Type-Based Resource-Guided Search

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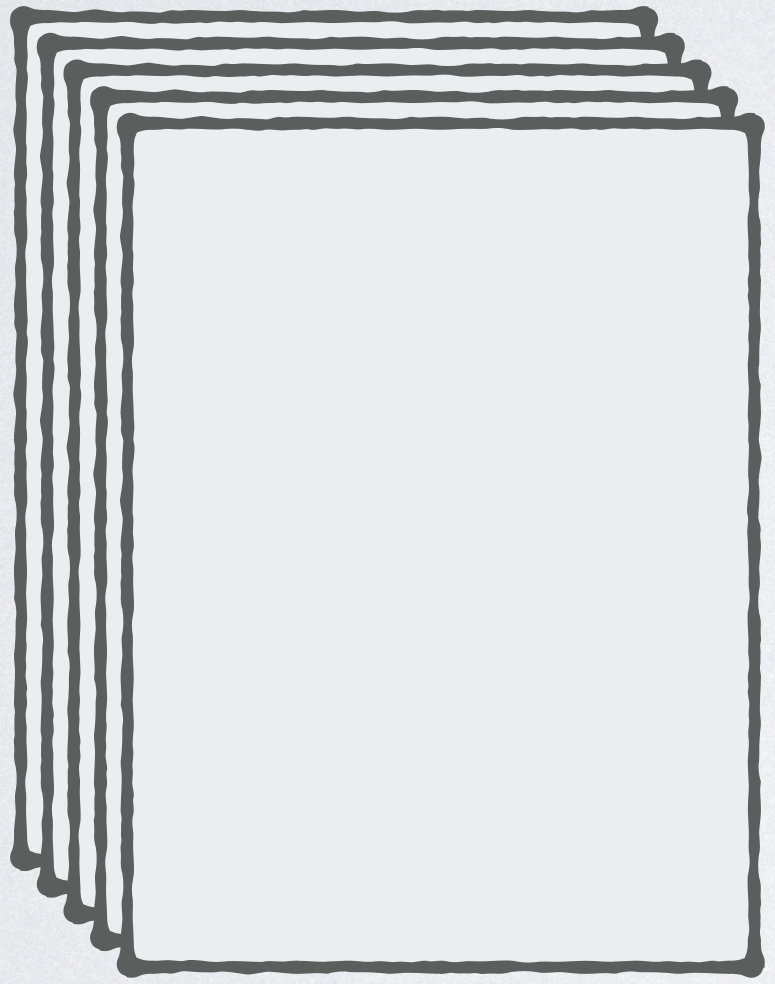
Carnegie Mellon University

About Me

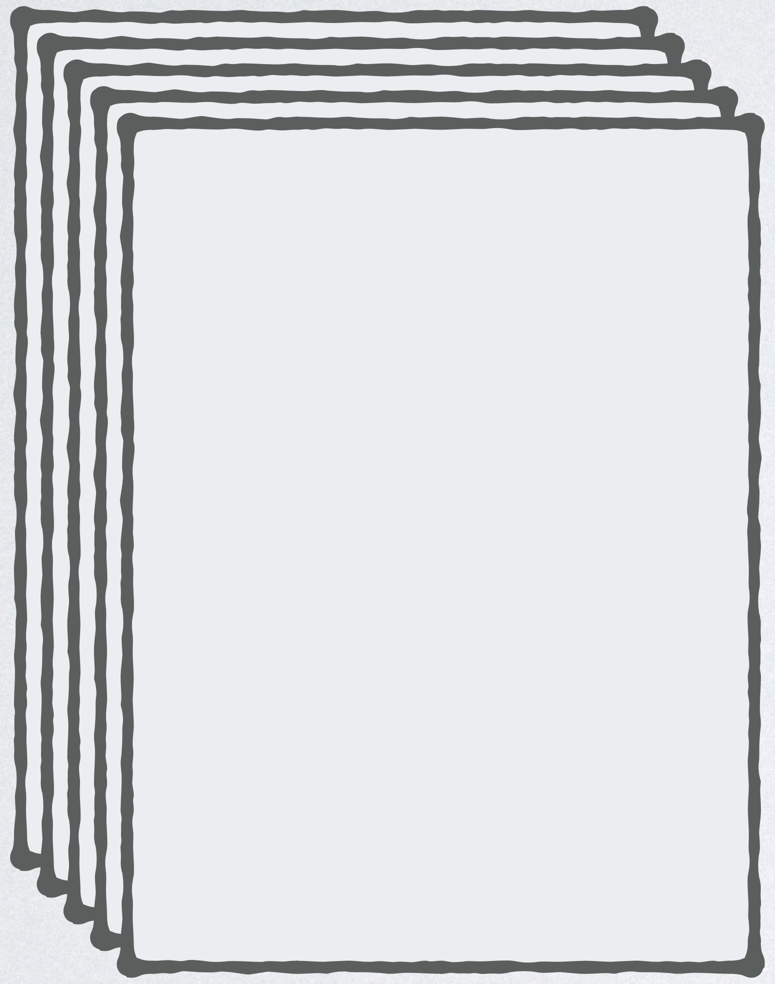
- I am a doctoral student at Carnegie Mellon University.
- I am interested in programming languages and software engineering.
- My focuses are **probabilistic programming** and **static resource analysis**.

Resource Analysis



Programs

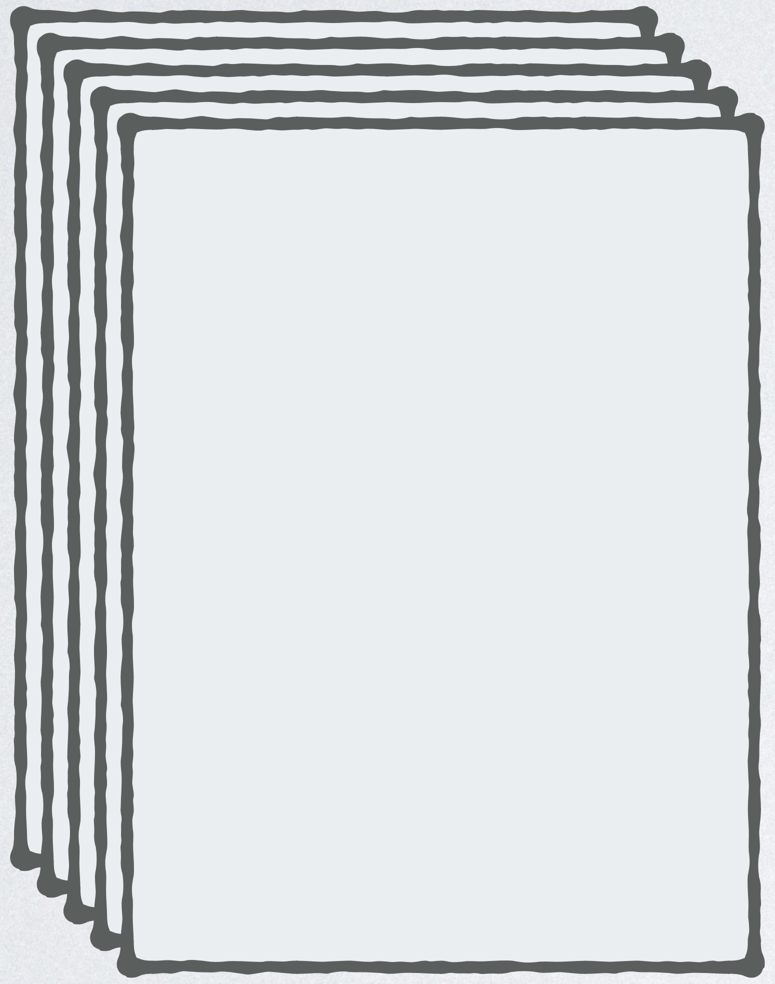
RESOURCE ANALYSIS



Programs

Performance

Resource Analysis



- Time
- Memory
- Power
- …

Programs

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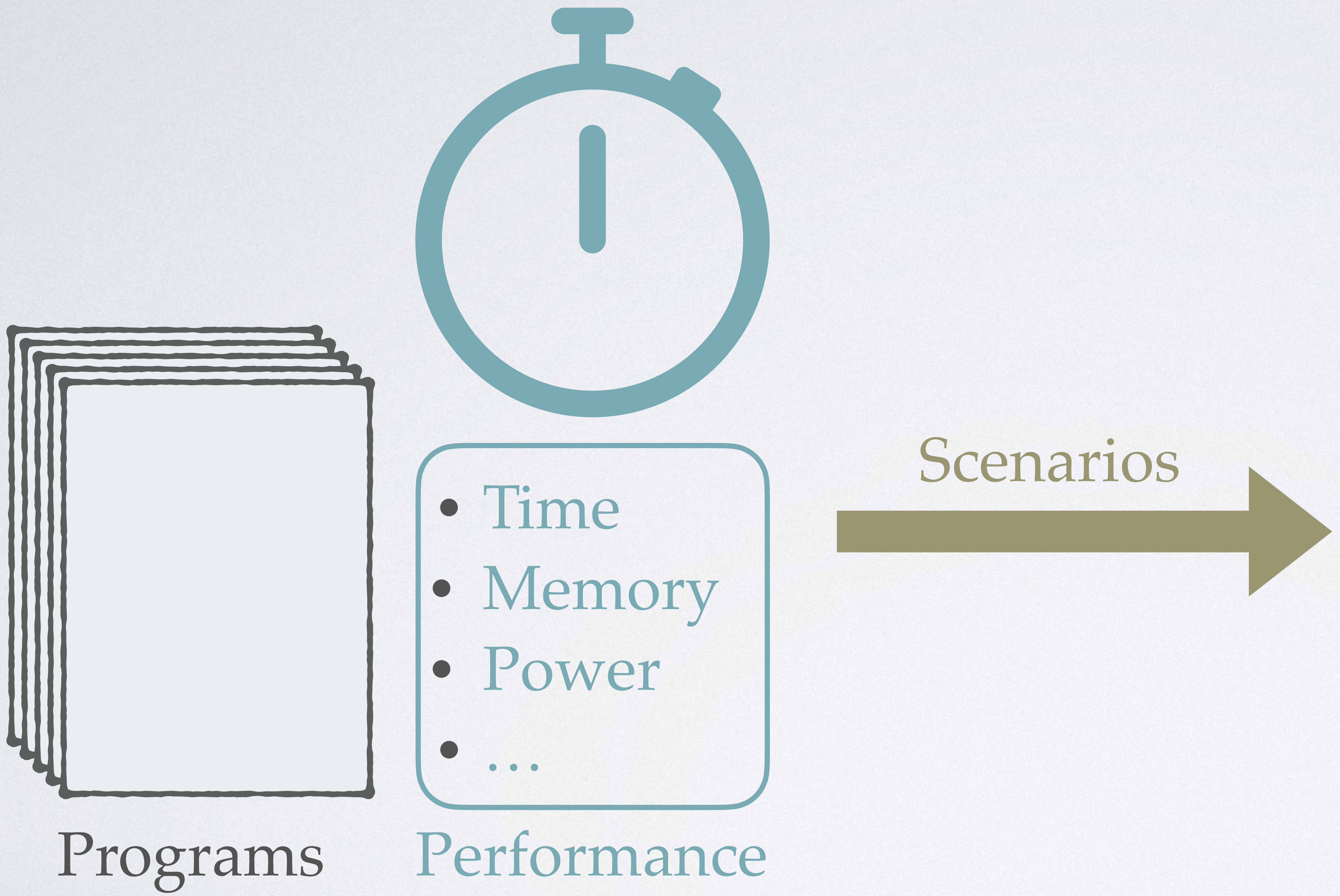

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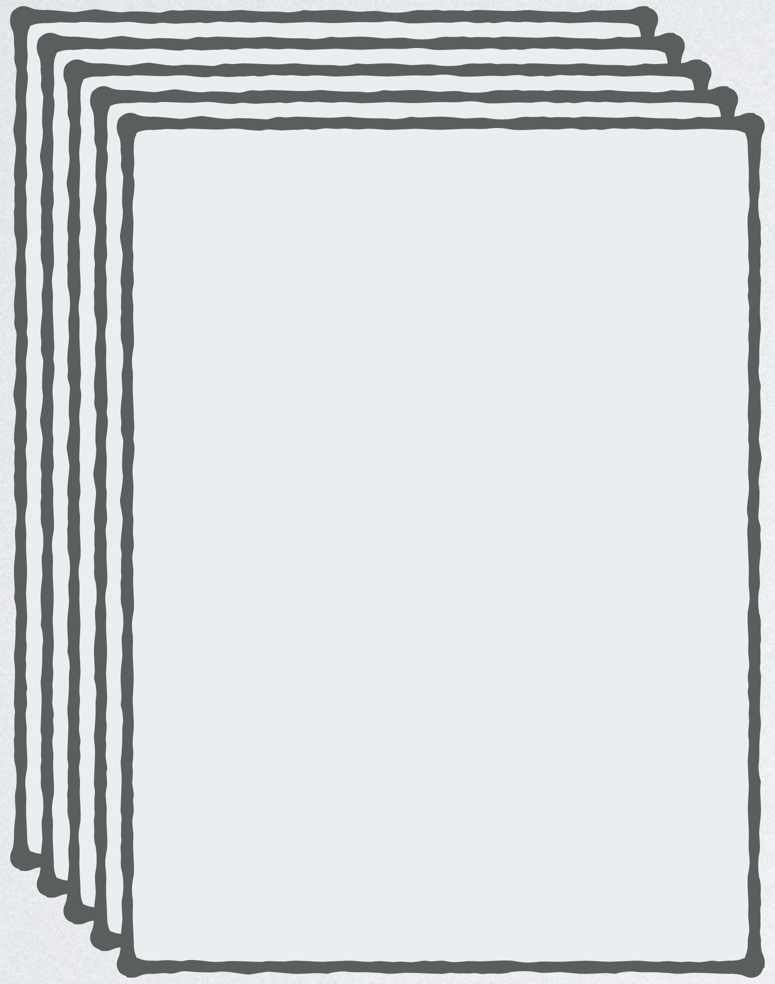

Programs

- Time
- Memory
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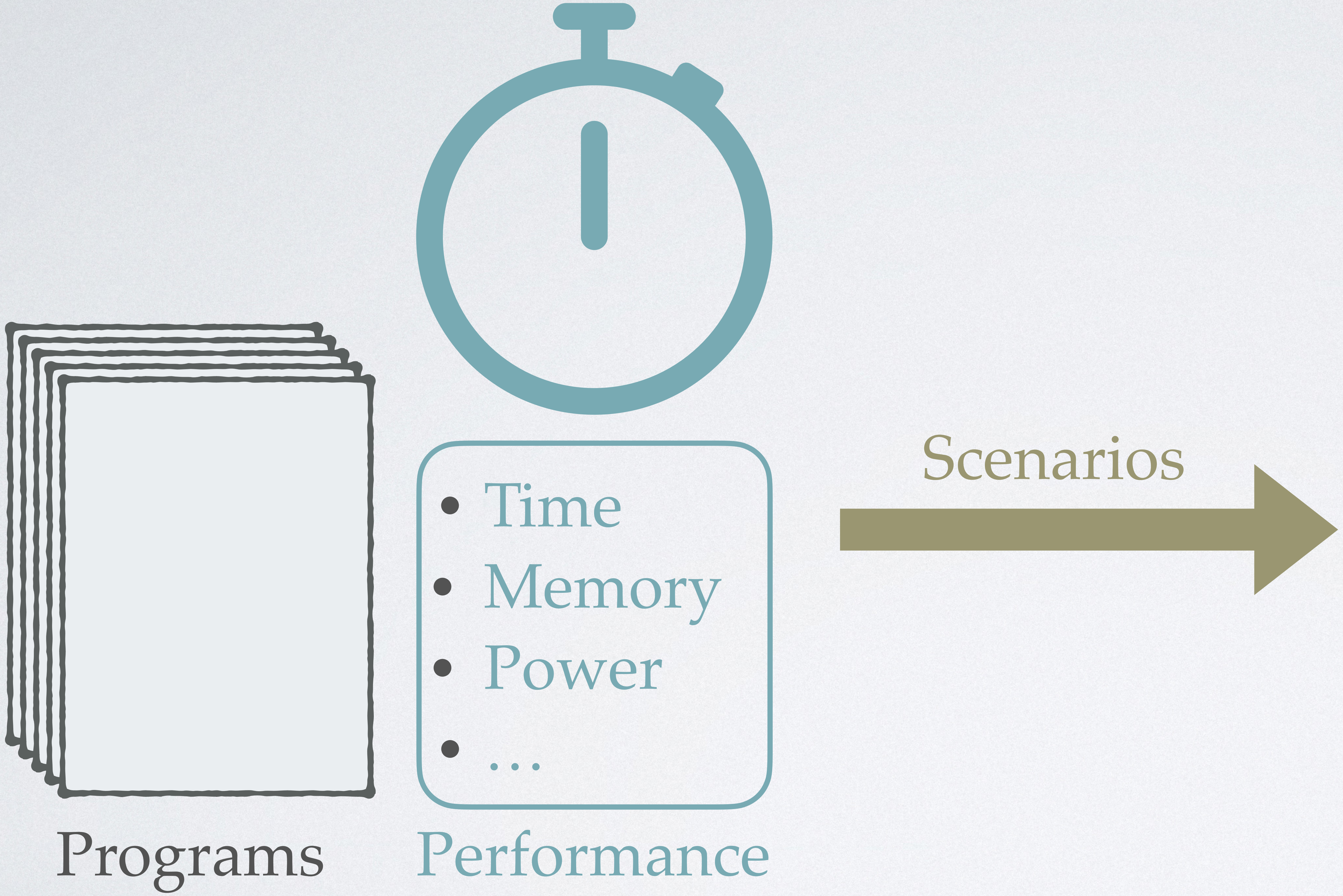
- …

Performance

Scenarios

- Identifying bottlenecks

Resource Analysis



- Identifying bottlenecks
- Timing side channels

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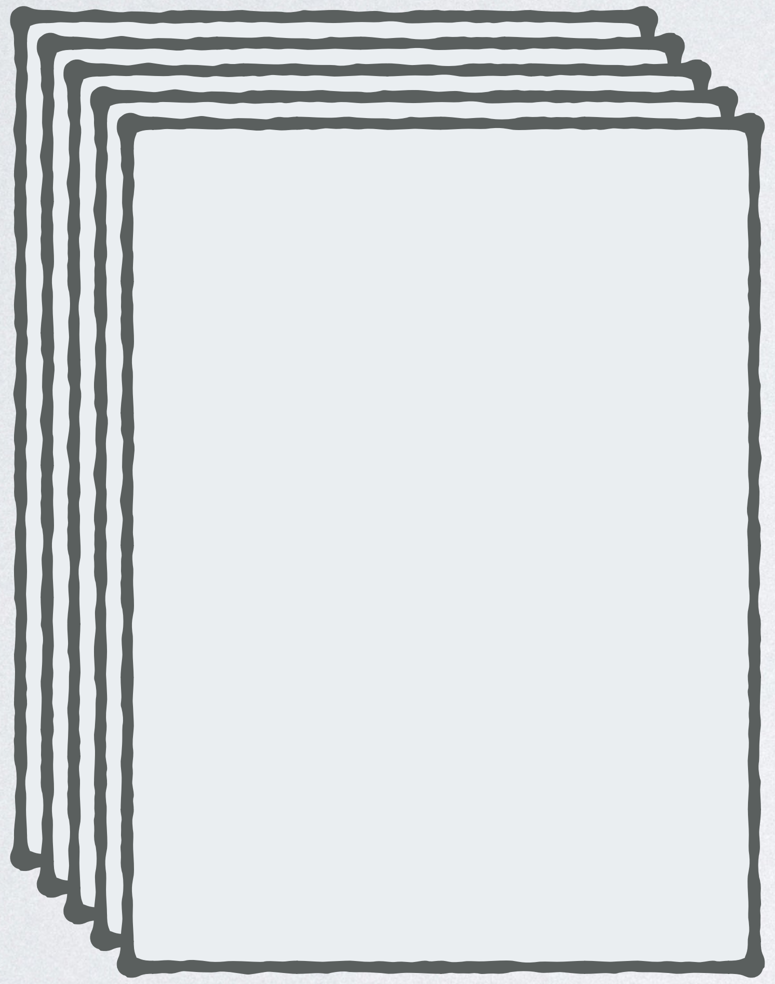

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Programs

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Scenarios →

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- Gas usage in blockchains

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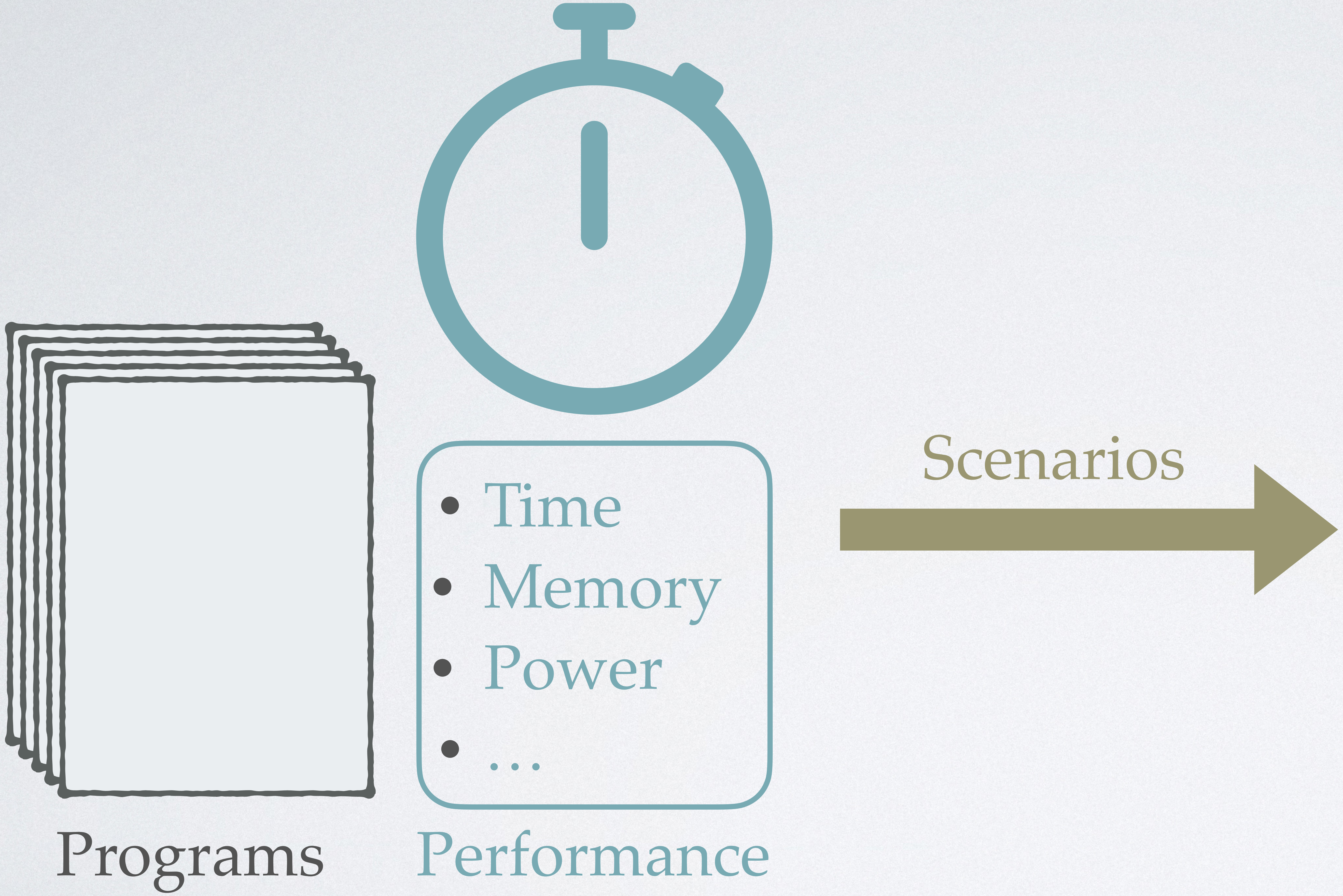
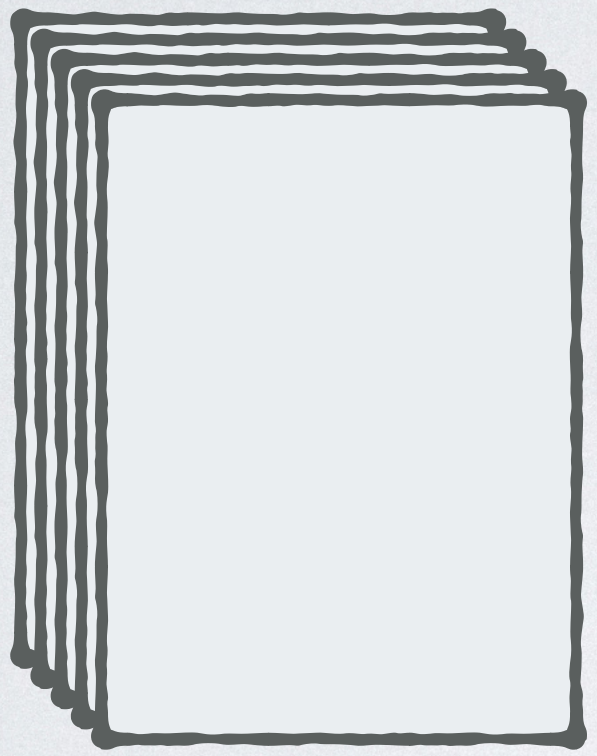


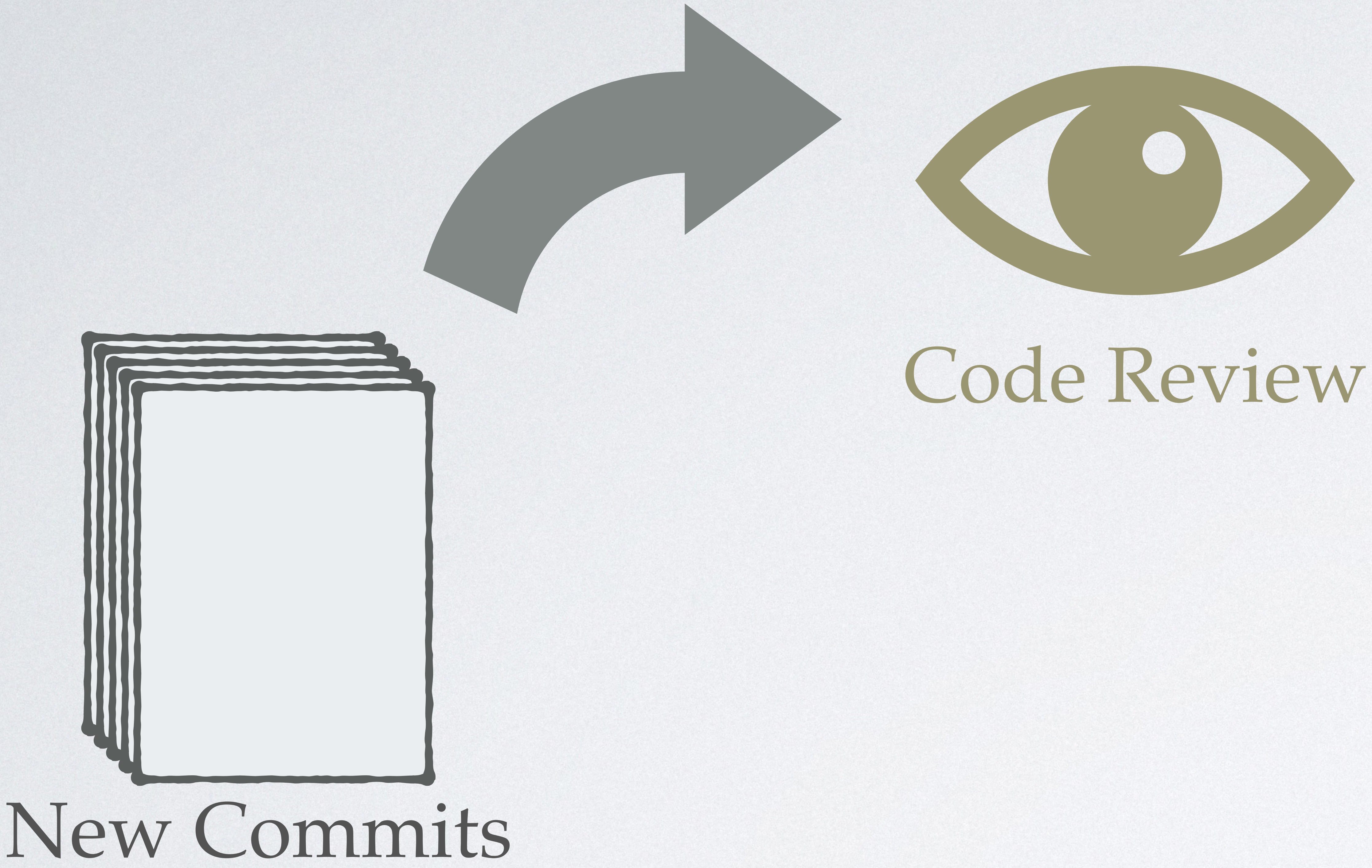
- Identifying bottlenecks
- Timing side channels
- Gas usage in blockchains
- Carbon footprint

STATIC RESOURCE ANALYSIS



New Commits

Static Resource Analysis



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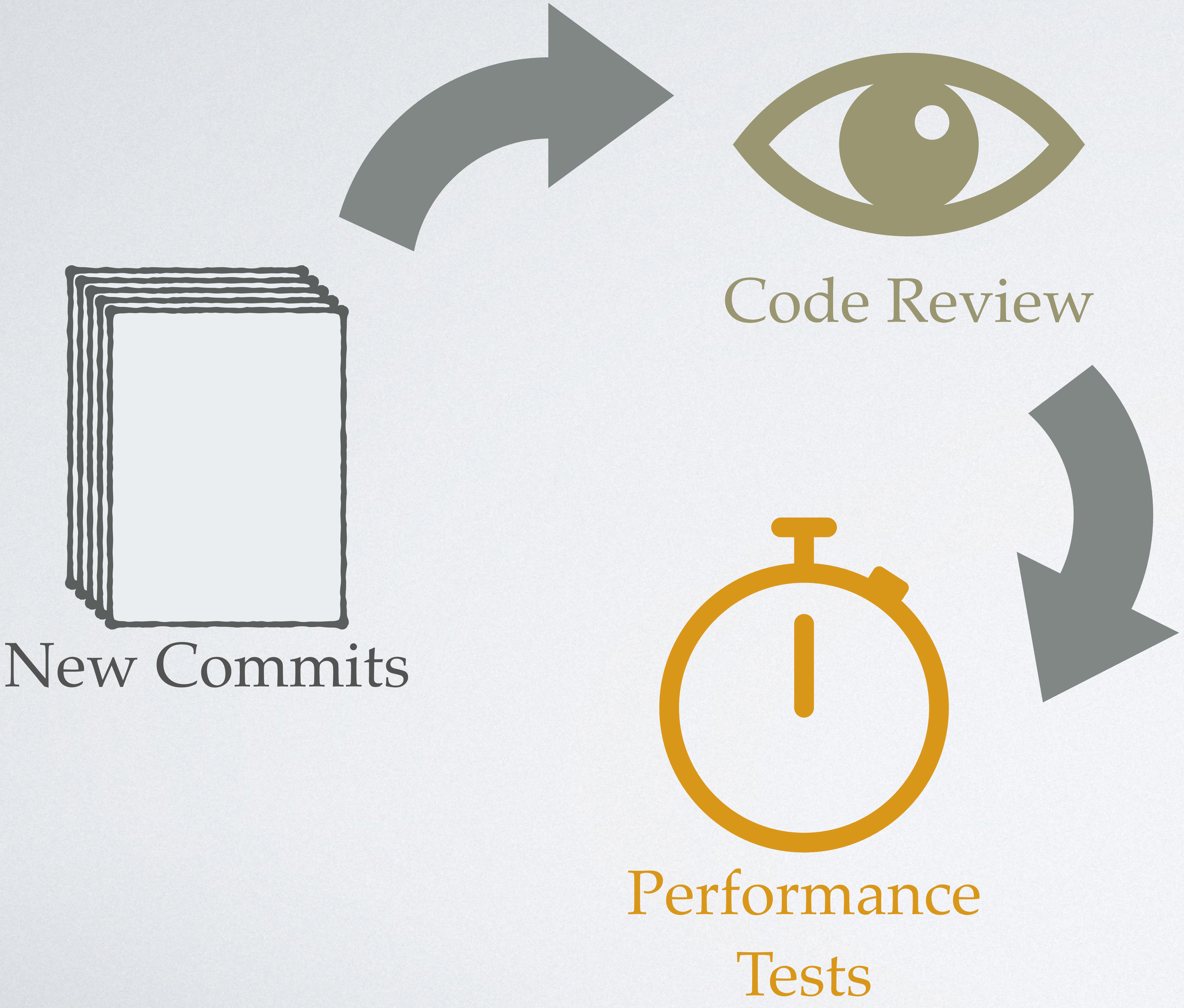

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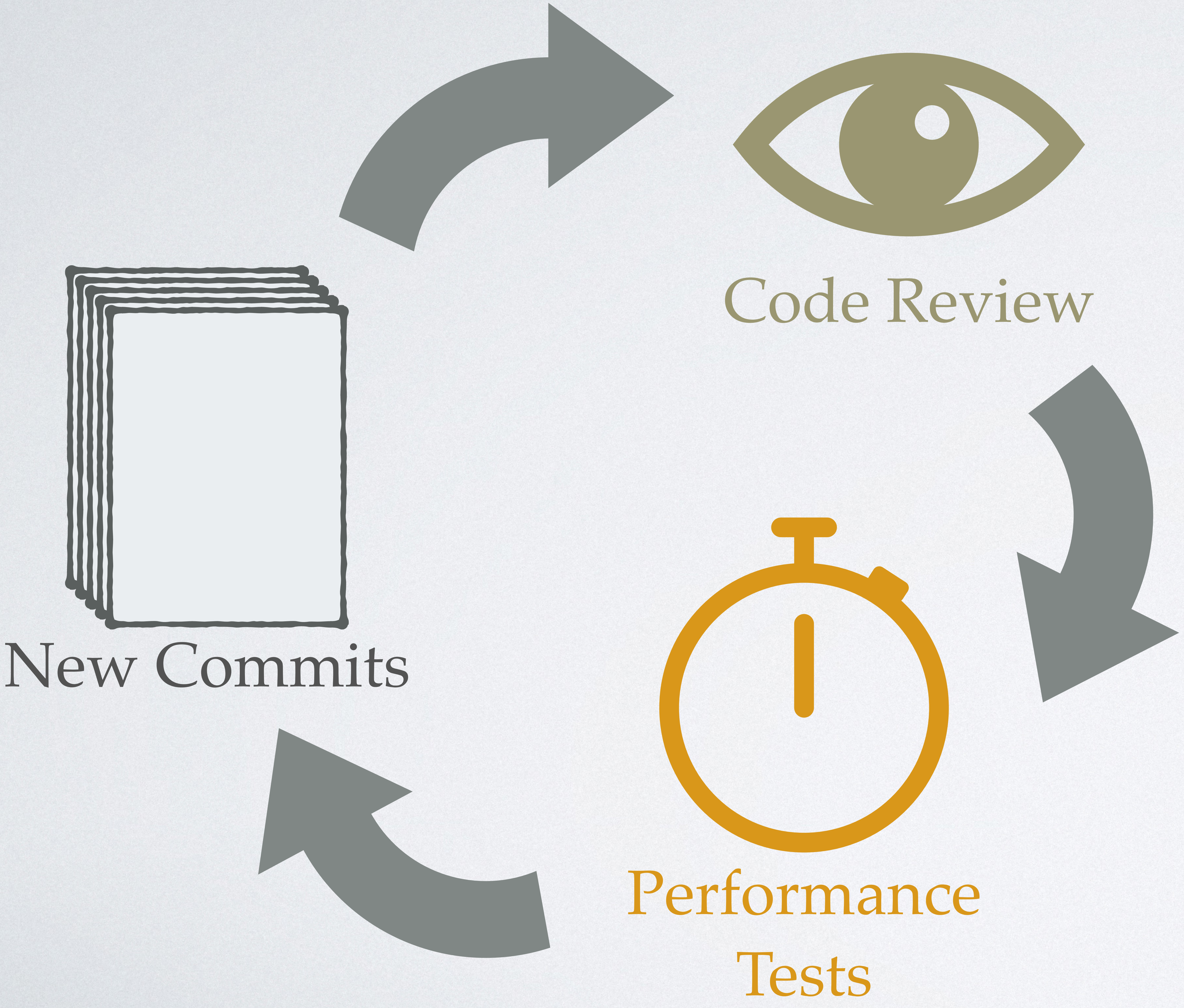

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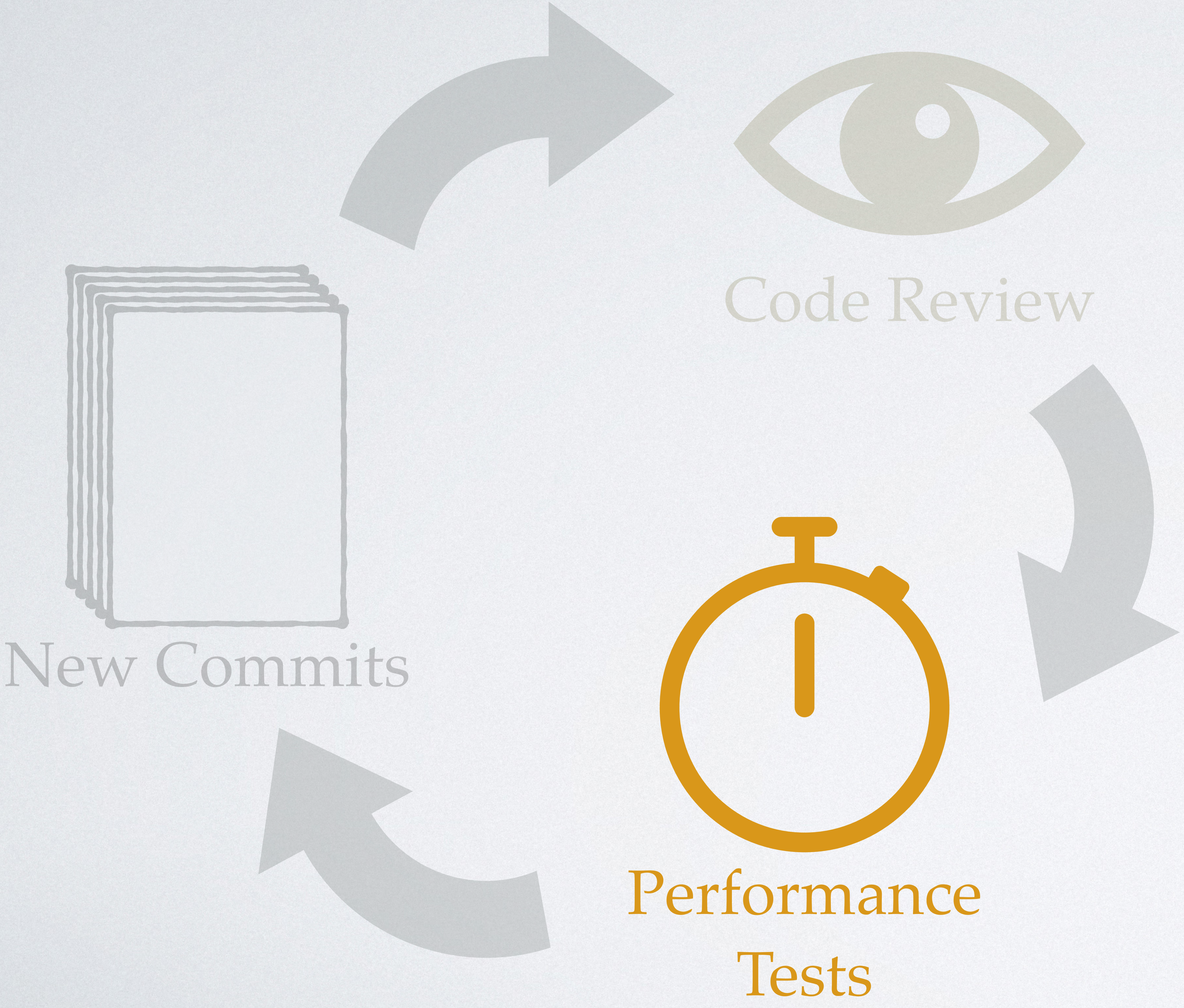
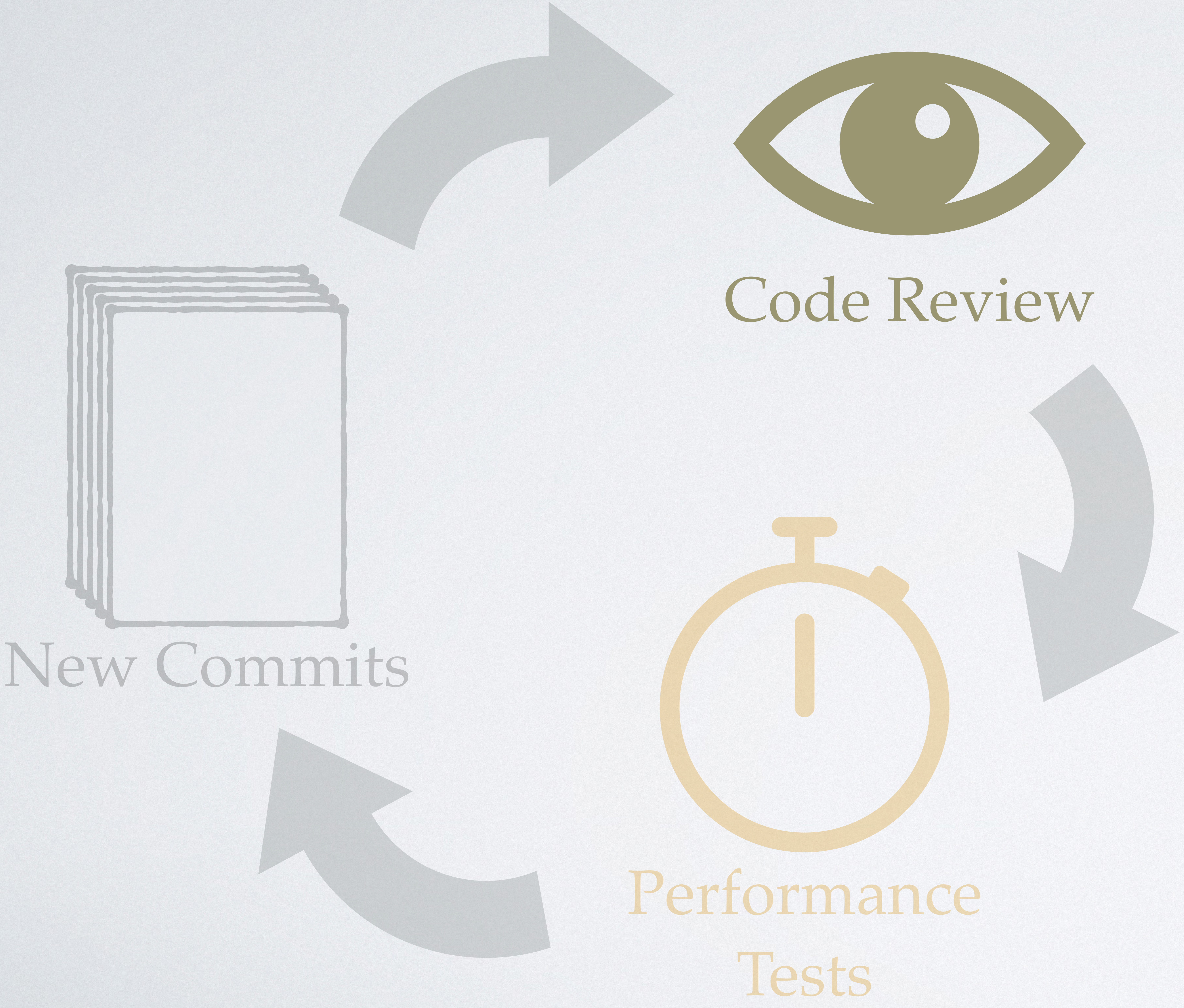


Possible drawbacks:

- Incomplete test coverage
- Time-consuming

STATIC RESOURCE ANALYSIS



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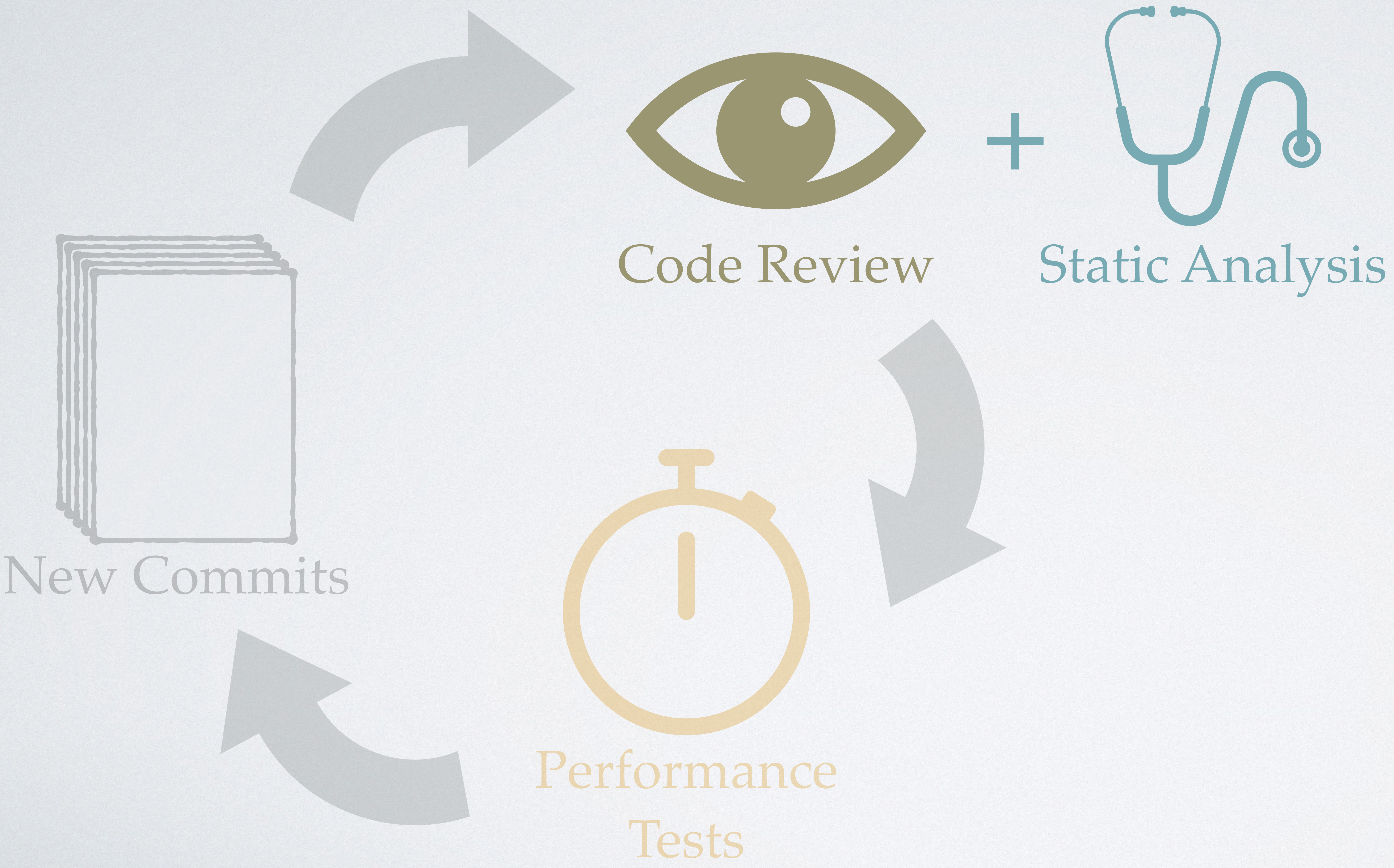

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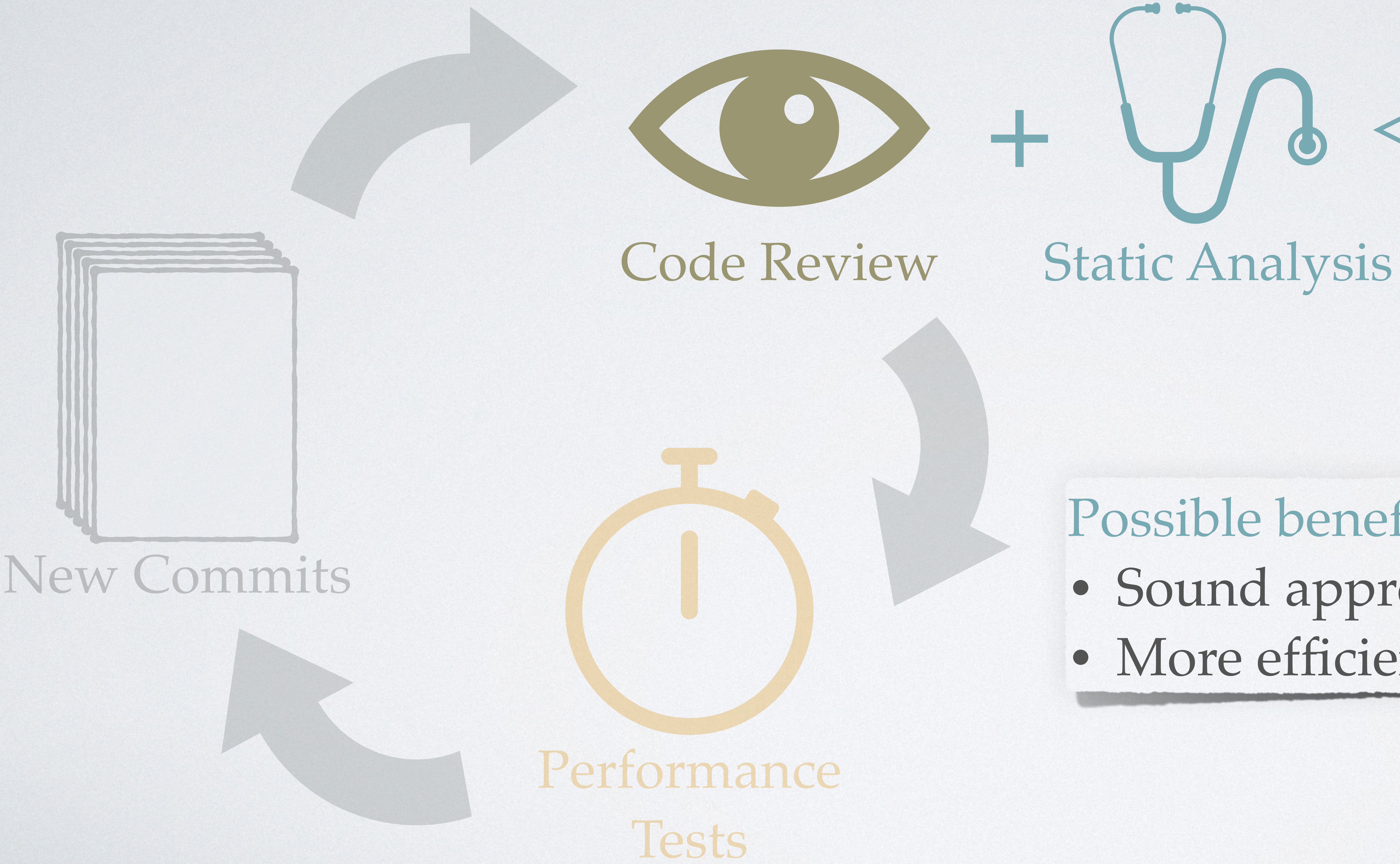

Analyze resource usage at compile time!

Possible benefits:

- Sound approximations for all inputs
- More efficient if analysis is incremental

Static Resource Analysis in Infer

Examples come from Infer's documentation. Available on: https://fbinfer.com/docs/next/checker-cost/.

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```java
void loop(ArrayList<Integer> list) {
  for (int i = 0; i <= list.size(); i++) {
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$$8\,|\,list\,| + 16 = O(\,|\,list\,|\,)$$

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Static Resource Analysis in RaML

RaML

[RaML17] J. Hoffmann, A. Das, and S.-C. Weng. 2017. Towards Automatic Resource Bound Analysis for OCaml. In *POPL'17*.

STATIC RESOURCE ANALYSIS IN RAML

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let rec append l1 l2 =
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RaML

append : $\langle L^9(\alpha) \times L^0(\alpha), 3\rangle \to \langle L^0(\alpha), 0\rangle$

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Simplified bound:

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This Talk: Type-Based Automatic Amortized Resource Analysis

[RaML17] J. Hoffmann, A. Das, and S.-C. Weng. 2017. Towards Automatic Resource Bound Analysis for OCaml. In *POPL'17*.

OUTLINE

- Automatic Amortized Resource Analysis
- Type-Guided Worst-Case Input Generation
- Resource-Guided Program Synthesis

AUTOMATIC AMORTIZED RESOURCE ANALYSIS

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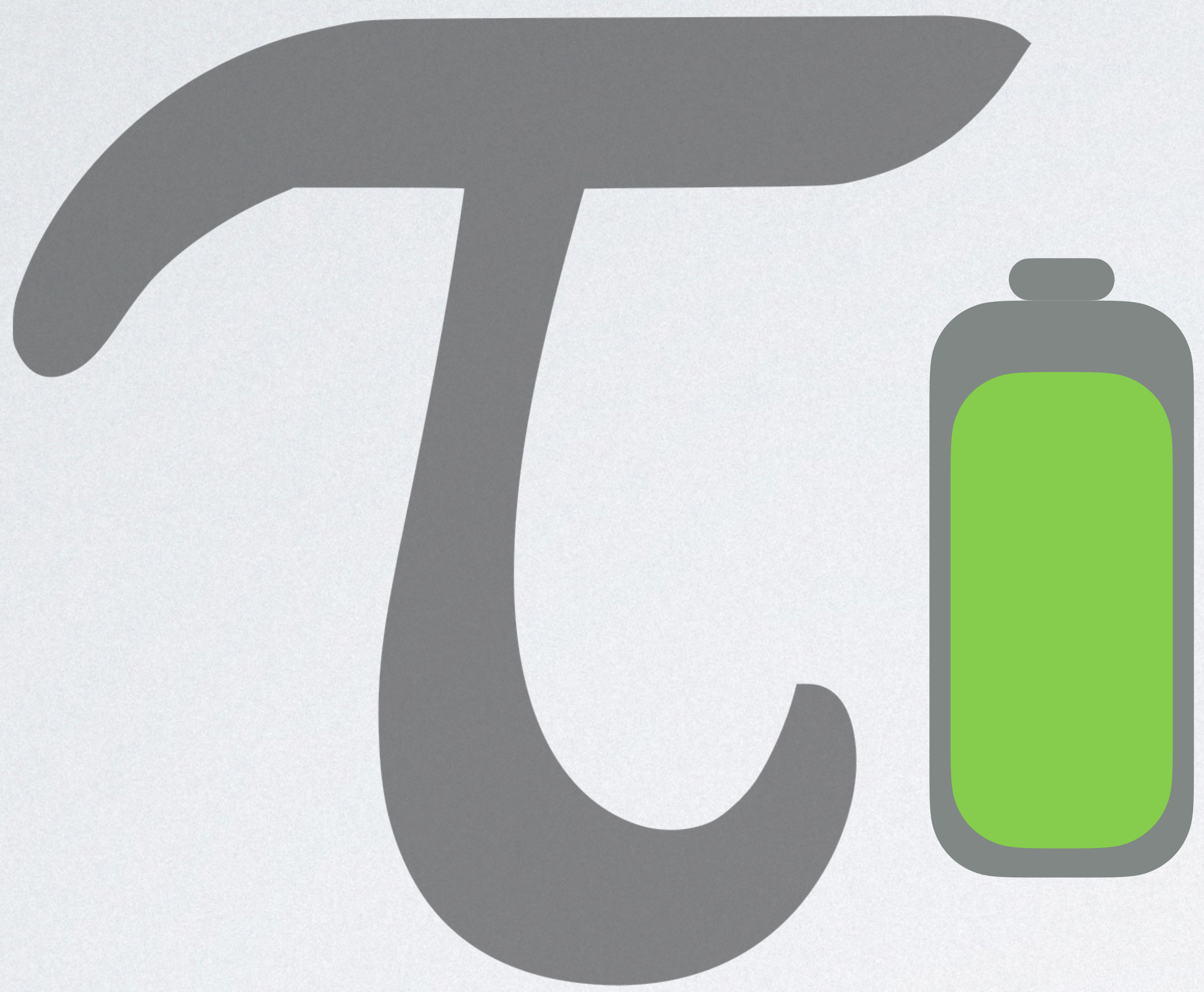
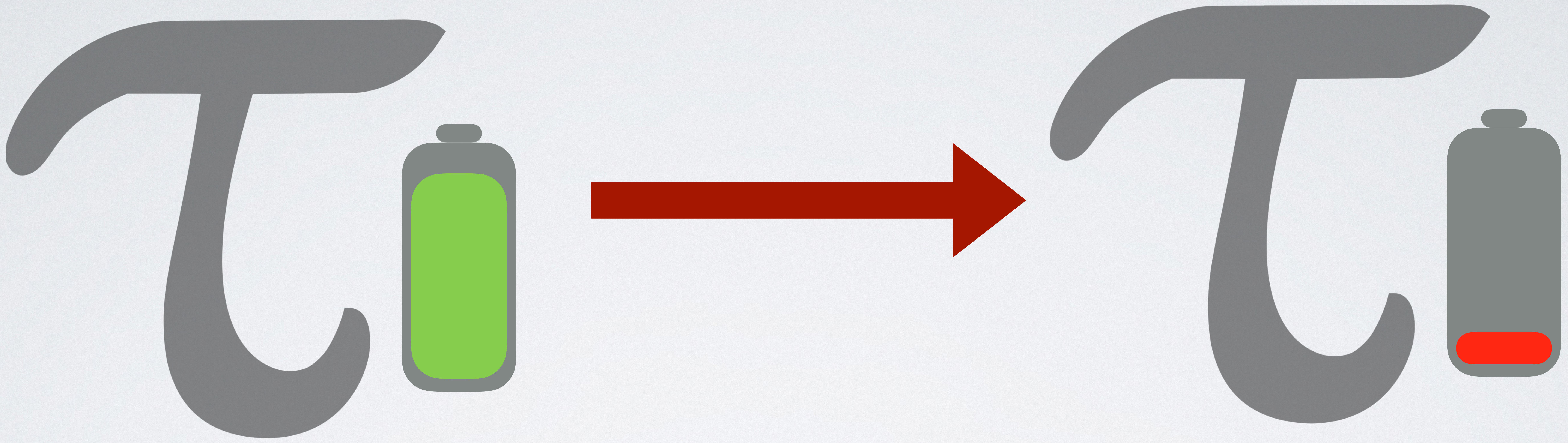


Automatic Amortized Resource Analysis



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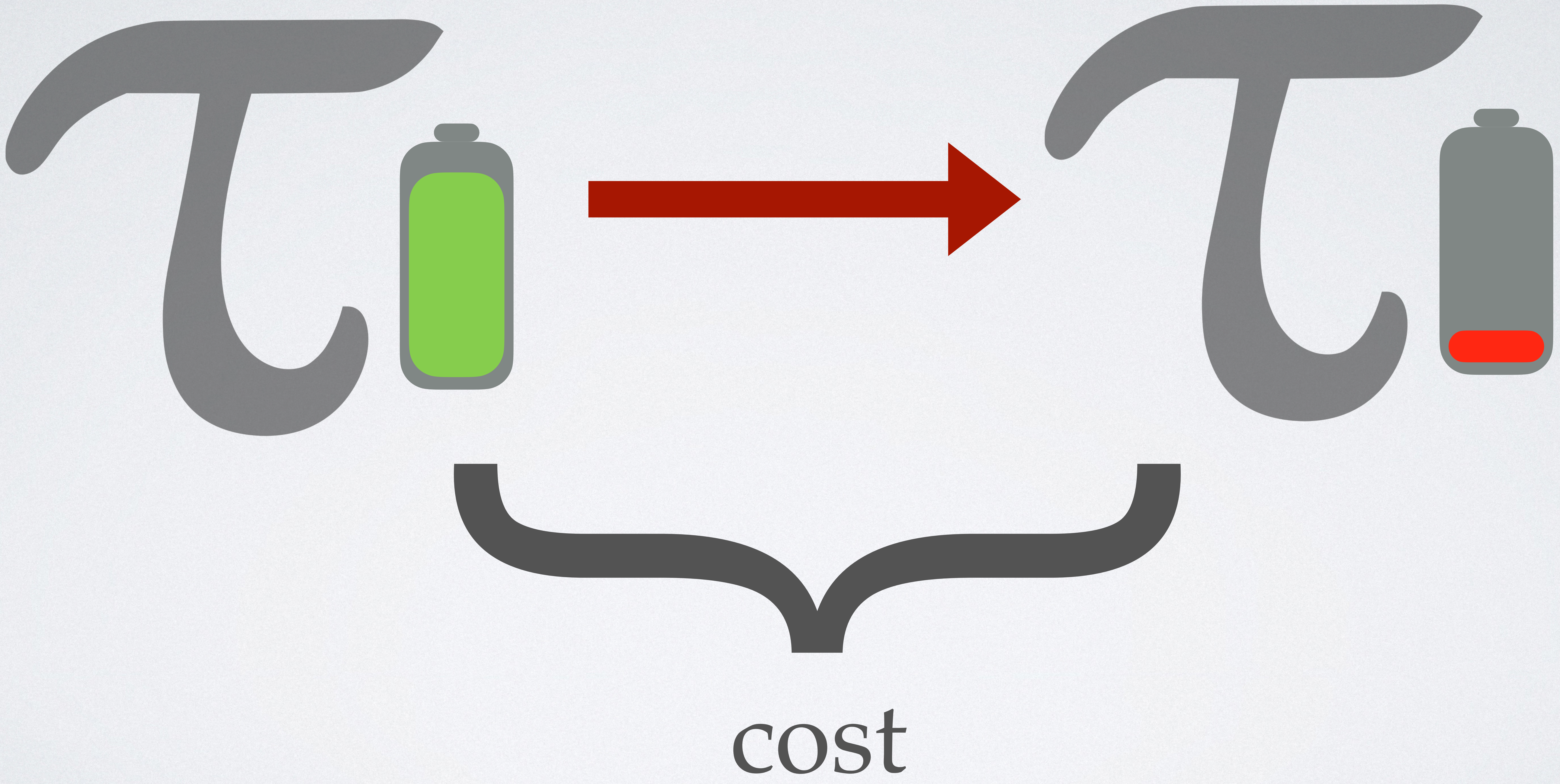

The Potential Method

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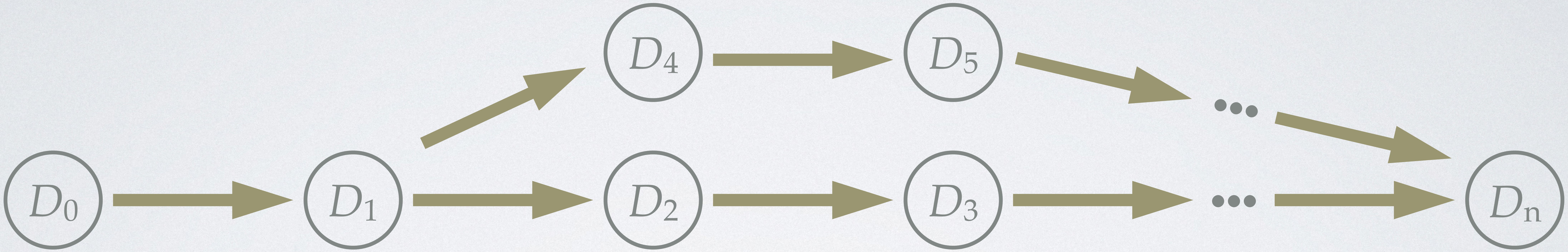

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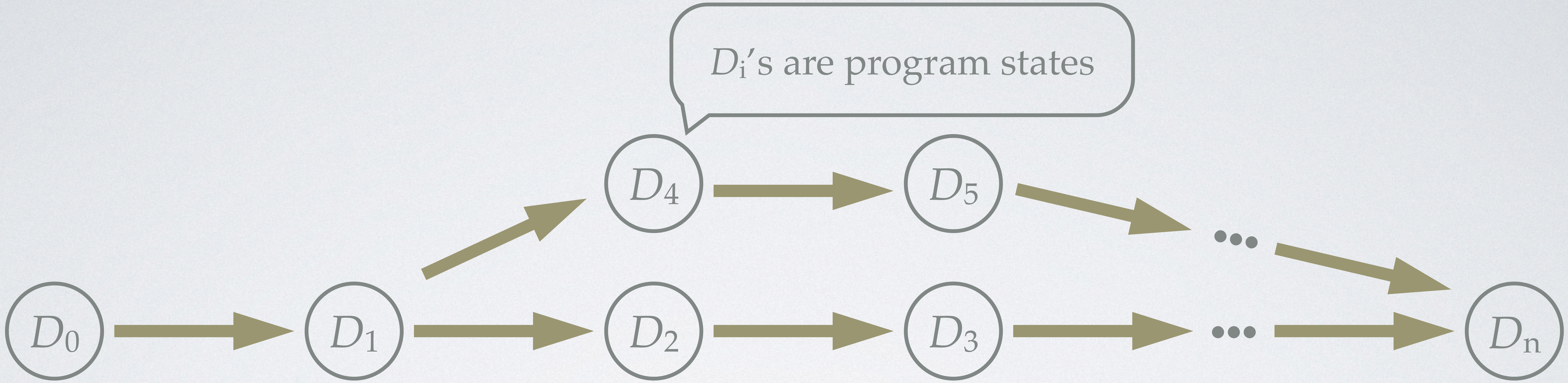

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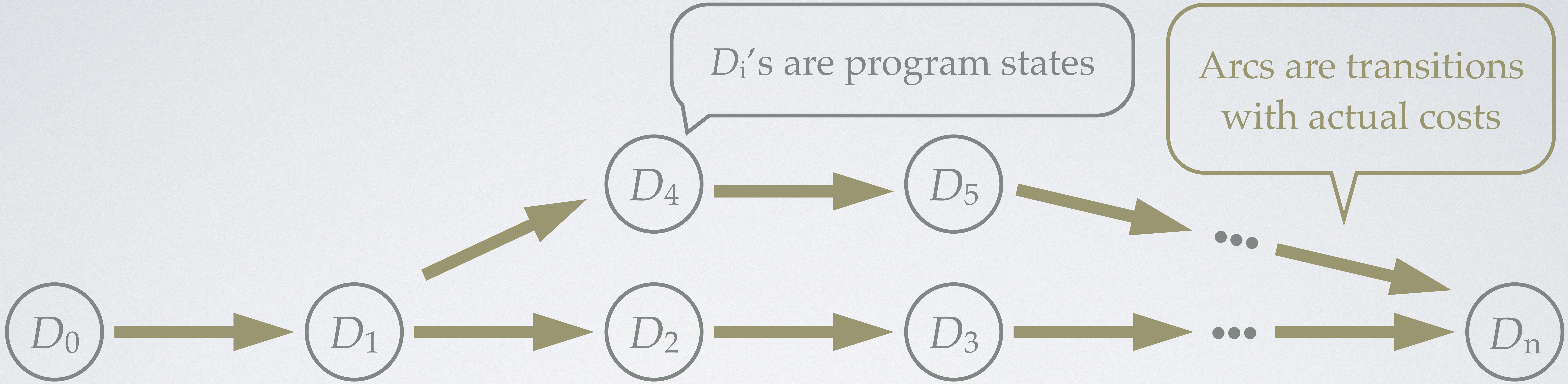

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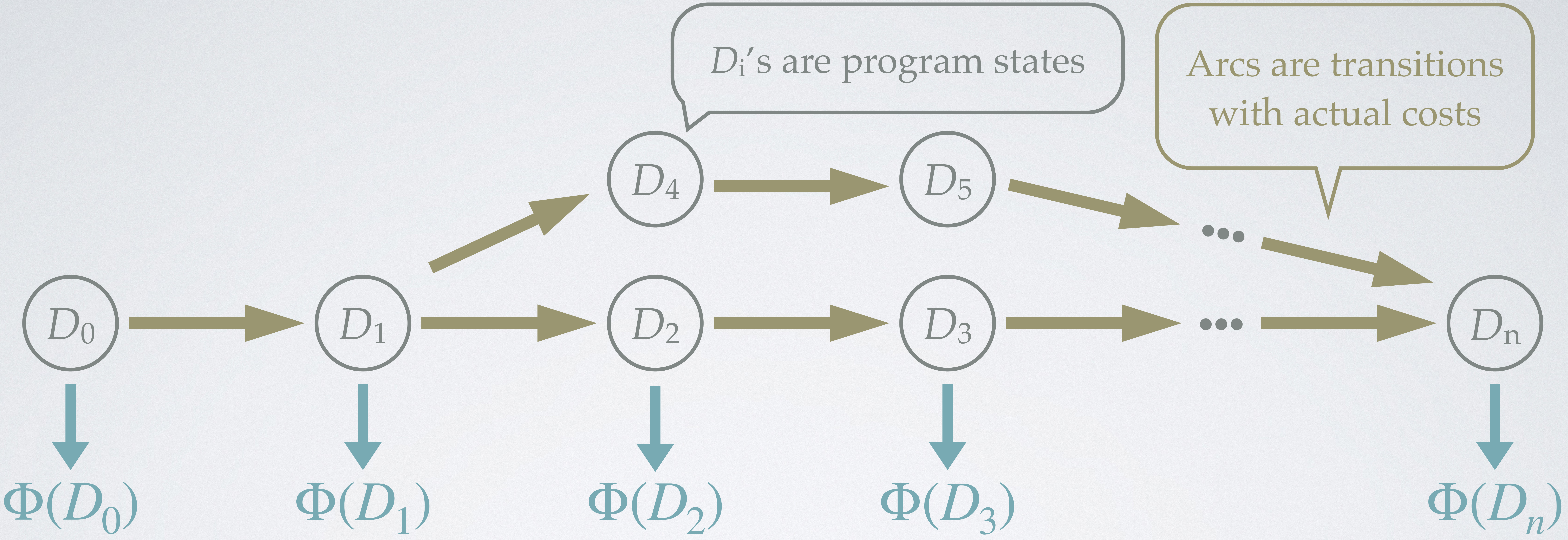

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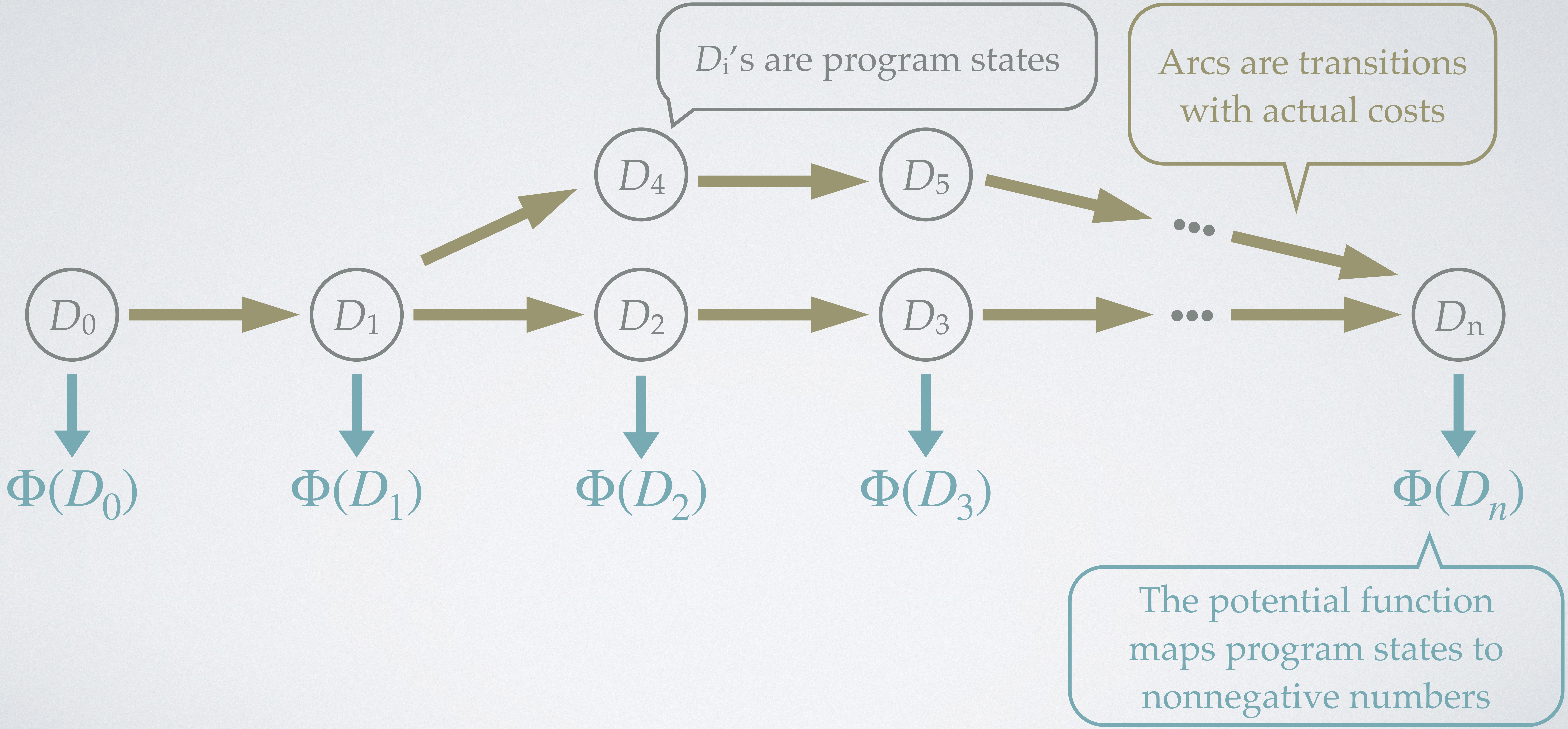

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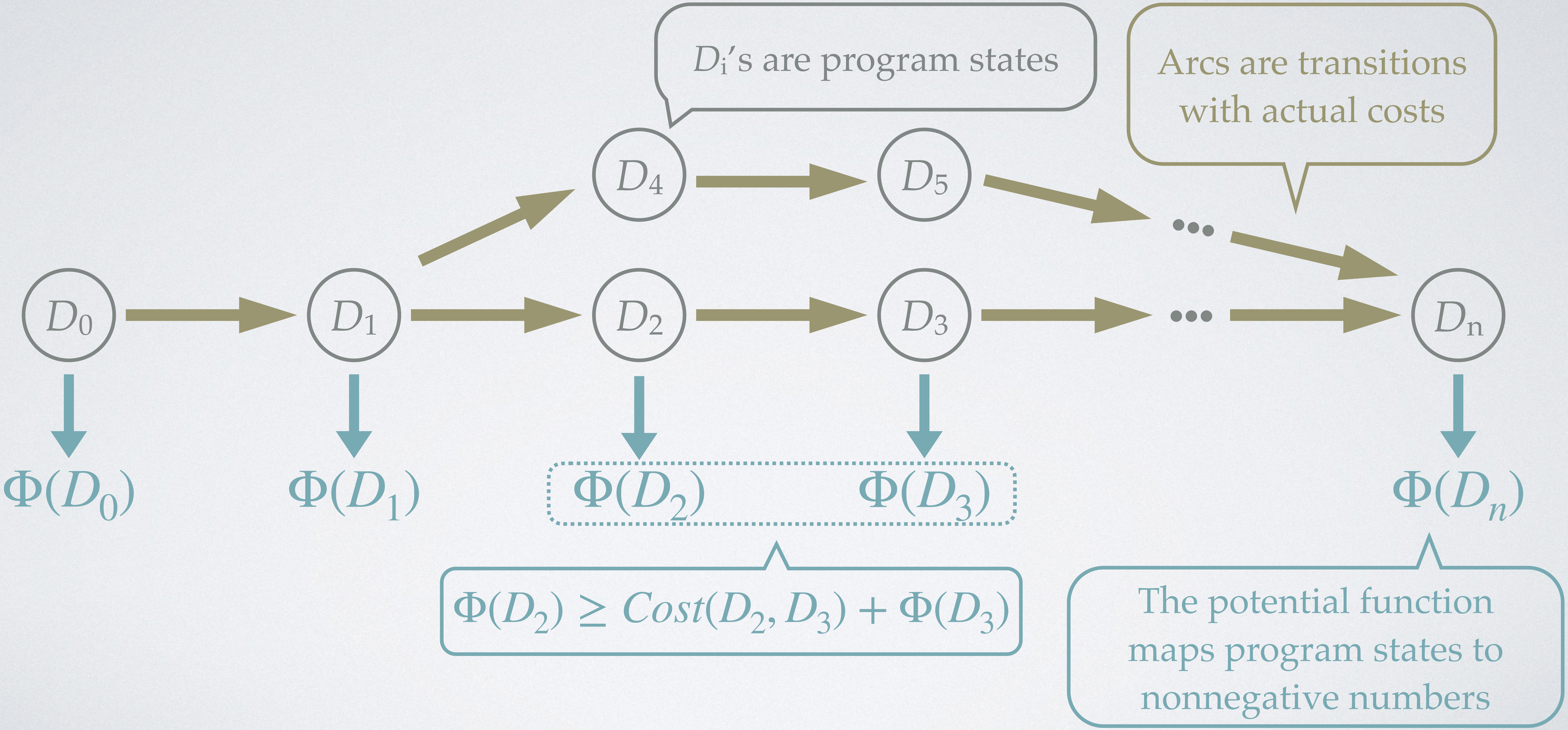

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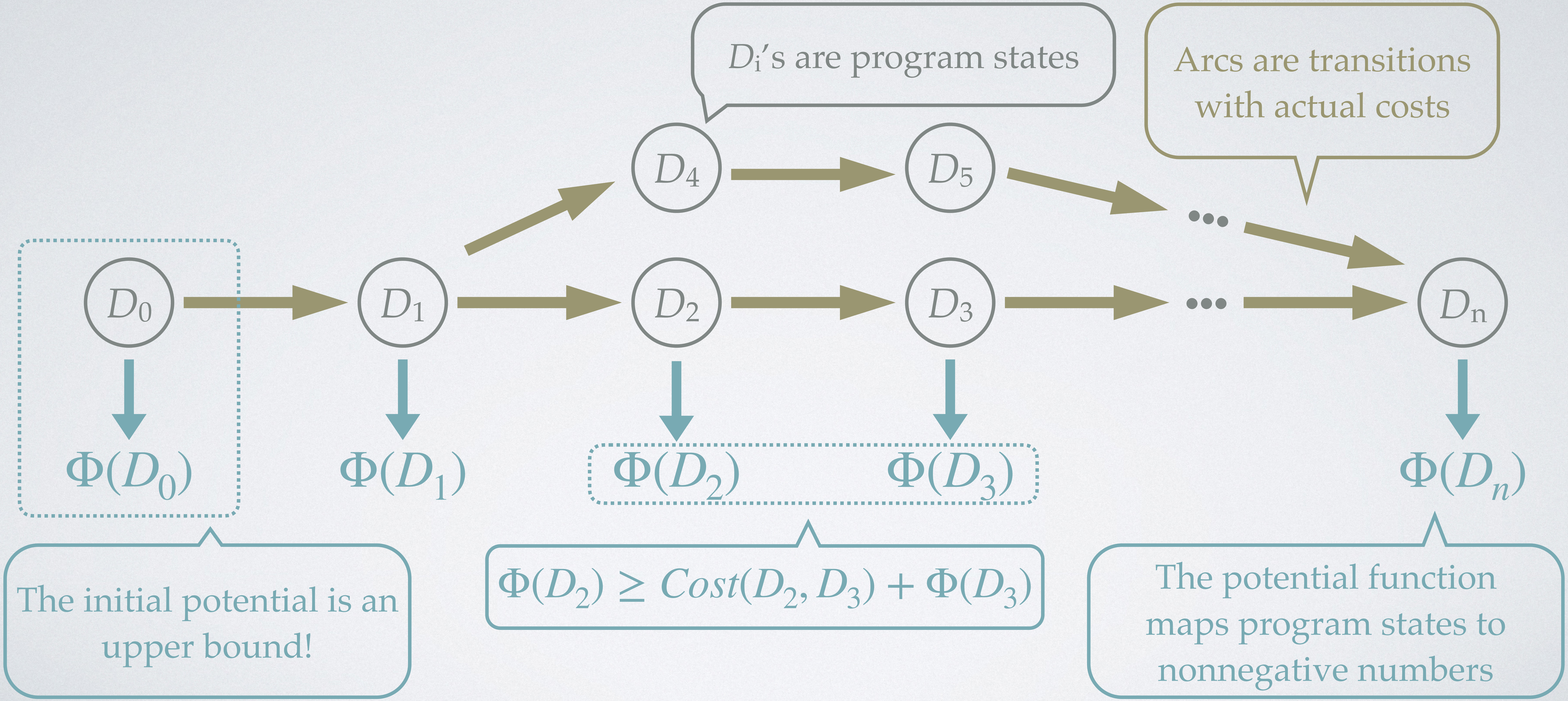

Potential-Augmented Types

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Resource metric:
count recursive calls

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append : $\langle L^1(\alpha) \times L^0(\alpha), 0\rangle \to \langle L^0(\alpha), 0\rangle$

$Cost = |\ell_1|$

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[l1: L¹(a), l2: L⁰(a)]; 0 units
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Resource metric: count recursive calls

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let rec append l1 l2 =            [l1: L¹(a), l2: L⁰(a)]; 0 units
  match l1 with                   // l1 is consumed
  | [] ->                         [l2: L⁰(a)]; 0 units
    l2                            // l2 is consumed. Type-checked!
  | x::xs ->                      [l2: L⁰(a), x: a, xs: L¹(a)]; 1 unit
    let () = tick(1) in           [l2: L⁰(a), x: a, xs: L¹(a)]; 0 units
    let rest = append xs l2 in    [x: a, rest: L⁰(a)]; 0 units
    x::rest                       // x and rest are consumed. Type-checked!
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  | [] ->                        [l2: L⁰(a)]; 0 units
    l2                           // l2 is consumed. Type-checked!
  | x::xs ->                     [l2: L⁰(a), x: a, xs: L¹(a)]; 1 unit
    let () = tick(1) in          [l2: L⁰(a), x: a, xs: L¹(a)]; 0 units
    let rest = append xs l2 in   [x: a, rest: L⁰(a)]; 0 units
    x::rest                      // x and rest are consumed. Type-checked!
```

Principle: The potential at a program point is defined by a static type annotation of data structures.

AUTOMATION VIA LP SOLVING

```
let rec append l1 l2 =
  match l1 with
  | [] ->
    l2
  | x::xs ->
    let () = tick(1) in
    let rest = append xs l2 in
    x::rest
```

Automation via LP Solving

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =
  match l1 with
  | [] ->
    l2
  | x::xs ->
    let () = tick(1) in
    let rest = append xs l2 in
    x::rest
```

AUTOMATION VIA LP SOLVING

p, q, r, s, t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =
  match l1 with
  | [] ->
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```

AUTOMATION VIA LP SOLVING

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

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let rec append l1 l2 =
  match l1 with
  | [] ->
    l2
  | x::xs ->
    let () = tick(1) in
    let rest = append xs l2 in
    x::rest
```

Linear Constraints

```
p≥0,q≥0,r≥0,s≥0,t≥0
```

AUTOMATION VIA LP SOLVING

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =
  match l1 with
  | [] ->
    l2
  | x::xs ->
    let () = tick(1) in
    let rest = append xs l2 in
    x::rest
```

```
[l1: Lp(a), l2: Lq(a)]; r units
```

Linear Constraints

p≥0, q≥0, r≥0, s≥0, t≥0

AUTOMATION VIA LP SOLVING

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =
  match l1 with
  | [] ->
    l2
  | x::xs ->
    let () = tick(1) in
    let rest = append xs l2 in
    x::rest
```

```
[l1: Lp(a), l2: Lq(a)]; r units
// l1 is consumed
```

Linear Constraints

```
p≥0,q≥0,r≥0,s≥0,t≥0
```

AUTOMATION VIA LP SOLVING

p,q,r,s,t are unknown numeric variables

Linear Constraints

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =
  match l1 with
  | [] ->
    l2
  | x::xs ->
    let () = tick(1) in
    let rest = append xs l2 in
    x::rest
```

```
[l1: Lp(a), l2: Lq(a)]; r units
// l1 is consumed
[l2: Lq(a)]; r units
```

```
p≥0,q≥0,r≥0,s≥0,t≥0
```

Automation via LP Solving

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =
  match l1 with
  | [] ->
    l2
  | x::xs ->
    let () = tick(1) in
    let rest = append xs l2 in
    x::rest
```

```
[l1: Lp(a), l2: Lq(a)]; r units
// l1 is consumed
[l2: Lq(a)]; r units
// l2 is consumed
```

Linear Constraints

p≥0,q≥0,r≥0,s≥0,t≥0

Automation via LP Solving

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =            [l1: Lp(a), l2: Lq(a)]; r units
  match l1 with                   // l1 is consumed
  | [] ->                         [l2: Lq(a)]; r units
    l2                            // l2 is consumed
  | x::xs ->
    let () = tick(1) in
    let rest = append xs l2 in
    x::rest
```

Linear Constraints

p≥0,q≥0,r≥0,s≥0,t≥0

q≥s,r≥t

AUTOMATION VIA LP SOLVING

p,q,r,s,t are unknown numeric variables

Linear Constraints

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =          [l1: Lp(a), l2: Lq(a)]; r units
  match l1 with                 // l1 is consumed
  | [] ->                       [l2: Lq(a)]; r units
    l2                          // l2 is consumed
  | x::xs ->                    [l2: Lq(a), x: a, xs: Lp(a)]; r+p units
    let () = tick(1) in
    let rest = append xs l2 in
    x::rest
```

```
p≥0,q≥0,r≥0,s≥0,t≥0

q≥s,r≥t
```

Automation via LP Solving

p,q,r,s,t are unknown numeric variables

Linear Constraints

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =            [l1: Lp(a), l2: Lq(a)]; r units
  match l1 with                   // l1 is consumed
  | [] ->                         [l2: Lq(a)]; r units
    l2                            // l2 is consumed
  | x::xs ->                      [l2: Lq(a), x: a, xs: Lp(a)]; r+p units
    let () = tick(1) in           [l2: Lq(a), x: a, xs: Lp(a)]; r+p-1 units
    let rest = append xs l2 in
    x::rest
```

```
p≥0,q≥0,r≥0,s≥0,t≥0

q≥s,r≥t
```

AUTOMATION VIA LP SOLVING

p,q,r,s,t are unknown numeric variables

Linear Constraints

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =          [l1: Lp(a), l2: Lq(a)]; r units       p≥0,q≥0,r≥0,s≥0,t≥0
  match l1 with                 // l1 is consumed
  | [] ->                       [l2: Lq(a)]; r units
    l2                          // l2 is consumed                      q≥s,r≥t
  | x::xs ->                    [l2: Lq(a), x: a, xs: Lp(a)]; r+p units
    let () = tick(1) in         [l2: Lq(a), x: a, xs: Lp(a)]; r+p-1 units   r+p-1≥0
    let rest = append xs l2 in
    x::rest
```

AUTOMATION VIA LP SOLVING

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \rightarrow \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =            [l1: Lp(a), l2: Lq(a)]; r units
  match l1 with                   // l1 is consumed
  | [] ->                         [l2: Lq(a)]; r units
    l2                            // l2 is consumed
  | x::xs ->                      [l2: Lq(a), x: a, xs: Lp(a)]; r+p units
    let () = tick(1) in           [l2: Lq(a), x: a, xs: Lp(a)]; r+p-1 units
    let rest = append xs l2 in    [x: a, rest: Ls(a)]; p-1+t units
    x::rest
```

Linear Constraints

```
p≥0,q≥0,r≥0,s≥0,t≥0
q≥s,r≥t
r+p-1≥0
```

Automation via LP Solving

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =            [l1: Lp(a), l2: Lq(a)]; r units
  match l1 with                   // l1 is consumed
  | [] ->                         [l2: Lq(a)]; r units
    l2                            // l2 is consumed
  | x::xs ->                      [l2: Lq(a), x: a, xs: Lp(a)]; r+p units
    let () = tick(1) in           [l2: Lq(a), x: a, xs: Lp(a)]; r+p-1 units
    let rest = append xs l2 in    [x: a, rest: Ls(a)]; p-1+t units
    x::rest
```

Linear Constraints

```
p≥0,q≥0,r≥0,s≥0,t≥0
q≥s,r≥t
r+p-1≥0
p≥p,q≥q,r+p-1≥r
```

Automation via LP Solving

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \rightarrow \langle L^s(\alpha), t\rangle$

Linear Constraints

```
let rec append l1 l2 =          [l1: Lp(a), l2: Lq(a)]; r units        p≥0,q≥0,r≥0,s≥0,t≥0
  match l1 with                 // l1 is consumed
  | [] ->                       [l2: Lq(a)]; r units
    l2                          // l2 is consumed                       q≥s,r≥t
  | x::xs ->                    [l2: Lq(a), x: a, xs: Lp(a)]; r+p units
    let () = tick(1) in         [l2: Lq(a), x: a, xs: Lp(a)]; r+p-1 units   r+p-1≥0
    let rest = append xs l2 in  [x: a, rest: Ls(a)]; p-1+t units        p≥p,q≥q,r+p-1≥r
    x::rest                     // x and rest are consumed
```

Automation via LP Solving

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

Linear Constraints

```
let rec append l1 l2 =           [l1: Lp(a), l2: Lq(a)]; r units           p≥0,q≥0,r≥0,s≥0,t≥0
  match l1 with                  // l1 is consumed
  | [] ->                        [l2: Lq(a)]; r units
    l2                           // l2 is consumed                         q≥s,r≥t
  | x::xs ->                     [l2: Lq(a), x: a, xs: Lp(a)]; r+p units
    let () = tick(1) in          [l2: Lq(a), x: a, xs: Lp(a)]; r+p-1 units r+p-1≥0
    let rest = append xs l2 in   [x: a, rest: Ls(a)]; p-1+t units           p≥p,q≥q,r+p-1≥r
    x::rest                      // x and rest are consumed                p-1+t≥s+t
```

AUTOMATION VIA LP SOLVING

p,q,r,s,t are unknown numeric variables

Linear Constraints

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

```
let rec append l1 l2 =           [l1: Lp(a), l2: Lq(a)]; r units          p≥0,q≥0,r≥0,s≥0,t≥0
  match l1 with                  // l1 is consumed
  | [] ->                        [l2: Lq(a)]; r units
    l2                           // l2 is consumed                         q≥s,r≥t
  | x::xs ->                     [l2: Lq(a), x: a, xs: Lp(a)]; r+p units
    let () = tick(1) in          [l2: Lq(a), x: a, xs: Lp(a)]; r+p-1 units r+p-1≥0
    let rest = append xs l2 in   [x: a, rest: Ls(a)]; p-1+t units          p≥p,q≥q,r+p-1≥r
    x::rest                      // x and rest are consumed                p-1+t≥s+t
```

p=1, q=r=s=t=0

AUTOMATION VIA LP SOLVING

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

Linear Constraints

```
let rec append l1 l2 =              [l1: Lp(a), l2: Lq(a)]; r units         p≥0,q≥0,r≥0,s≥0,t≥0
  match l1 with                     // l1 is consumed
  | [] ->                           [l2: Lq(a)]; r units
    l2                              // l2 is consumed                       q≥s,r≥t
  | x::xs ->                        [l2: Lq(a), x: a, xs: Lp(a)]; r+p units
    let () = tick(1) in             [l2: Lq(a), x: a, xs: Lp(a)]; r+p-1 units  r+p-1≥0
    let rest = append xs l2 in      [x: a, rest: Ls(a)]; p-1+t units        p≥p,q≥q,r+p-1≥r
    x::rest                         // x and rest are consumed              p-1+t≥s+t
```

append : $\langle L^1(\alpha) \times L^0(\alpha), 0\rangle \to \langle L^0(\alpha), 0\rangle$ ← p=1, q=r=s=t=0

Automation via LP Solving

p,q,r,s,t are unknown numeric variables

append : $\langle L^p(\alpha) \times L^q(\alpha), r\rangle \to \langle L^s(\alpha), t\rangle$

Linear Constraints

```
let rec append l1 l2 =          [l1: Lp(a), l2: Lq(a)]; r units          p≥0,q≥0,r≥0,s≥0,t≥0
  match l1 with                 // l1 is consumed
  | [] ->                       [l2: Lq(a)]; r units
    l2                          // l2 is consumed                          q≥s,r≥t
  | x::xs ->                    [l2: Lq(a), x: a, xs: Lp(a)]; r+p units
    let () = tick(1) in         [l2: Lq(a), x: a, xs: Lp(a)]; r+p-1 units  r+p-1≥0
    let rest = append xs l2 in  [x: a, rest: Ls(a)]; p-1+t units           p≥p,q≥q,r+p-1≥r
    x::rest                     // x and rest are consumed                 p-1+t≥s+t
```

append : $\langle L^2(\alpha) \times L^1(\alpha),3\rangle \to \langle L^1(\alpha),3\rangle$ ← p=2, q=s=1, r=t=3

The Frontier of AARA

[RaML17]	Multivariate polynomial bounds, amortized complexity (binary counters, …)
[Atkey10]	Imperative programs, heaps, separation logic
[JHL+10]	Higher-order functions
[HM18]	Logarithmic amortized complexity (splay trees, …)
[KH20]	Exponential bounds

[Atkey10] R. Atkey. 2010. Amortised Resource Analysis with Separation Logic. In *ESOP'10*.
[JHL+10] S. Jost, K. Hammond, H.-W. Loidl, and M. Hofmann. 2010. Static Determination of Quantitative Resource Usage for Higher-Order Programs. In *POPL'10*.
[HM18] M. Hofmann and G. Moser. 2018. Analysis of Logarithmic Amortised Complexity. Available on: https://arxiv.org/abs/1807.08242.
[KH20] D. M. Kahn and J. Hoffmann. 2020. Exponential Automatic Amortized Resource Analysis. In *FoSSaCS'20*.

Can we use the type information from AARA to guide other tasks?

Type-Based Resource-Guided Search

Type-Based Resource-Guided Search

- **Search** algorithms are used in many PL-related tasks.

Type-Based Resource-Guided Search

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- **Symbolic Execution**: **search** for an execution path that satisfies constraints.

Type-Based Resource-Guided Search

- **Search** algorithms are used in many PL-related tasks.
- **Symbolic Execution**: **search** for an execution path that satisfies constraints.
- **Program Synthesis**: **search** for a program that satisfies specifications.

Type-Based Resource-Guided Search

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Type-Based Resource-Guided Search

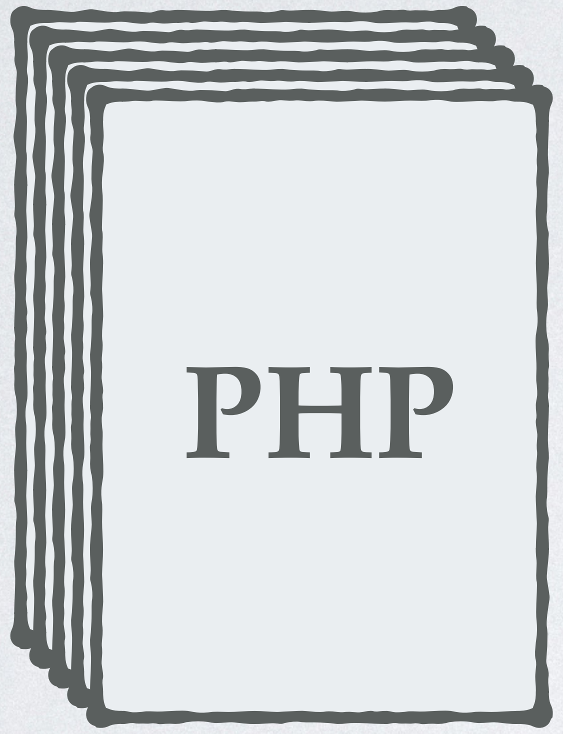
- Search algorithms are used in many PL-related tasks.
- **Symbolic Execution**: search for an execution path that satisfies constraints.
- **Program Synthesis**: search for a program that satisfies specifications.

- **Idea**: Resource information can be used to prune the search space.

OUTLINE

- [x] Automatic Amortized Resource Analysis
- [ ] Type-Guided **Worst-Case Input** Generation
- [ ] Resource-Guided Program Synthesis

Example of Worst-Case Analysis

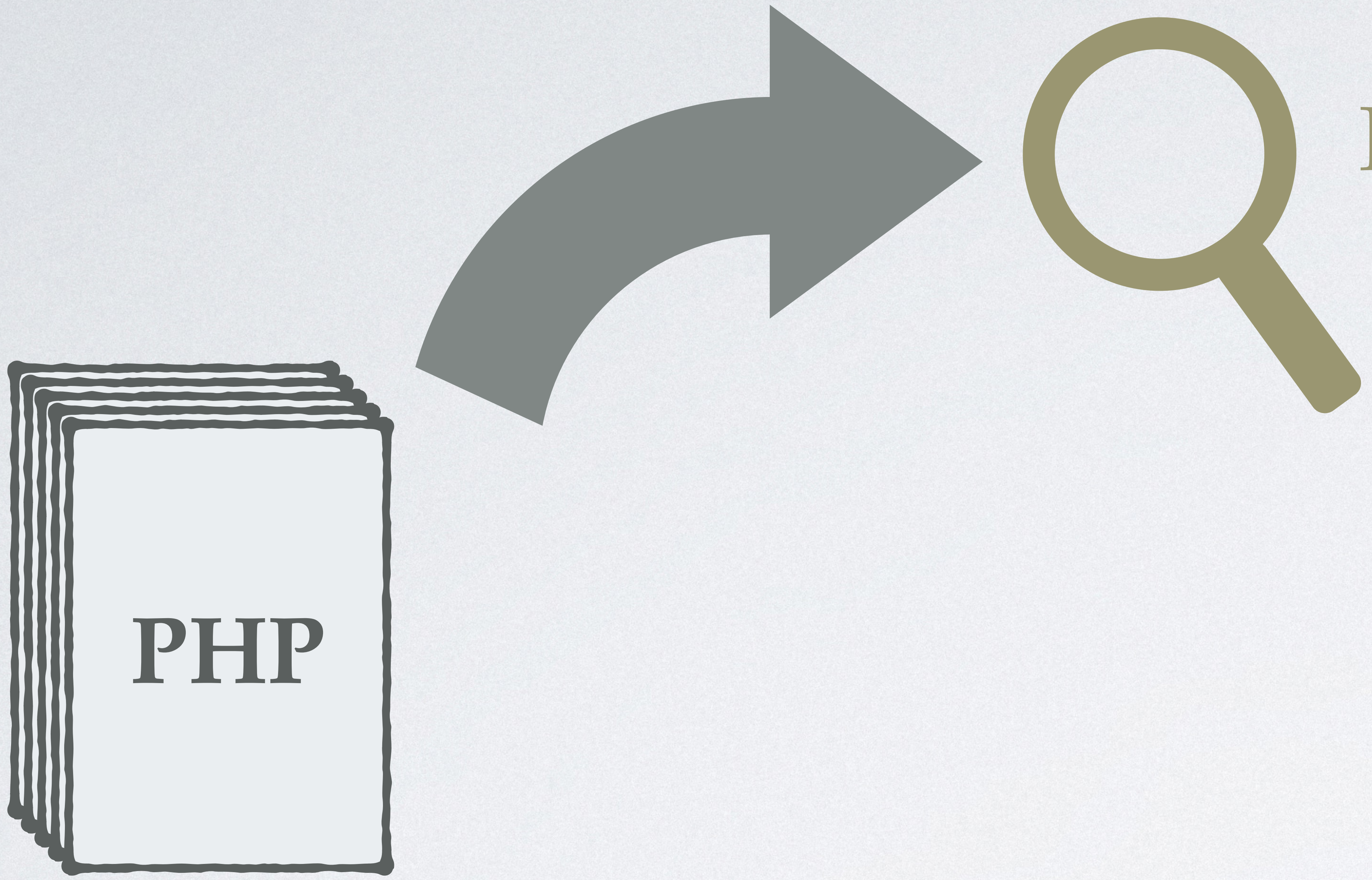


[1] CVE - CVE-2011-4885. Available on: https://cve.mitre.org/cgi-bin/cvename.cgi?name=CVE-2011-4885.

[2] PHP 5.3.8 - Hashtables Denial of Service. Available on https://www.exploit-db.com/exploits/18296/.

[3] PHP: PHP 5 ChangeLog. Available on http://www.php.net/ChangeLog-5.php#5.3.9.

EXAMPLE OF WORST-CASE ANALYSIS



Potential Denial-of-Service attack[1]

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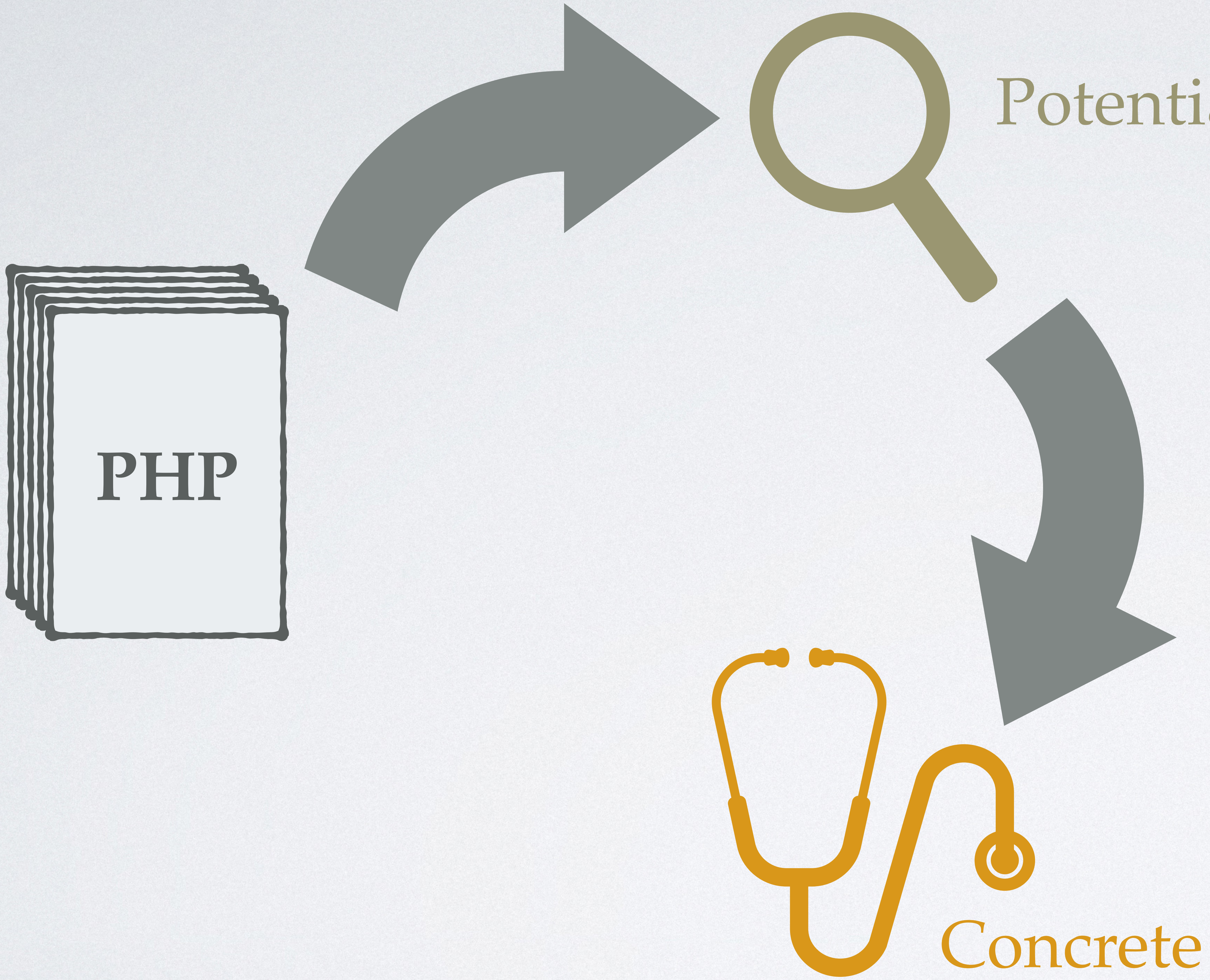
EXAMPLE OF WORST-CASE ANALYSIS



Potential Denial-of-Service attack[1]

Concrete exploits (by hash collisions)[2]

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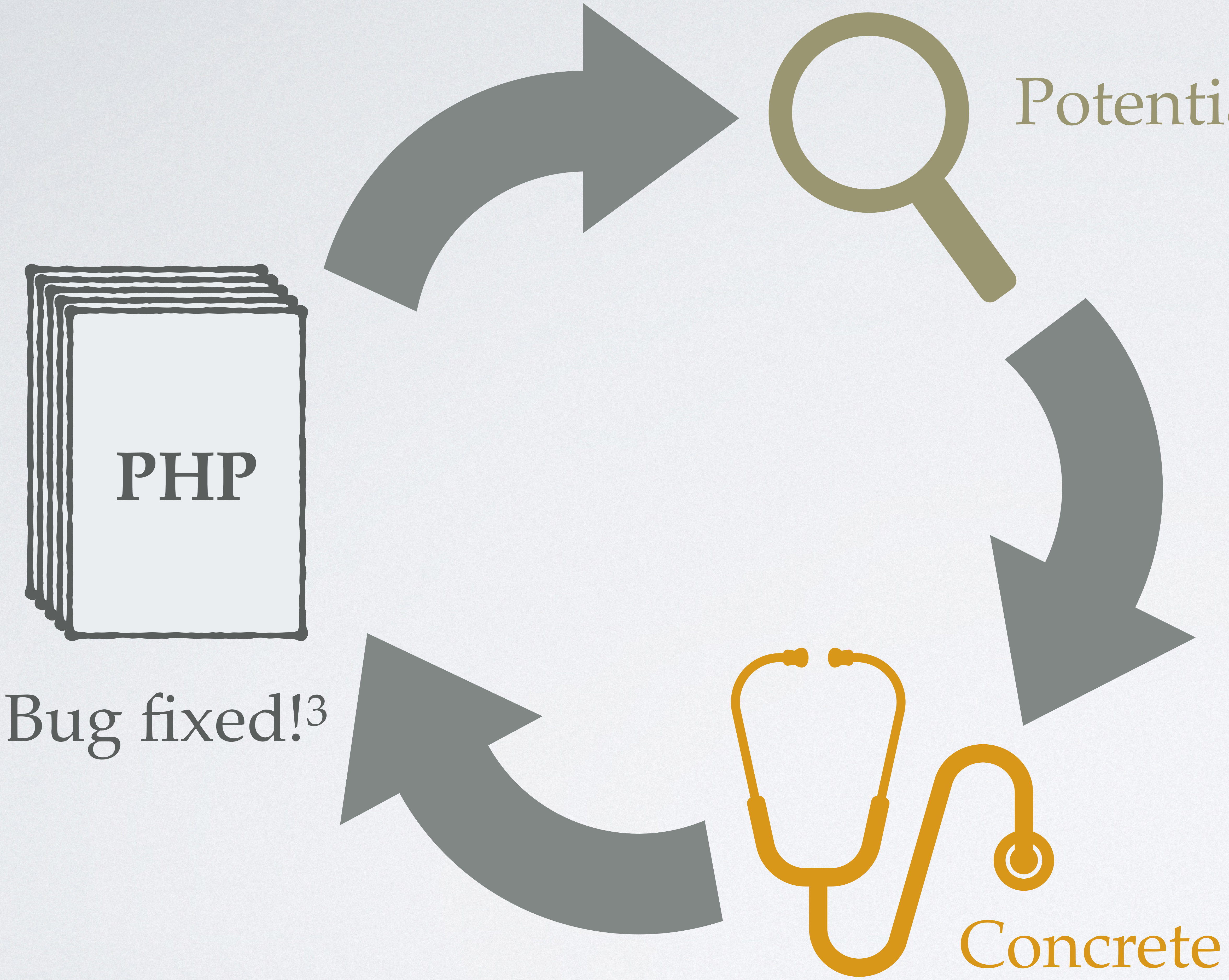

Potential Denial-of-Service attack[1]

Concrete exploits (by hash collisions)[2]

Bug fixed![3]

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Example of Worst-Case Analysis

Worst-case inputs are instrumental to understand and fix performance bugs!

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[2] PHP 5.3.8 - Hashtables Denial of Service. Available on https://www.exploit-db.com/exploits/18296/.

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Existing Approaches

EXISTING APPROACHES

Dynamic

- Fuzz testing
- Symbolic execution
- Dynamic worst-case analysis
- …

- Flexible & universal
- Potentially unsound: The resulting inputs might not expose the worst-case behavior.

Existing Approaches

Dynamic

- Fuzz testing
- Symbolic execution
- Dynamic worst-case analysis
- …

- Flexible & universal
- Potentially unsound: The resulting inputs might not expose the worst-case behavior.

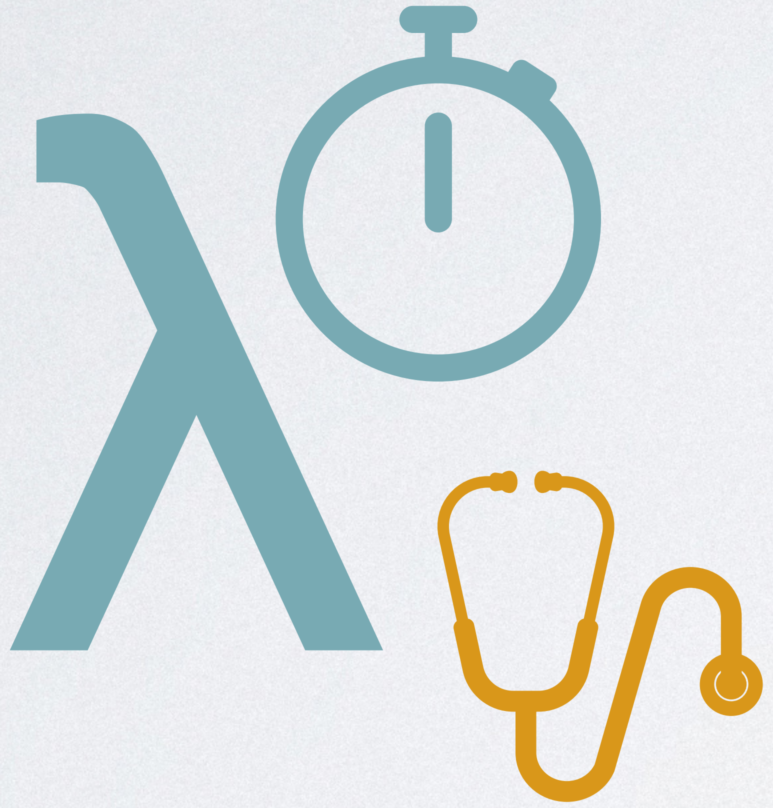
Static

- Type systems
- Abstract interpretation
- …

- Sound upper bounds
- Potentially not tight: No concrete witness — the bound might be too conservative.

Type-Guided Worst-Case Input Generation

D. Wang and J. Hoffmann. 2019. Type-Guided Worst-Case Input Generation. In *POPL'19*.



TYPE-GUIDED WORST-CASE INPUT GENERATION

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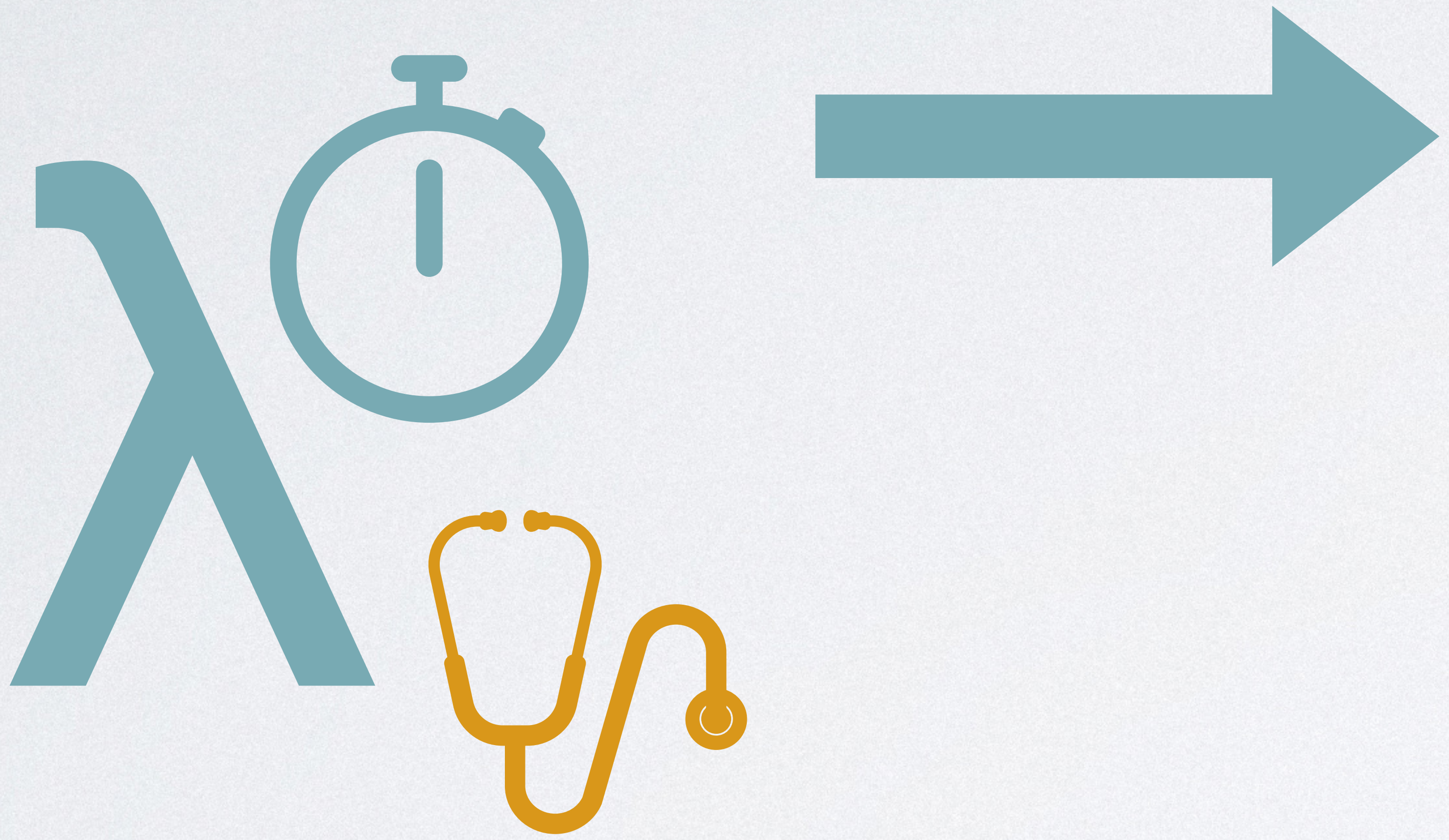

Resource Aware ML (RaML)

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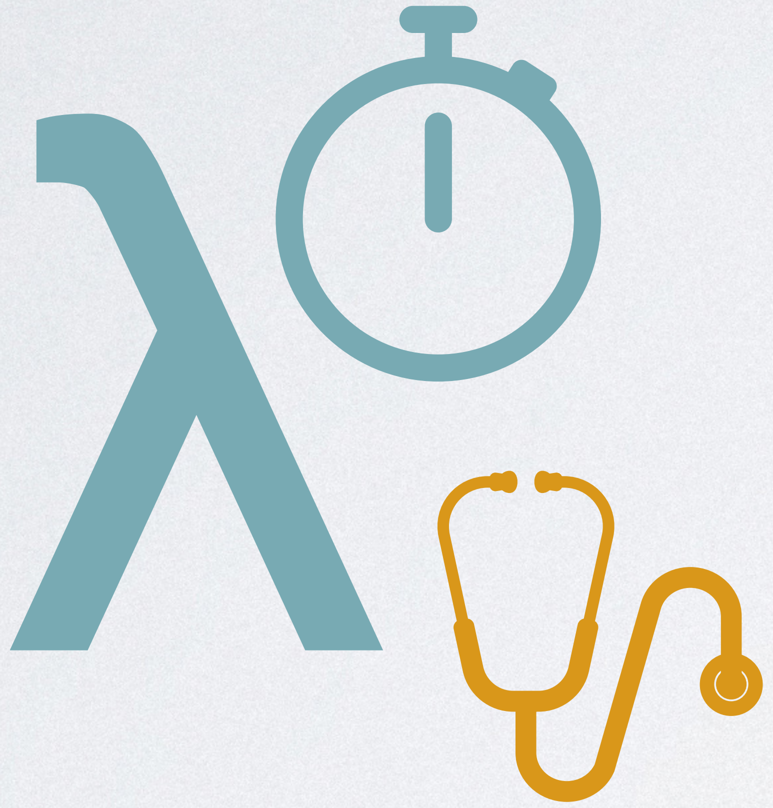

Resource Aware ML (RaML)

Type-Guided Worst-Case Input Generation

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Resource Aware ML (RaML)

Symbolic Execution

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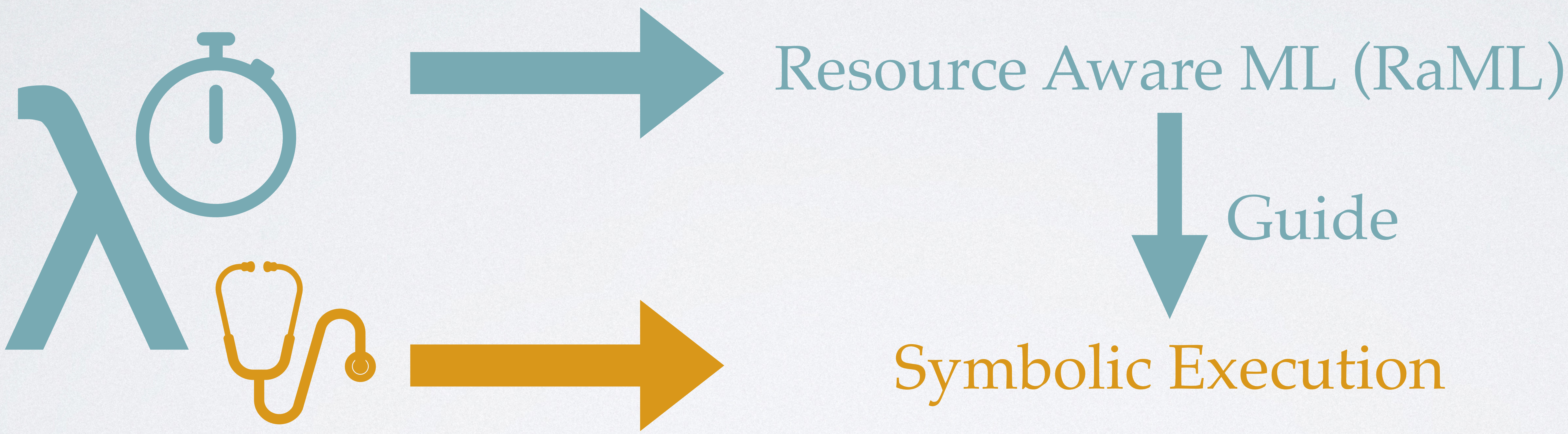

Symbolic Execution

- **Idea**: **search** all execution paths, **record** path constraints, and **compute** resource usage.

$$\gamma \vdash e \Rightarrow \langle \psi, S \rangle$$

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symbolic environment

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symbolic environment

expression

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- **Idea**: <u>**search**</u> all execution paths, <u>**record**</u> path constraints, and <u>**compute**</u> resource usage.

$$\gamma \vdash e \Rightarrow \langle \psi, S \rangle$$

symbolic environment

expression

path constraints

Symbolic Execution

- **Idea**: **search** all execution paths, **record** path constraints, and **compute** resource usage.

$$\gamma \vdash e \Rightarrow \langle \psi, S \rangle$$

symbolic environment

expression

path constraints

symbolic evaluation result

Symbolic Execution

- **Idea**: **search** all execution paths, **record** path constraints, and **compute** resource usage.

$$\gamma \vdash e \Rightarrow \langle \psi, S \rangle$$

symbolic environment

expression

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symbolic evaluation result

- Symbolic execution rules for conditional expressions:

SYMBOLIC EXECUTION

- **Idea**: **search** all execution paths, **record** path constraints, and **compute** resource usage.

$$\underbrace{\gamma}_{\text{symbolic environment}} \vdash \underbrace{e}_{\text{expression}} \Rightarrow \underbrace{\langle \underbrace{\psi}_{\text{path constraints}}, S \rangle}_{\text{symbolic evaluation result}}$$

- Symbolic execution rules for conditional expressions:

$$\boxed{\text{Then}} \quad \frac{\gamma \vdash e_1 \Rightarrow \langle \psi, S \rangle}{\gamma \vdash \text{if } e \text{ then } e_1 \text{ else } e_2 \Rightarrow \langle \gamma(e) \land \psi, S \rangle}$$

$$\boxed{\text{Else}} \quad \frac{\gamma \vdash e_2 \Rightarrow \langle \psi, S \rangle}{\gamma \vdash \text{if } e \text{ then } e_1 \text{ else } e_2 \Rightarrow \langle \neg\gamma(e) \land \psi, S \rangle}$$

Symbolic Execution

```
let rec lpairs l =
  match l with
  | [] -> []
  | x1::xs ->
    match xs with
    | [] -> []
    | x2::xs' ->
      if x1 < x2 then
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Symbolic Execution

```
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  match l with
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Worst case: always enter the then-branch

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Worst case: always enter the then-branch

- An example of worst-case execution paths for input lists of length 4:

$$\ell \mapsto [\mathsf{int}^1, \mathsf{int}^2, \mathsf{int}^3, \mathsf{int}^4] \vdash$$
$$\mathsf{lpairs}\ \ell \Rightarrow \langle (\mathsf{int}^1 < \mathsf{int}^2) \wedge (\mathsf{int}^3 < \mathsf{int}^4),$$
$$[(\mathsf{int}^1, \mathsf{int}^2), (\mathsf{int}^3, \mathsf{int}^4)] \rangle$$

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$$\mathsf{lpairs}\ \ell \Rightarrow \langle (\mathsf{int}^1 < \mathsf{int}^2) \wedge (\mathsf{int}^3 < \mathsf{int}^4), [(\mathsf{int}^1, \mathsf{int}^2), (\mathsf{int}^3, \mathsf{int}^4)] \rangle$$

- Invoke an SMT solver to find a model, e.g., [0,1,0,1].

Type-Guided Symbolic Execution

- **Nondeterminism** leads to state explosion:

$$\frac{\boxed{\text{Then}} \quad \gamma \vdash e_1 \Rightarrow \langle \psi, S \rangle}{\gamma \vdash \mathsf{if}\ e\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 \Rightarrow \langle \gamma(e) \land \psi, S \rangle} \qquad \frac{\boxed{\text{Else}} \quad \gamma \vdash e_2 \Rightarrow \langle \psi, S \rangle}{\gamma \vdash \mathsf{if}\ e\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 \Rightarrow \langle \neg\gamma(e) \land \psi, S \rangle}$$

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Use the information about **potentials** obtained from **resource aware type checking** to **prune the search space** of symbolic execution.

Type-Guided Symbolic Execution

$\langle L^1(\text{int}),0\rangle \rightarrow \langle L^0(\text{int} \times \text{int}),0\rangle$

```
let rec lpairs l =
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Waste!

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If an execution path does not have **potential waste**, it must expose the worst-case resource usage.

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$\Phi' = |xs'| = 2$

If an execution path does not have **potential waste**, it must expose the worst-case resource usage.

Prune the search space!

Theoretical Results

Soundness: If the algorithm generates ***an input***, then the input will cause the program to consume ***exactly*** the same amount of resource as the inferred ***upper bound*** (by RaML).

Speed up Input Generation

$$\boxed{\text{Then}}\quad \frac{\gamma \vdash e_1 \Rightarrow \langle \psi, S \rangle}{\gamma \vdash \texttt{if}\ e\ \texttt{then}\ e_1\ \texttt{else}\ e_2 \Rightarrow \langle \gamma(e) \wedge \psi, S \rangle}$$

$$\boxed{\text{Else}}\quad \frac{\gamma \vdash e_2 \Rightarrow \langle \psi, S \rangle}{\gamma \vdash \texttt{if}\ e\ \texttt{then}\ e_1\ \texttt{else}\ e_2 \Rightarrow \langle \neg\gamma(e) \wedge \psi, S \rangle}$$

Speed up Input Generation

- How about **eliminating** some generation rules?

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Still Sound!

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Still Sound!

- **Generalization**: enforce all the calls with the ***same*** shape of inputs execute the ***same*** path in the function body.

Example: Quicksort

Terminalizer

```
The default interactive shell is now zsh.
To update your account to use zsh, please run `chsh -s /bin/zsh`.
For more details, please visit https://support.apple.com/kb/HT208050.
Dis-iMac:raml di$
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```

OUTLINE

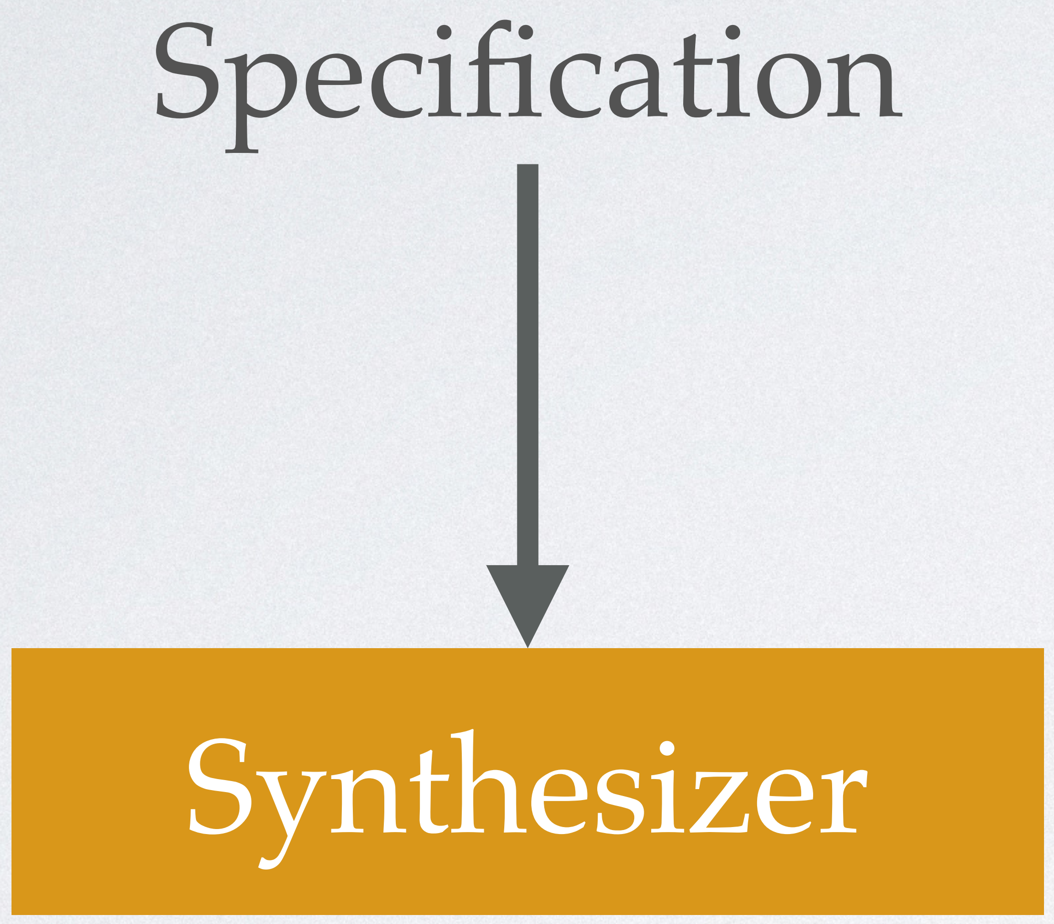
- ☑ Automatic Amortized Resource Analysis
- ☑ Type-Guided Worst-Case Input Generation
- ☐ Resource-Guided **Program Synthesis**

Type-Directed Synthesis

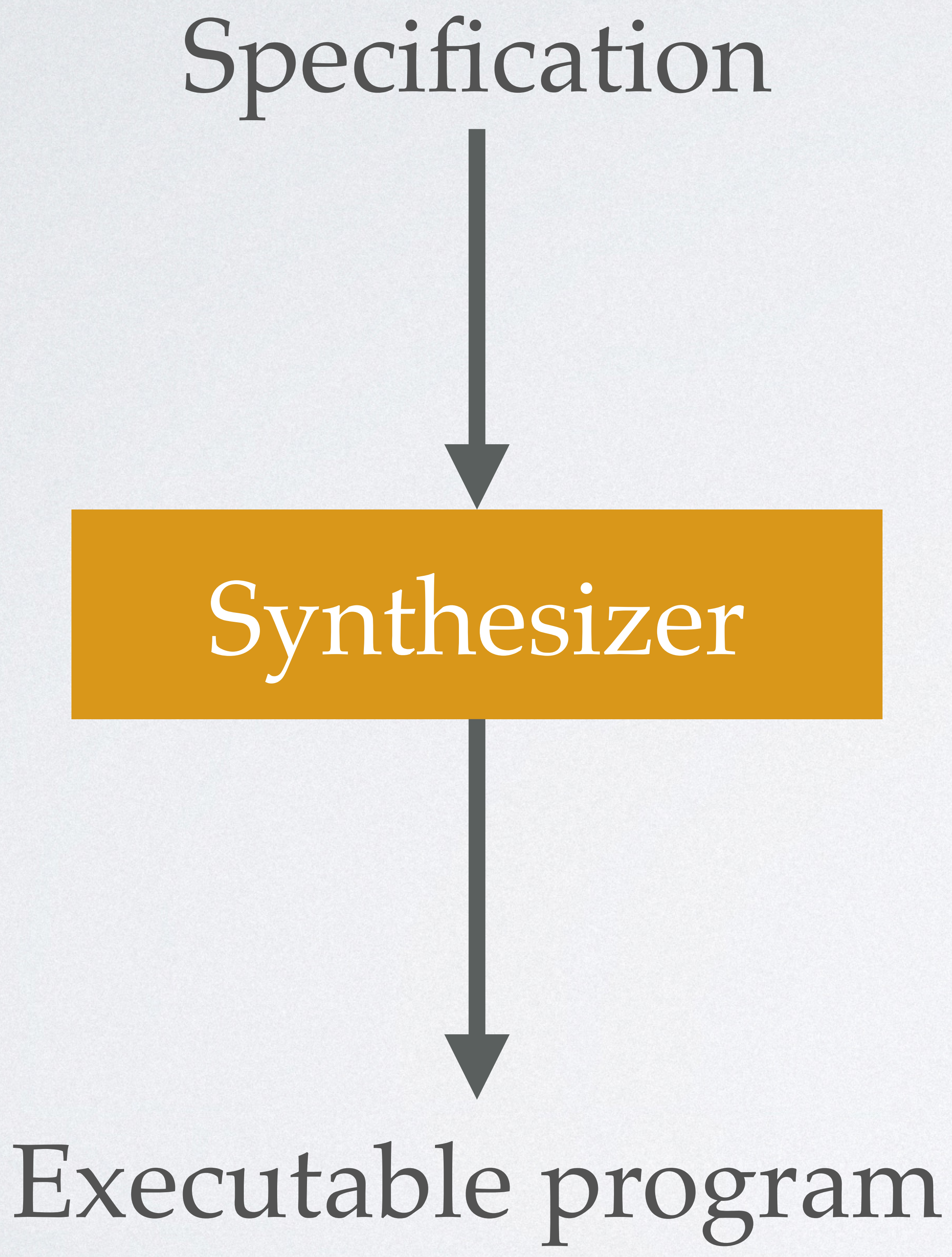
Type-Directed Synthesis

Specification

TYPE-DIRECTED SYNTHESIS



Type-Directed Synthesis



TYPE-DIRECTED SYNTHESIS

Specification

Synthesizer

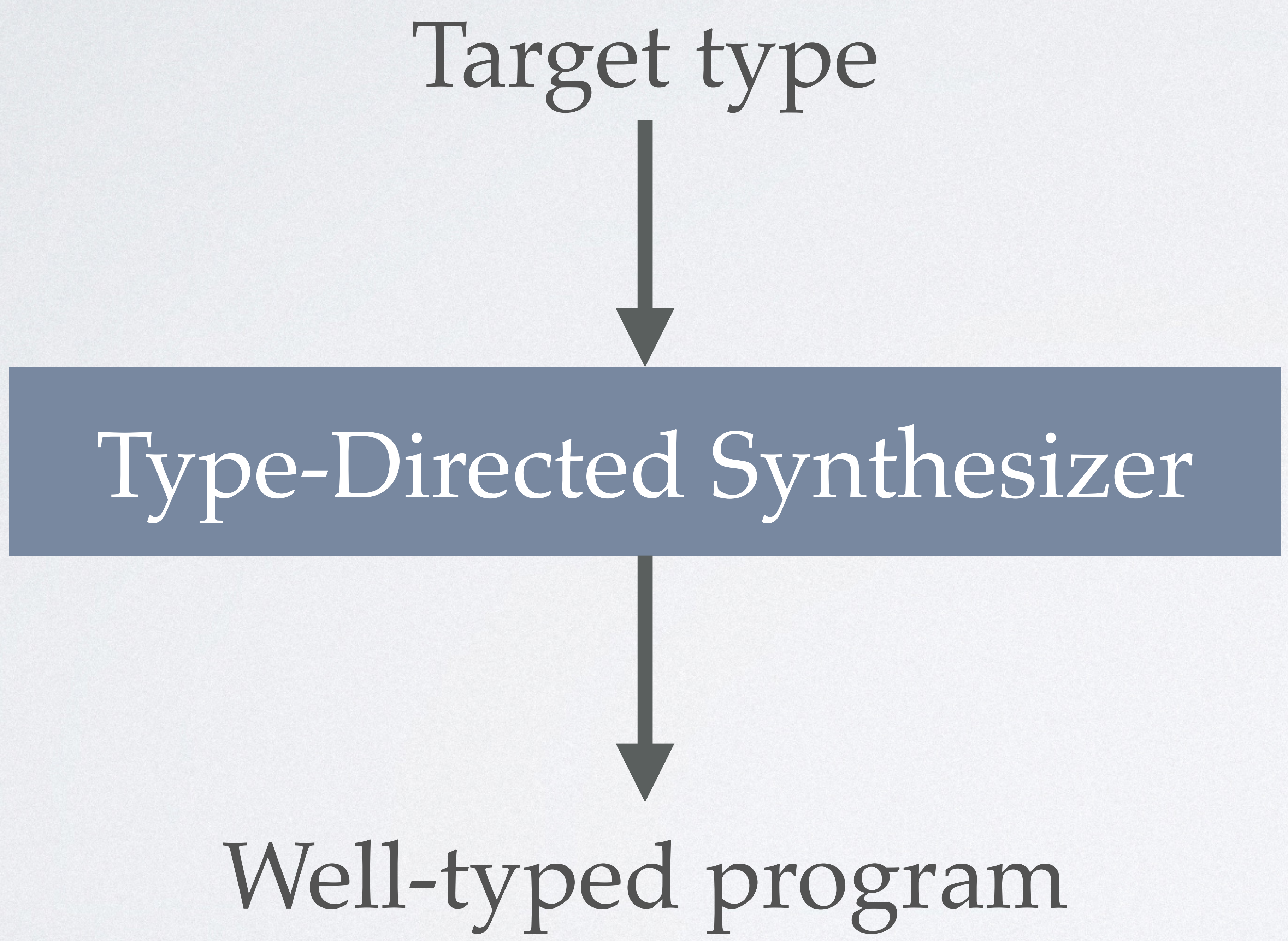
Executable program

Target type

Type-Directed Synthesizer

Well-typed program

Type-Directed Synthesis



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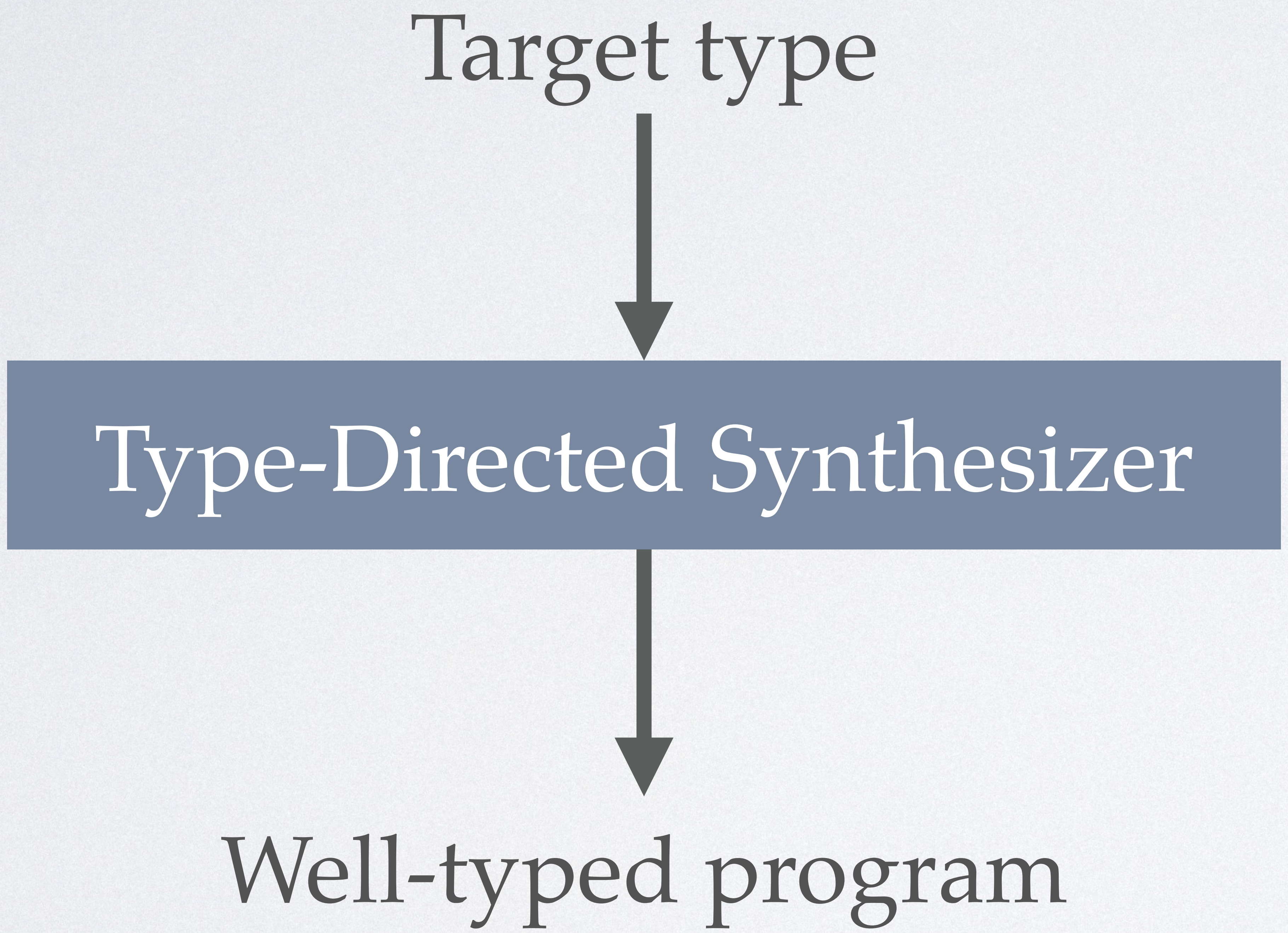



```
id : a -> a
```

Type-Directed Synthesis

Target type

↓

Type-Directed Synthesizer

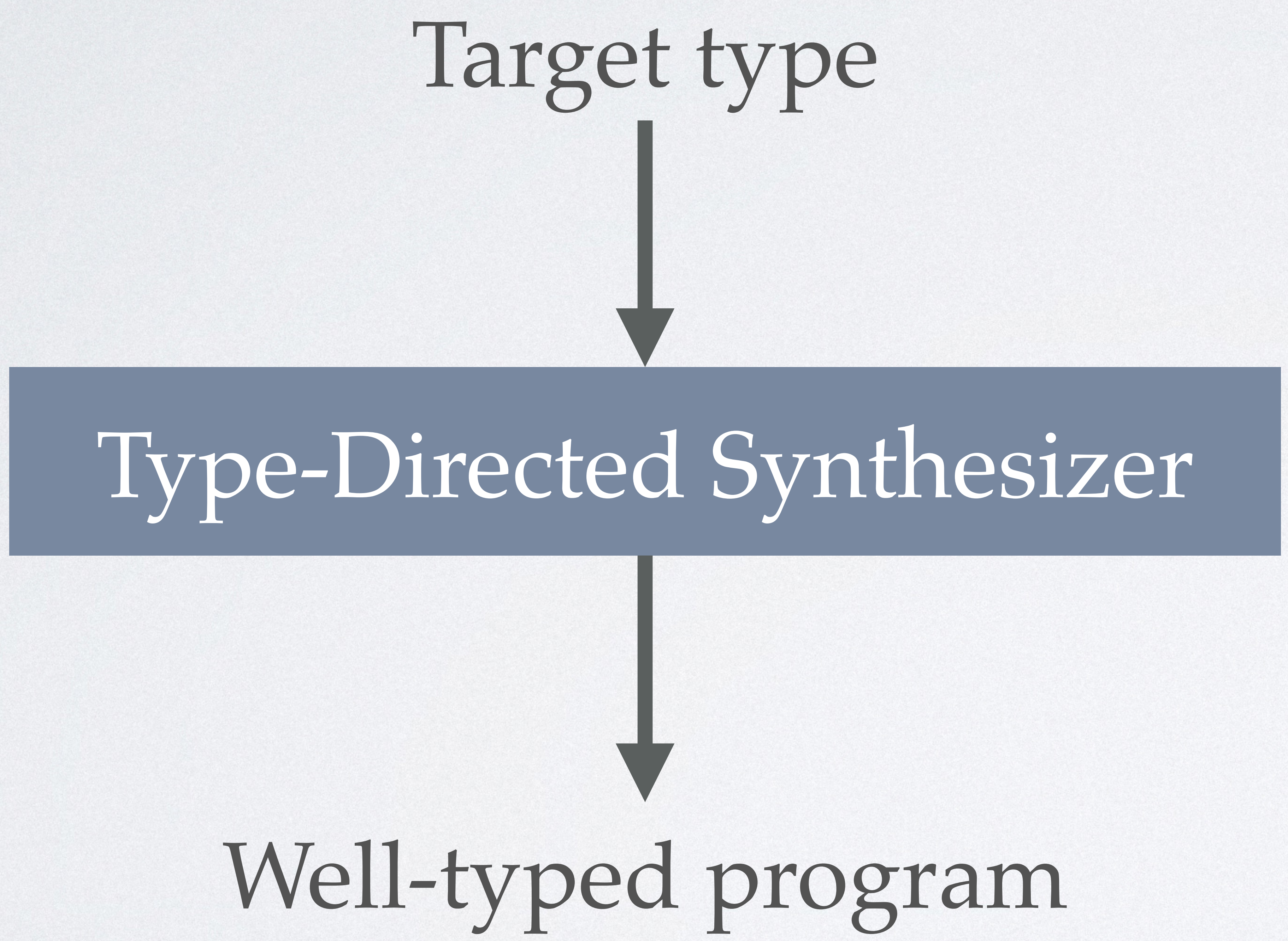
↓

Well-typed program

```
id : a -> a
```

```
let id x = x
```

Type-Directed Synthesis



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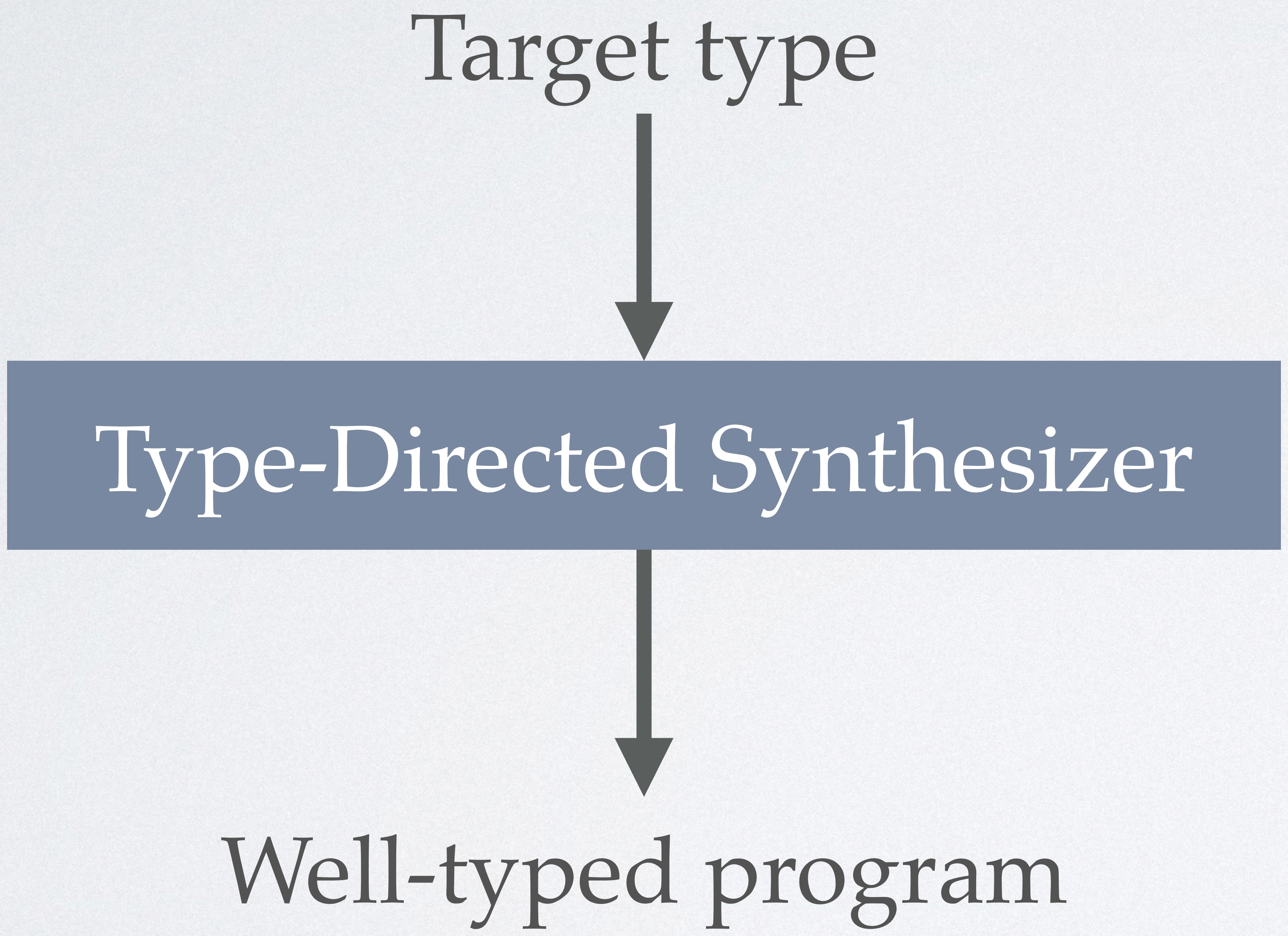
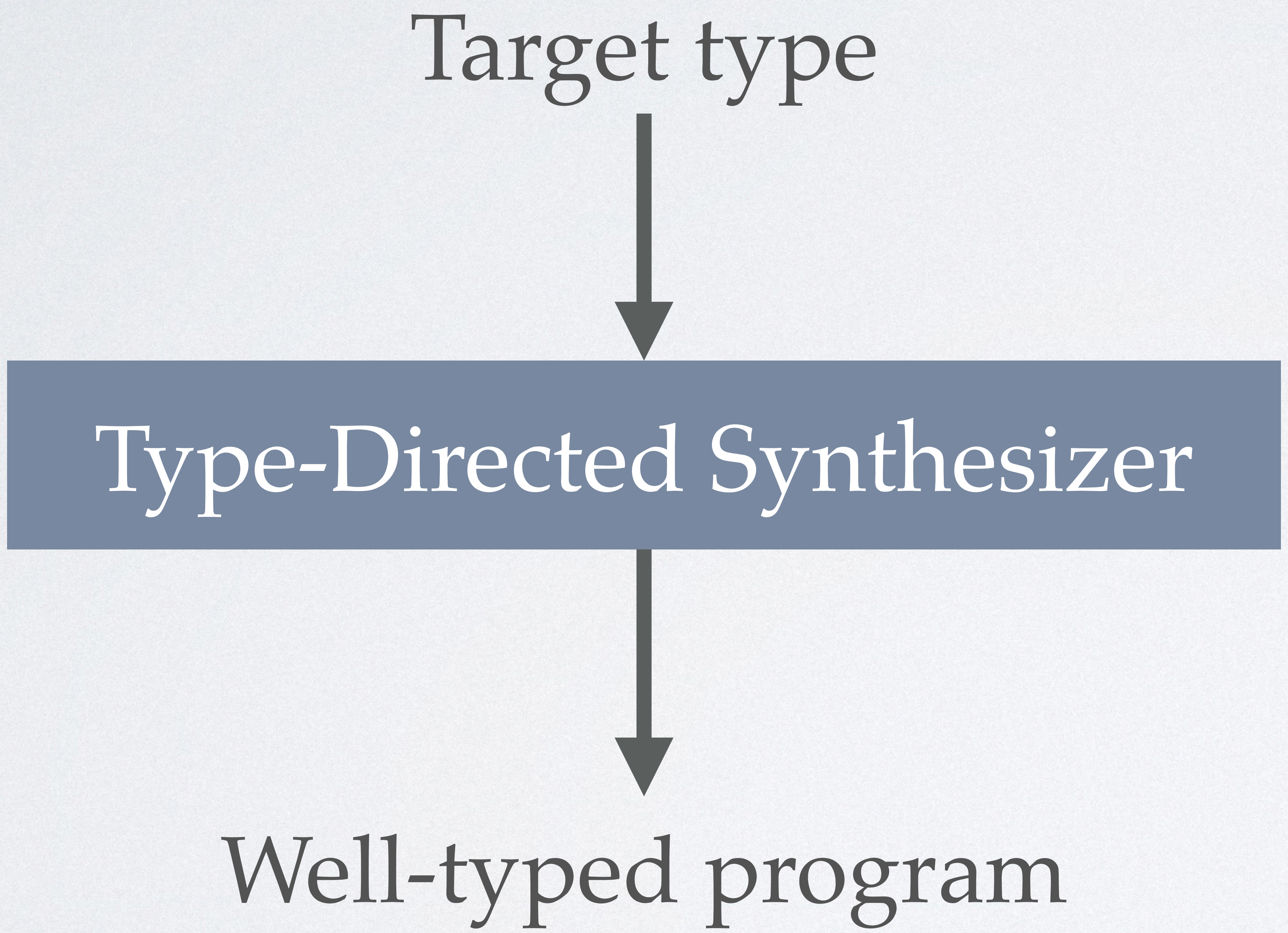



```
rep : int -> a -> List a
```

TYPE-DIRECTED SYNTHESIS




```
rep : int -> a -> List a
```

```
let rep n x = []
```

TYPE-DIRECTED SYNTHESIS

Target type

Type-Directed Synthesizer

Well-typed program

```
rep : int -> a -> List a
```

does not implement the replicate function

```
let rep n x = []
```

LIQUID TYPES

[RKJ08] P. M. Rondon, M. Kawaguchi, and R. Jhala. 2008. Liquid Types. In *PLDI'08*.

Liquid Types

`{ v: B | ψ }`

A value `v` of type `B` that satisfies `ψ`

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`{ Int | v ≥ 0 }` A non-negative integer

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Liquid Types

```
{ v: B | ψ }
```

A value `v` of type `B` that satisfies `ψ`

```
{ Int | v ≥ 0 }
```

A non-negative integer

```
(xs: List a) -> { List a | len(v) = len(xs) + 1 }
```

A function that returns a list whose length is one plus the length of its input

[RKJ08] P. M. Rondon, M. Kawaguchi, and R. Jhala. 2008. Liquid Types. In *PLDI'08*.

Synthesis with Liquid Types

```
rep : (n: int) -> a ->
{ List a | len(v) = n }
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Synthesis with Liquid Types

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let rec rep n x =
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  then []
  else x::(rep (n - 1) x)
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Reduce the synthesis problem to
finding an inhabitant of the target type

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SYNTHESIS WITH LIQUID TYPES

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Reduce the synthesis problem to finding an inhabitant of the target type

Use **type rules** to **reject** incomplete programs during the **search**

Synthesis with Liquid Types

```
  common : (xs: SList a) -> (ys: SList a) ->
{ SList a | elems(v) = elems(xs) ∩ elems(ys) }
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Type-Directed Synthesizer

```
let rec common xs ys =
  match xs with
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    if not (member x ys)
    then common xt ys
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```

SYNTHESIS WITH LIQUID TYPES

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Quadratic Complexity!
(#function calls)

ReSyn: Liquid Types + Linear Potentials

T. Knoth, D. Wang, N. Polikarpova, and J. Hoffmann. 2019. Resource-Guided Program Synthesis. In *PLDI'19*.

Potential: numeric

$$\{\ v : B \mid \psi\ \}^{\phi}$$

Refinement: boolean

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$$\{\ \texttt{Int}\ |\ v \geq 0\ \}^{5\cdot v}$$

A non-negative integer carrying potential equal to 5 times of its value

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A non-negative integer carrying potential equal to 5 times of its value

$\mathtt{List}\ a^{\mathtt{ite}(v \geq 0, 1, 0)}$

A list of numbers carrying potential equal to #non-negative elements in it

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Refinement: boolean

Type-checking is reduced to constraint solving in Presburger arithmetic.

`{ Int | v ≥ 0 }`$^{5\cdot v}$

A non-negative integer carrying potential equal to 5 times of its value

`List a`$^{\mathtt{ite(v\geq 0,1,0)}}$

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Resource-Guided Synthesis

```
common : (xs: SList a¹) -> (ys: SList a¹) ->
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member : (z: a) -> (zs: SList a¹) ->
{ Bool | v = (z ∈ elems(zs) }
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```

each element in the list carries **one unit** of potential,
thus the complexity must be **linear** in the list length

Resource-Guided Synthesis

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  ??
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    then common xt ys
    else ??
```

RESOURCE-GUIDED SYNTHESIS

```
 common : (xs: SList a¹) -> (ys: SList a¹) ->
{ SList a | elems(v) = elems(xs) ∩ elems(ys) }


 let rec common xs ys =
   match xs with
   | [] -> []
   | x::xt ->
     if not (member x ys)    ys: List aᵖ <: List a¹
     then common xt ys       ys: List aᑫ <: List a¹
     else ??
```

Resource-Guided Synthesis

```
common : (xs: SList a^1) -> (ys: SList a^1) ->
{ SList a | elems(v) = elems(xs) ∩ elems(ys) }
```

```
let rec common xs ys =
  match xs with
  | [] -> []
  | x::xt ->
    if not (member x ys)    ys: List a^p <: List a^1      [p ≥ 1]
    then common xt ys       ys: List a^q <: List a^1      [q ≥ 1]
    else ??
```

Resource-Guided Synthesis

```
common : (xs: SList a¹) -> (ys: SList a¹) ->
{ SList a | elems(v) = elems(xs) ∩ elems(ys) }
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    else ??
```

Potential Sharing

```
ys: List aᵖ <: List a¹     [p ≥ 1]
ys: List aᵠ <: List a¹     [q ≥ 1]
```

RESOURCE-GUIDED SYNTHESIS

```
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{ SList a | elems(v) = elems(xs) ∩ elems(ys) }
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    else ??
```

Potential Sharing

$[1 \geq p+q]$

ys: List a^p <: List a^1 $[p \geq 1]$

ys: List a^q <: List a^1 $[q \geq 1]$

Resource-Guided Synthesis

```
common : (xs: SList a¹) -> (ys: SList a¹) ->
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```

Potential Sharing

ys: List a^p <: List a^1

ys: List a^q <: List a^1

[1 ≥ p+q]

[p ≥ 1]

[q ≥ 1]

Infeasible!

Resource-Guided Synthesis

```
common : (xs: SList a¹) -> (ys: SList a¹) ->
{ SList a | elems(v) = elems(xs) ∩ elems(ys) }
```

```
let rec common xs ys =
  match xs with
  | [] -> []
  | x::xt ->
    match ys with
    | [] -> []
    | y::yt ->
      if x < y then common xt ys
      else if y < x then common xs yt
      else x::(common xt yt)
```

Example: List Append

ReSyn - resource-guided program synthesis

Triple | List Compress | List Intersect | List Insert | List Range | More ReSyn

```

-- | Let's ask ReSyn to generate a function
-- that returns a list three times the size of the input list `xs`,
-- and it only allowed two linear traversals over `xs`.
-- Try uncommenting different versions of the append
-- and observe how ReSyn associates the calls to append differently
-- to make sure they only perform two traversals of `xs`.
-- Also, try adding `-r=False` in the command line to disable resource analysis,
-- and observe that calls are always associated to the right.
triple :: xs: List {a| |2} -> {List a | len _v == 3 * (len xs)}
triple = ??

-- | A regular version of append.
-- | Its type signature requires 1 unit of potential for every element of `xs`,
-- | indicating that it performs a linear traversal over its first argument.
-- append :: xs: List {a| |1} -> ys: List a -> {List a | len _v == len xs + len ys}

-- | An append-and-swap function.
-- | Its type signature requires 1 unit of potential for every element of `ys`,
-- | indicating that it performs a linear traversal over its second(!) argument.
```

Run... -m 0

```
triple = \xs . appendSwap (appendSwap xs
                              xs) xs
```

EXAMPLE: LIST APPEND

ReSyn - resource-guided program synthesis

Triple | List Compress | List Intersect | List Insert | List Range | More ReSyn

```
-- | Let's ask ReSyn to generate a function
-- that returns a list three times the size of the input list `xs`,
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```

Run... -m 0

```
triple = \xs . appendSwap (appendSwap xs
                          xs) xs
```

OUTLINE

- ☑ Automatic Amortized Resource Analysis
- ☑ Type-Guided Worst-Case Input Generation
- ☑ Resource-Guided Program Synthesis