_0}{b}, \quad G = \frac{gT_0\beta_0}{b}, \quad M = \frac{\sigma B_0^2}{\rho b}$$

$$\delta = \frac{16\sigma^* T_\infty^3 A_1 x}{3\kappa^* bl^2 T_0^2 \alpha_\infty (1-\gamma t)^3}, \quad \beta = \frac{Q_0 A_1 x}{b^5 l^2 T_0 \alpha_\infty}, \quad J = \frac{\sigma B_0^2 x^2 A_1}{b^4 l^2 T_0^2 \alpha_\infty (1-\gamma t)},$$

$$\mathrm{Ec} = \frac{(\mu_0)^{n+1}(1-\gamma t)^2 x^{n+1/n} A_1}{b^6 l^2 T_0 \alpha_\infty}$$

(20) If the errors in the fundamental eqs. (2) and (3) are typing errors of the author with $\eta, f(\eta)$ and $\theta(\eta)$ in eq. (9) are correct, eqs. (10)-(12) are wrong and should be:

$$\frac{(-\mu_0)^n}{\rho} nf'''(f'')^{n-1} - (M+S)f' + ff'' - f'^2 - A(f' + 2\eta f'') + G\theta = 0$$

$$(k_0 + \delta + \varepsilon\theta)\theta' + \varepsilon\theta'^2 + \beta\theta - \Pr A(\theta + 2\eta\theta') + \Pr(f\theta' - \theta f') + Jf'^2 + \mathrm{Ec}(f'')^{n+1} = 0$$

with the boundary conditions:

$$f' = 1, \quad f = R, \quad \theta = k_0, \quad \text{at} \quad \eta = 0$$
$$f' \to 0, \quad \theta \to 0, \quad \text{as} \quad \eta \to \infty$$

where

$$k_0 = \frac{A_1 x}{bl^2 T_0}, \quad \Pr = \frac{A_1 x \rho c_p}{b^4 l^2 T_0 \alpha_4}, \quad A = \frac{\gamma}{b}, \quad S = \frac{S_0}{b}, \quad G = \frac{gT_0\beta_0}{b}, \quad M = \frac{\sigma B_0^2}{\rho b}$$

$$\delta = \frac{16\sigma^* T_\infty^3 A_1 x}{3\kappa^* bl^2 T_0^2 \alpha_\infty (1-\gamma t)^3}, \quad \beta = \frac{Q_0 A_1 x}{b^5 l^2 T_0 \alpha_\infty}, \quad J = \frac{\sigma B_0^2 x^2 A_1}{b^4 l^2 T_0^2 \alpha_\infty (1-\gamma t)},$$

$$\mathrm{Ec} = \frac{(\mu_0)^2 (1-\gamma t)^2 x^{n+1/n} A_1}{b^6 l^2 T_0 \alpha_\infty}$$

(21) The values of k_0 and ε are not defined in tabs. 2 and 3 .

(22) At the same time $\beta(x, t)$ is defined as heat source/sink parameter and as the expansion coefficient of temperature in Nomenclature and eq. (5), respectively.

(23) However, the transformed equations and the boundary conditions (9)-(12) (in dimensional form) are wrong. These equations must be in non-dimensional form.

(24) Prasad and Vajravelu [1] compared their results with those obtained by Andersson *et al.* [3] in the absence of the magnetic field ($M = 0$) and permeable stretching sheet ($R = 0$). In this case the momentum equation takes the following form:

$$nf'''(f'')^{n-1} + \left(\frac{2n}{n+1}\right) ff'' - f'^2 = 0$$

with the boundary conditions:

$$f' = 1, \quad f = 0, \quad \text{at} \quad \eta = 0,$$
$$f' \to 0, \quad \text{as} \quad \eta \to \infty$$

Gamal cited in *Results and discussion* section that "In order to verify the accuracy of our present method, we have compared our results with those of Prasad and Vajravelu [1] and Andersson *et al.* [3]. Table 1 shows the values of $-f''(0)$ for various values of n. The comparisons in all above cases are found to be excellent and agreed, also, the results are found to be similar to Prasad and Vajravelu [1] and Andersson *et al.* [3], so it is good".

This comparison is false because the second term of the momentum equation given by Gamal is ff'' (see eq. (10) with $A = M = S + G = R = 0$) not $(2n/n + 1)ff''$ as written in Andersson *et al.* [3] and Prasad and Vajravelu [1] .

In regards to the above arguments it is obvious that the mathematical analysis and the obtained results are also wrong.

References

[1] Prasad, K. V., Vajravelu, K., Heat Transfer in the MHD Flow of a Power Law Fluid over a Non-Isothermal Stretching Sheet, *Int. J. Heat and Mass Transfer, 52* (2009), 21-22, pp. 4956-4965
[2] Shenoy, A.V., Non-Newtonian Fluid Heat Transfer in Porous Media, *Adv. Heat Transfer, 24* (1994), pp. 101-190
[3] Andersson, H. I., *et al.*, Magnetohydrodynamic Flow of a Power Law Fluid over a Stretching Sheet, *Int. J. Nonlinear Mech., 72* (1992), 4 pp. 929-936

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