Sound Probabilistic Inference via Guide Types

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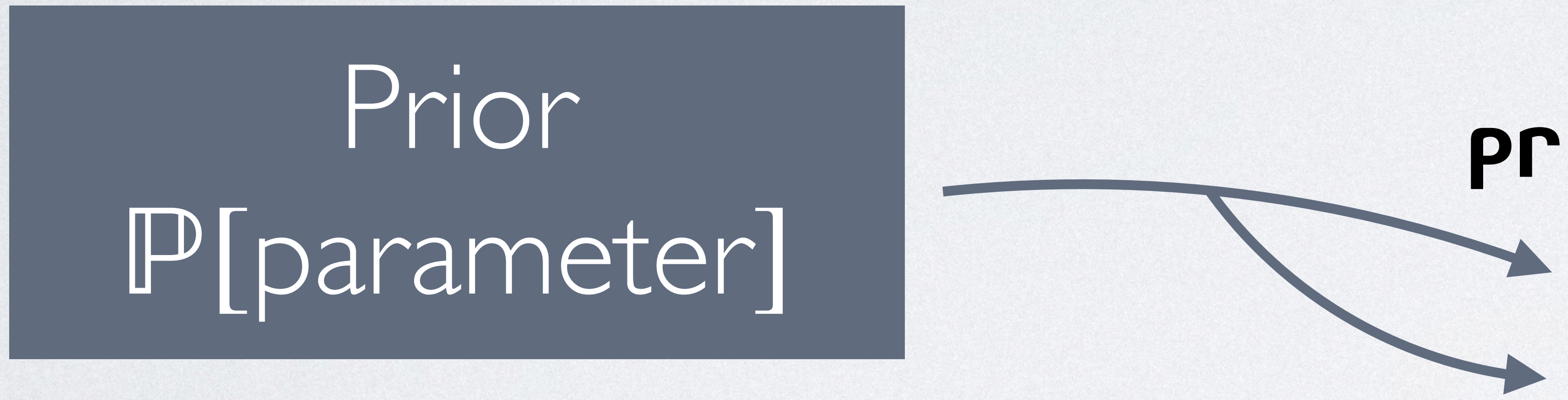
Probabilistic Programming

A Flexible Way of Describing Statistical Models

```
proc model() {
  param1 <- sample(Normal(2, 1));
  param2 <- sample(Normal(-2, 1));
  data <- sample(Normal(param1 * param2, 10));
  return
}
```

Probabilistic Programming

A Flexible Way of Describing **Statistical Models**



Prior

ℙ[parameter]

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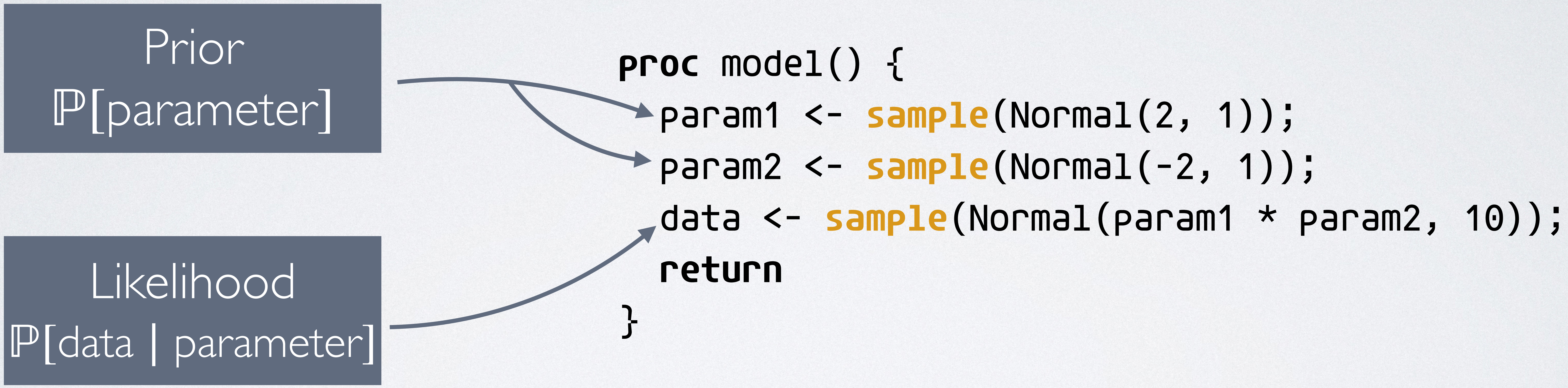

Prior

$\mathbb{P}$[parameter]

Likelihood

$\mathbb{P}$[data | parameter]

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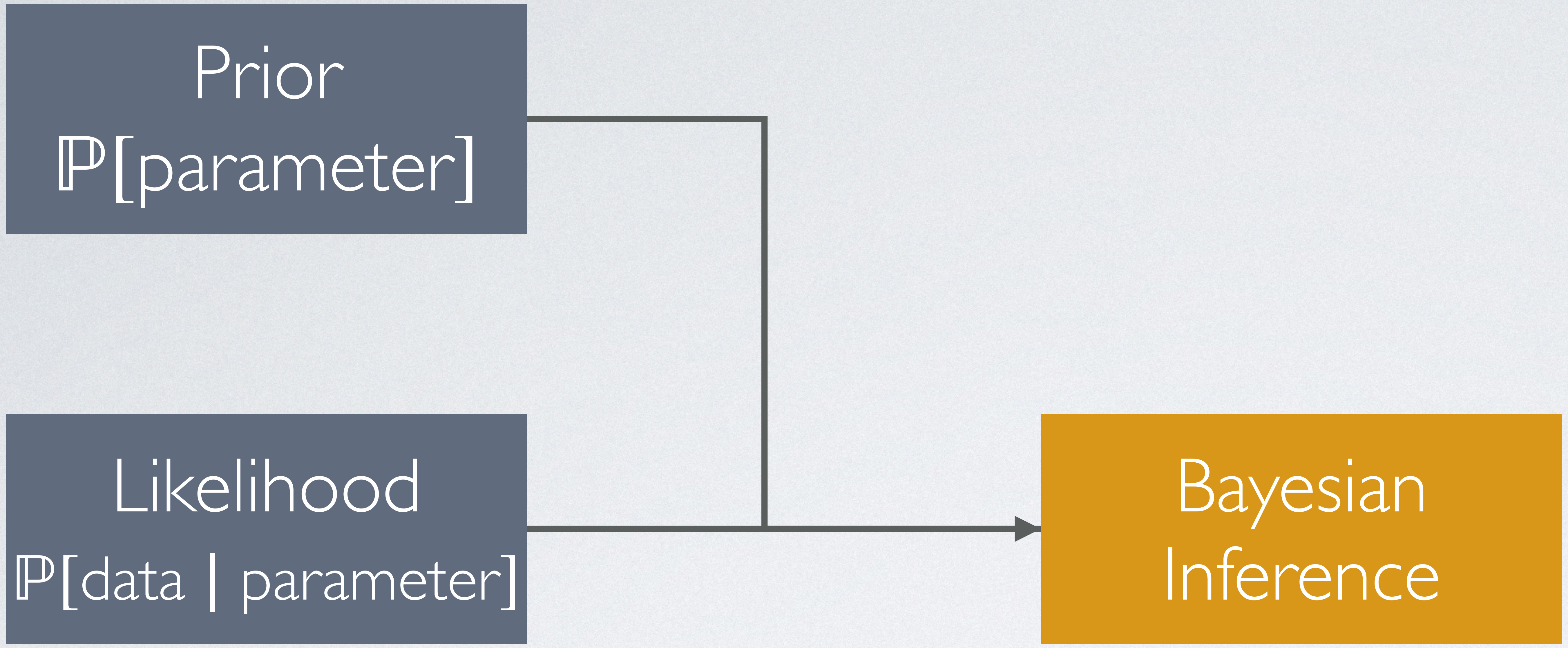
Bayesian Inference



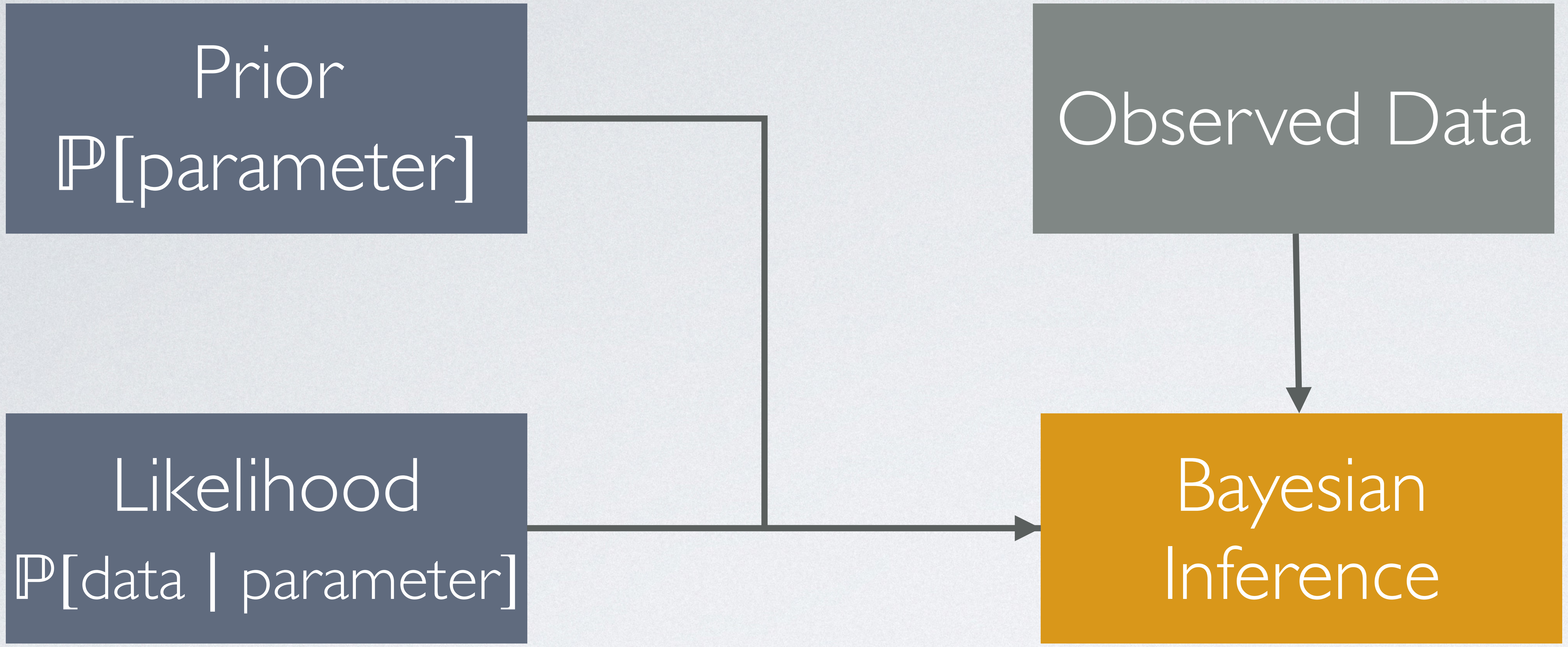
Prior

$\mathbb{P}$[parameter]

Likelihood

$\mathbb{P}$[data | parameter]

Bayesian Inference



Prior

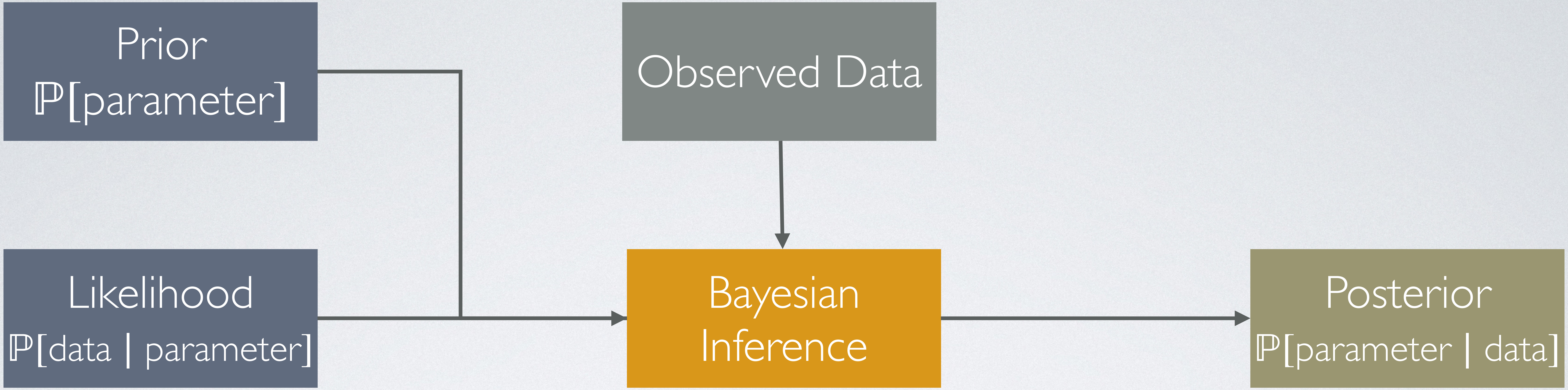
$\mathbb{P}[\text{parameter}]$

Likelihood

$\mathbb{P}[\text{data} \mid \text{parameter}]$

Observed Data

Bayesian Inference



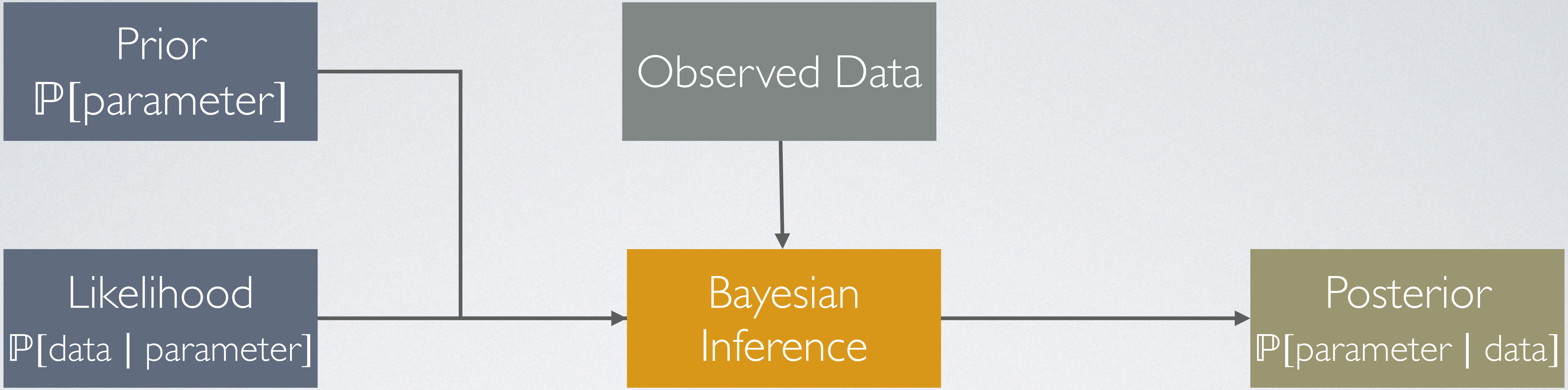
Prior
$\mathbb{P}[\text{parameter}]$

Observed Data

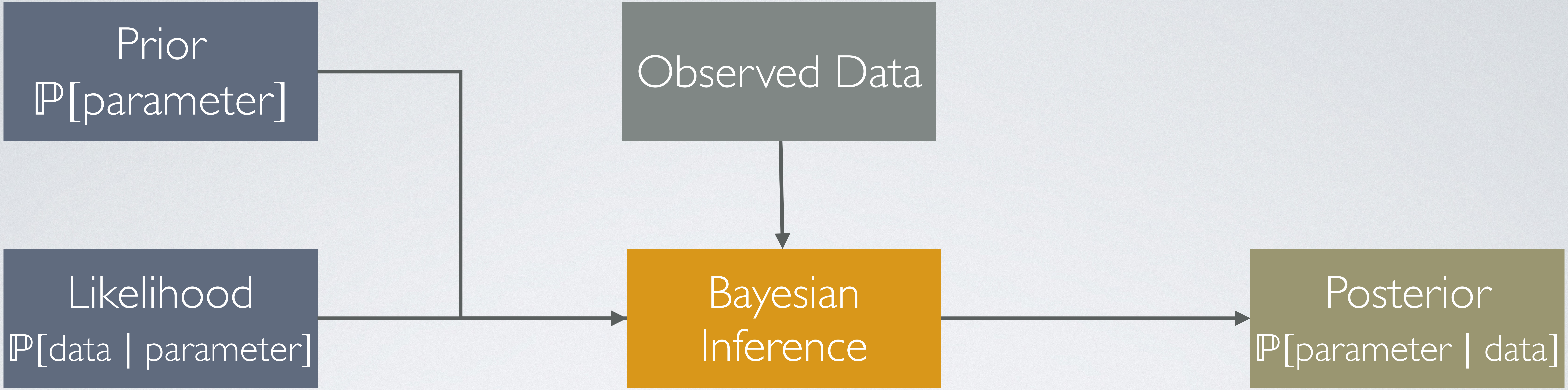
Likelihood
$\mathbb{P}[\text{data} \mid \text{parameter}]$

Bayesian Inference

Posterior
$\mathbb{P}[\text{parameter} \mid \text{data}]$

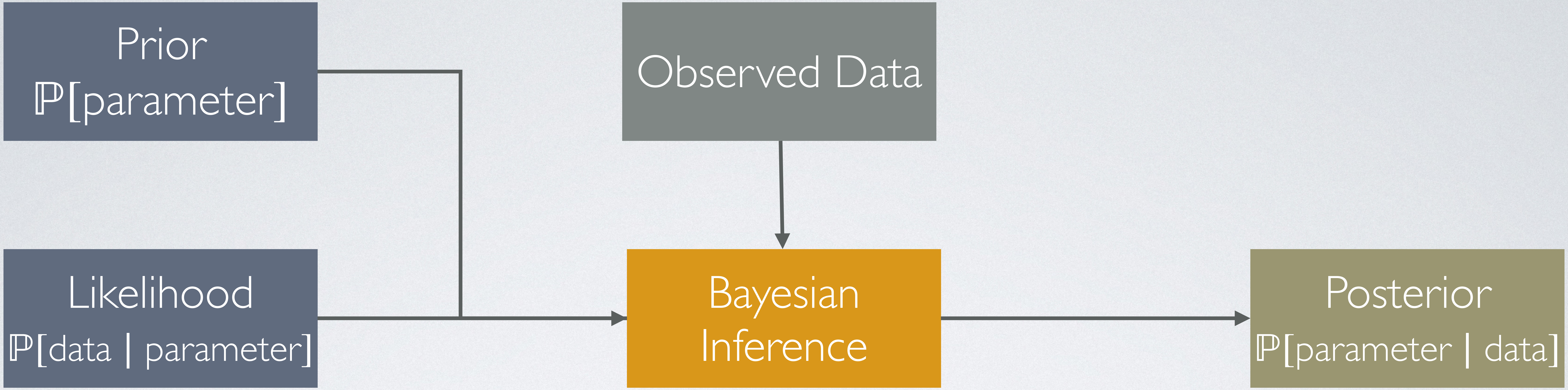


- Bayesian inference is good at reasoning about **uncertainty** in model parameters



- Bayesian inference is good at reasoning about **uncertainty** in model parameters

- **Downside:** No single algorithm works well for all models



- Bayesian inference is good at reasoning about **uncertainty** in model parameters

- **Downside:** No single algorithm works well for all models

- **This paper:** **Customize** Bayesian inference while maintaining **soundness**

GUIDE-BASED INFERENCE

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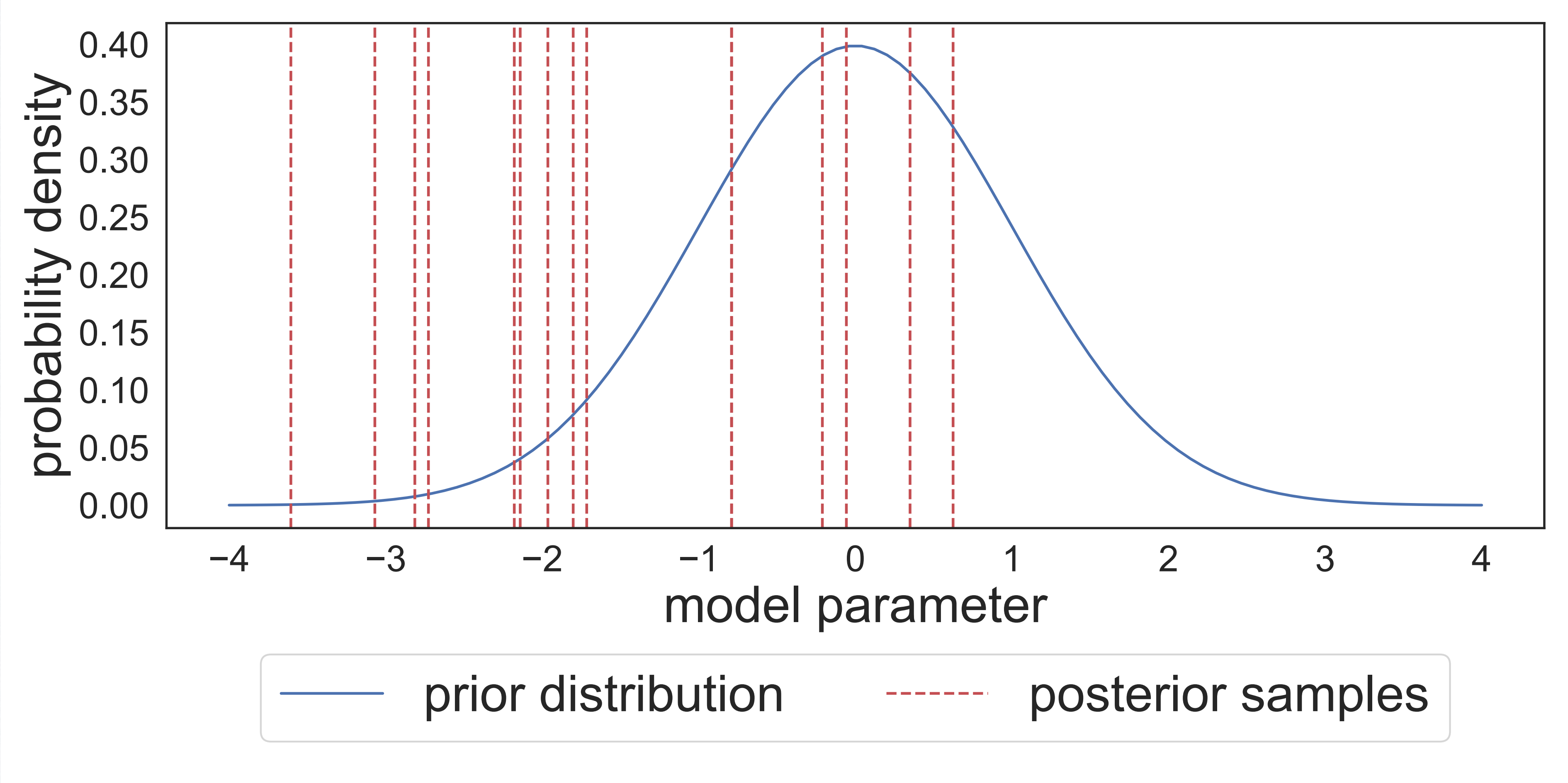
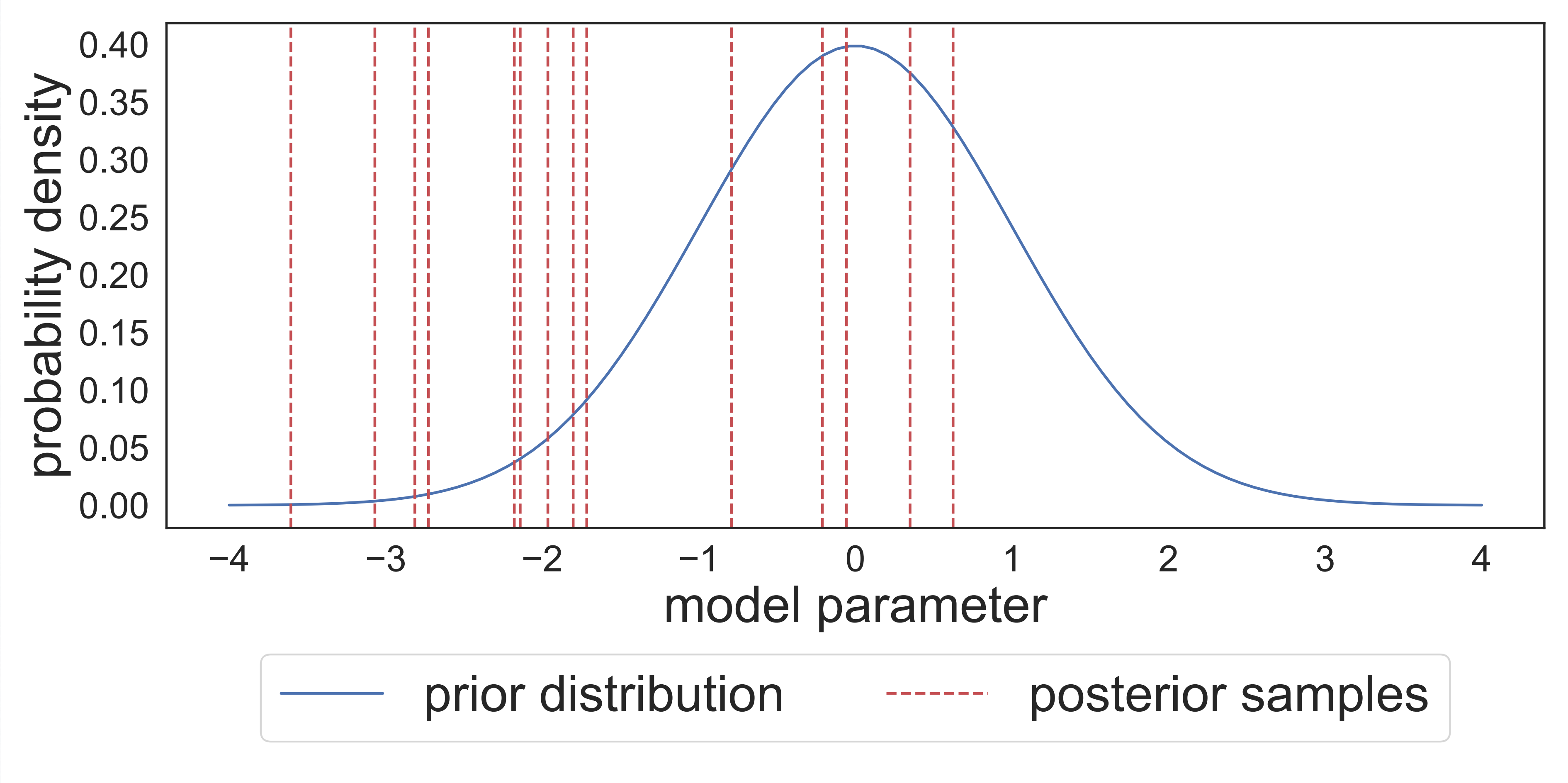
Monte-Carlo Methods



GUIDE-BASED INFERENCE

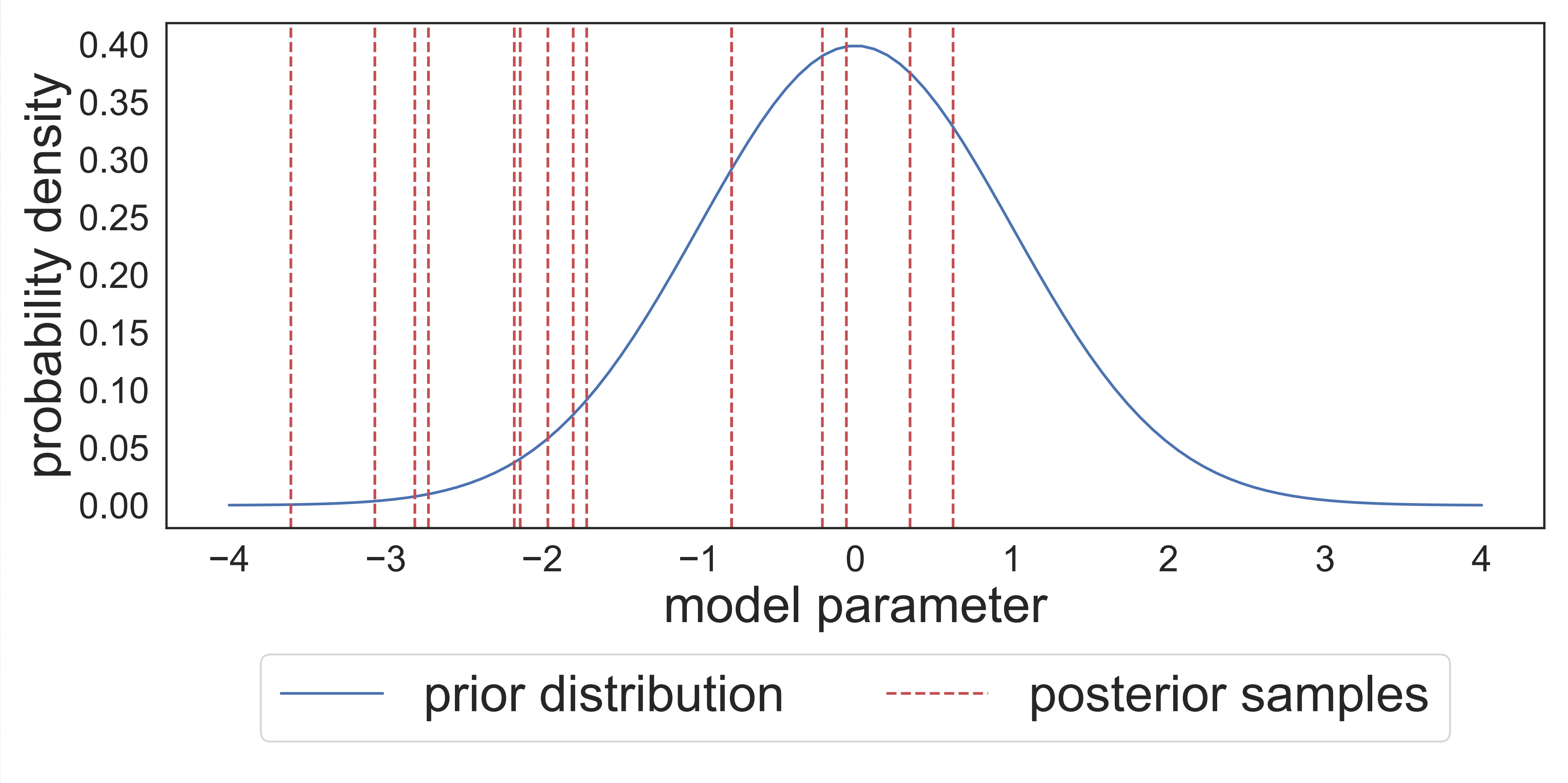
Monte-Carlo Methods



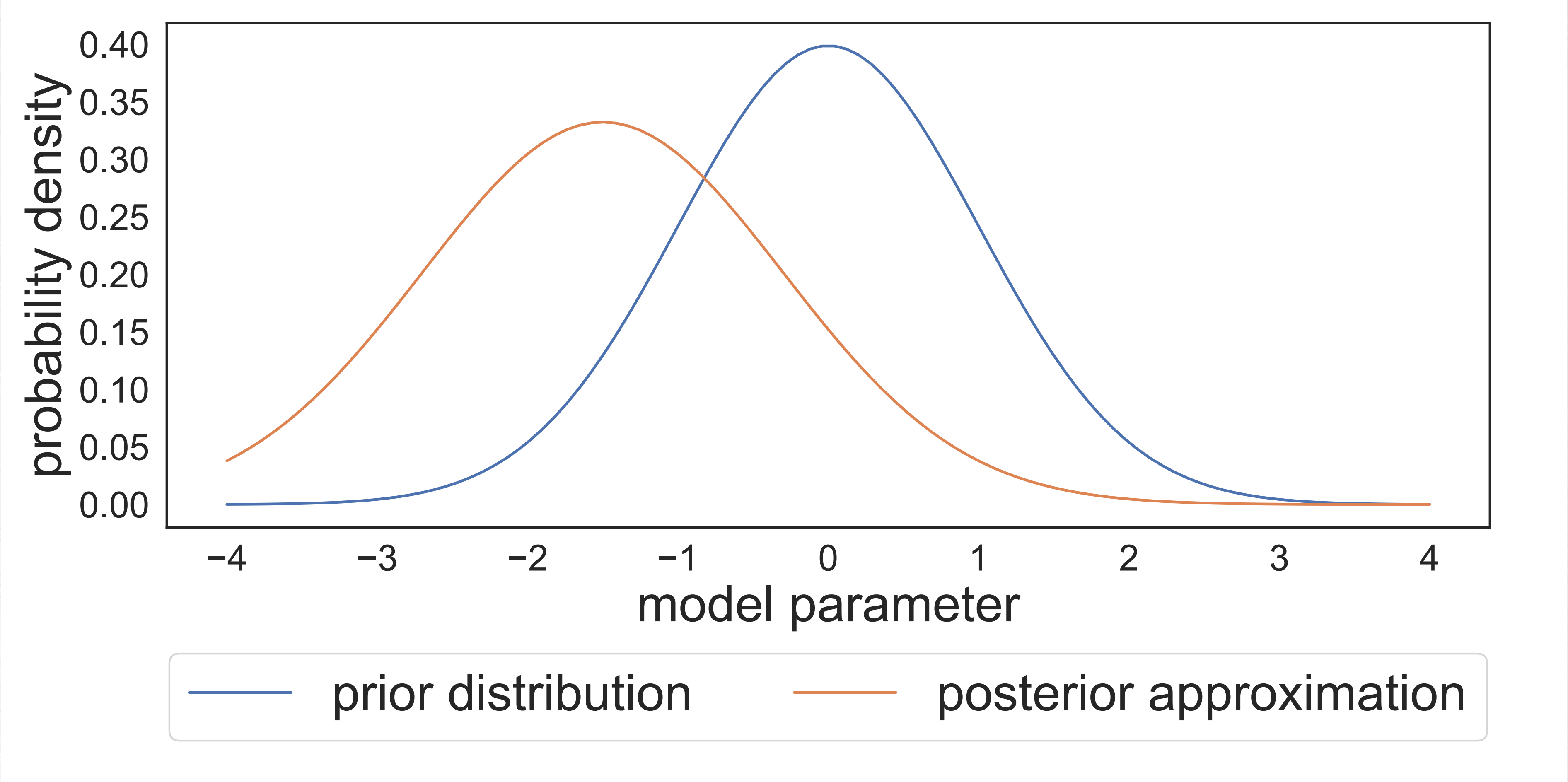
How to **guide** Monte-Carlo methods to explore the sample space for parameters?

GUIDE-BASED INFERENCE

Monte-Carlo Methods



Variational Methods



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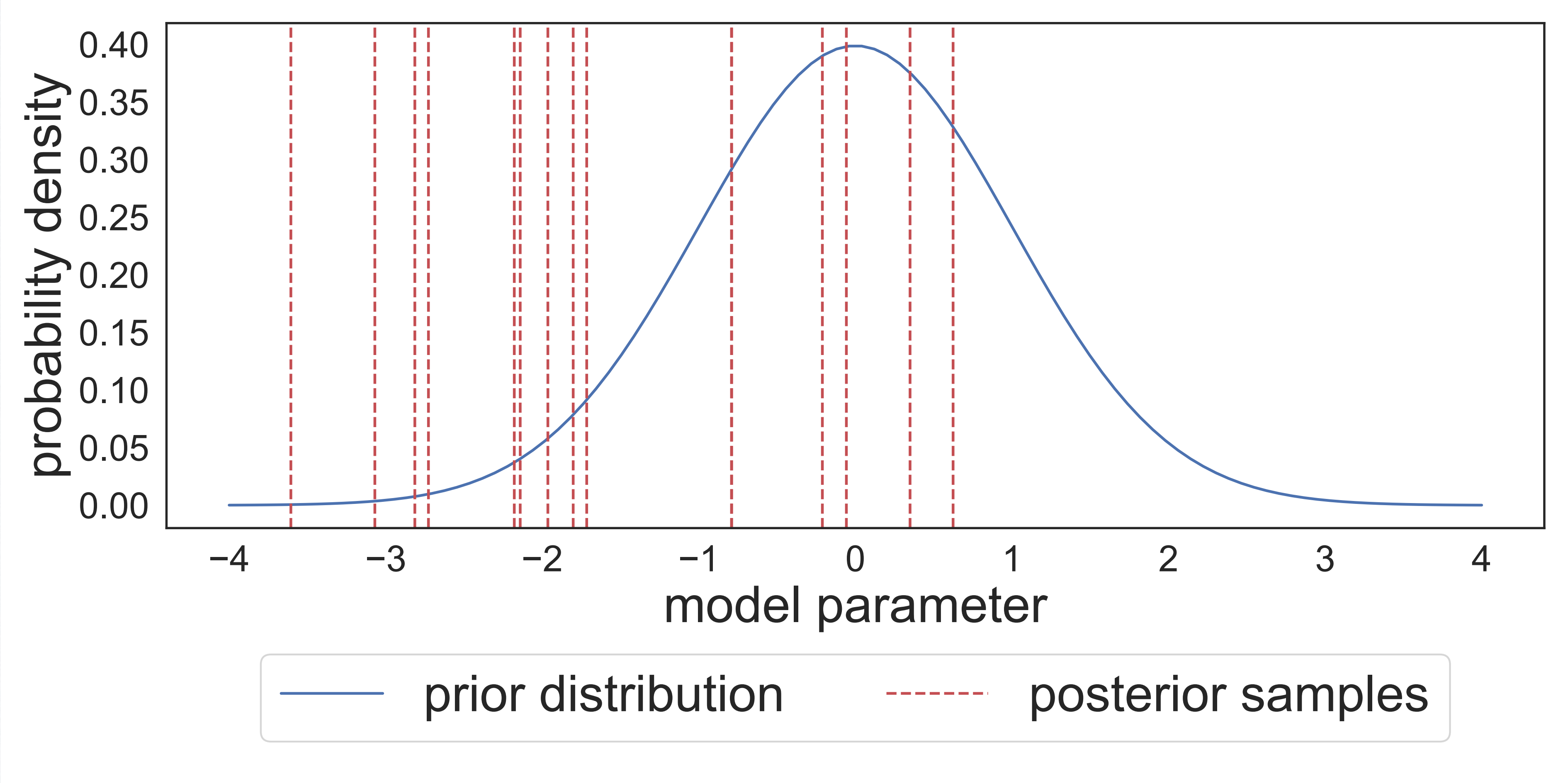
GUIDE-BASED INFERENCE

Monte-Carlo Methods



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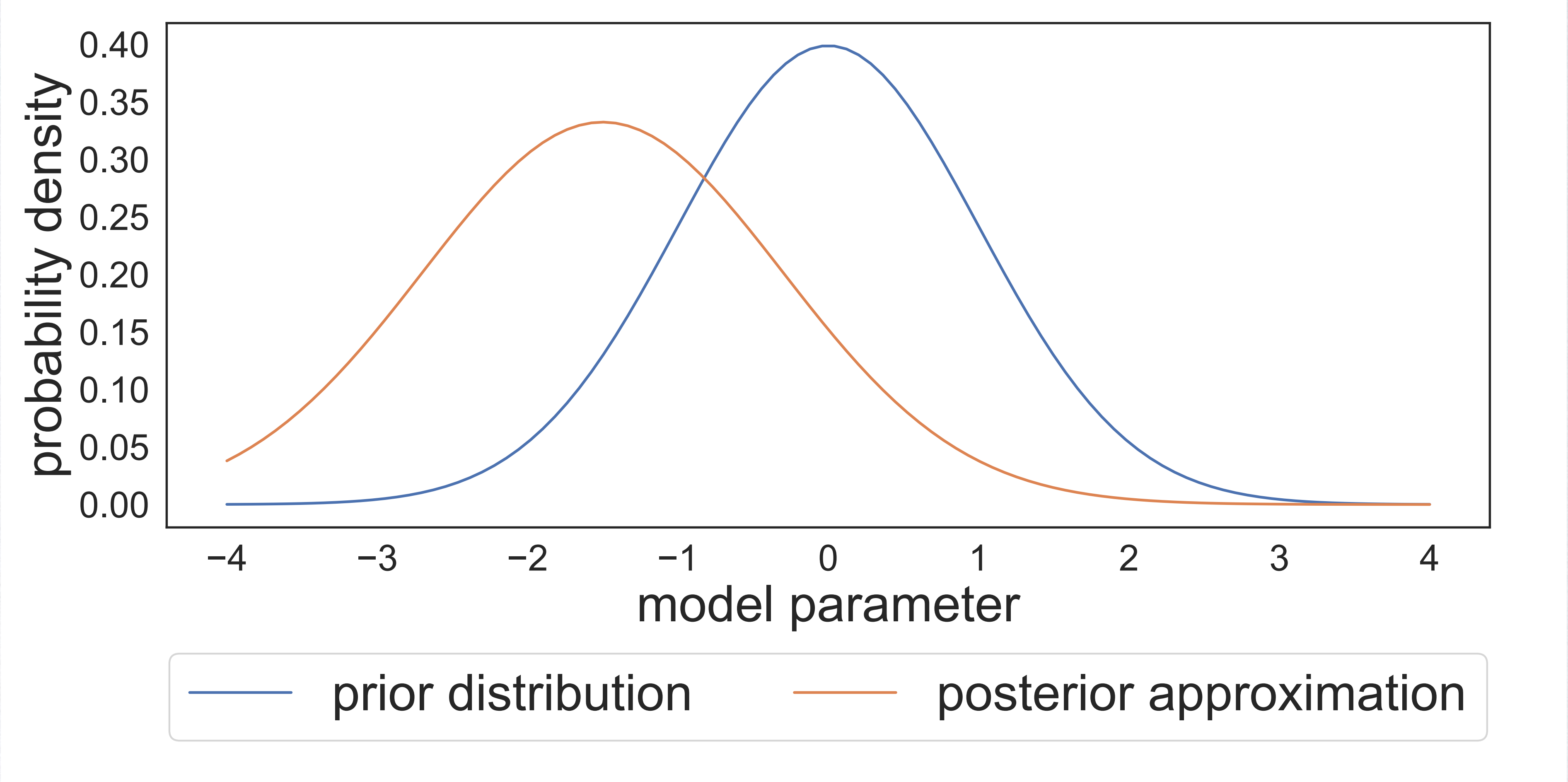
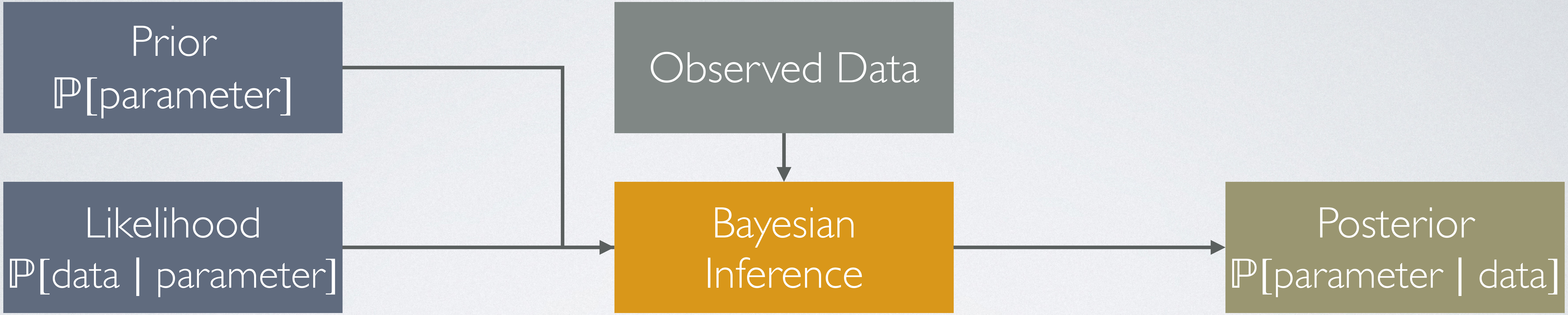


How to specify the **guide** distributions used by variational methods for approximation?

Guide-Based Inference



Prior
ℙ[parameter]

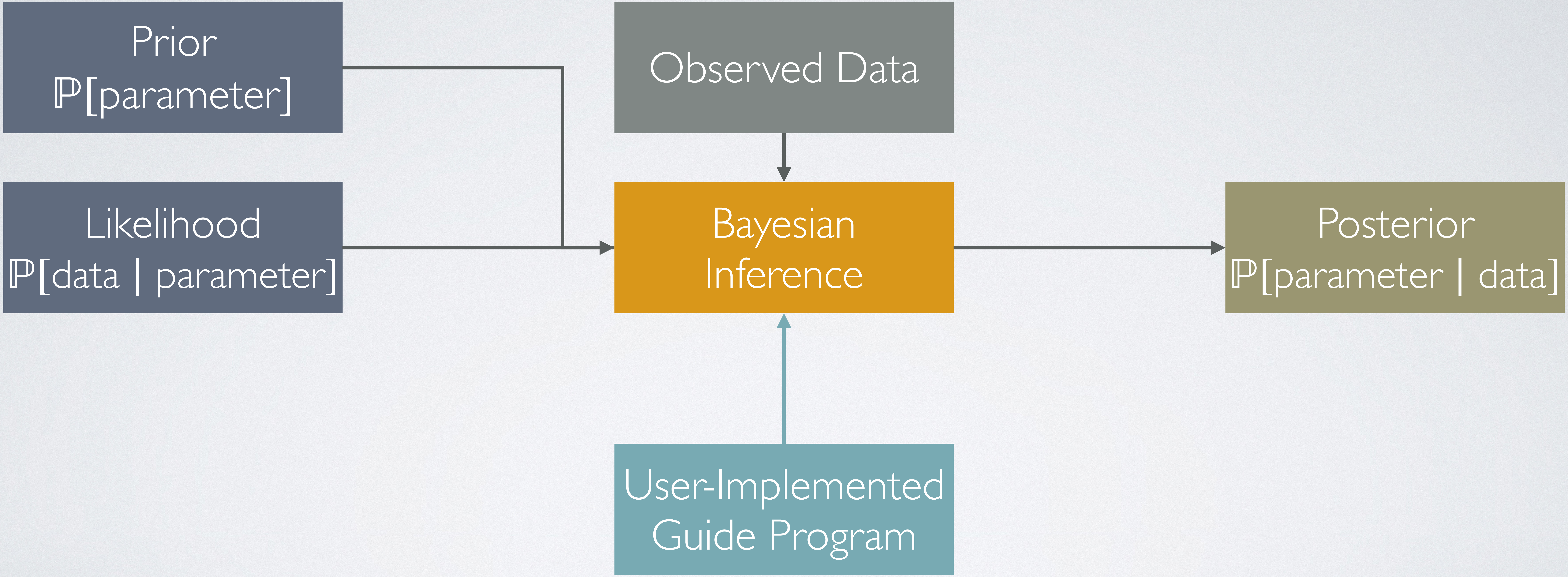
Likelihood
ℙ[data | parameter]

Observed Data

Bayesian Inference

Posterior
ℙ[parameter | data]

GUIDE-BASED INFERENCE



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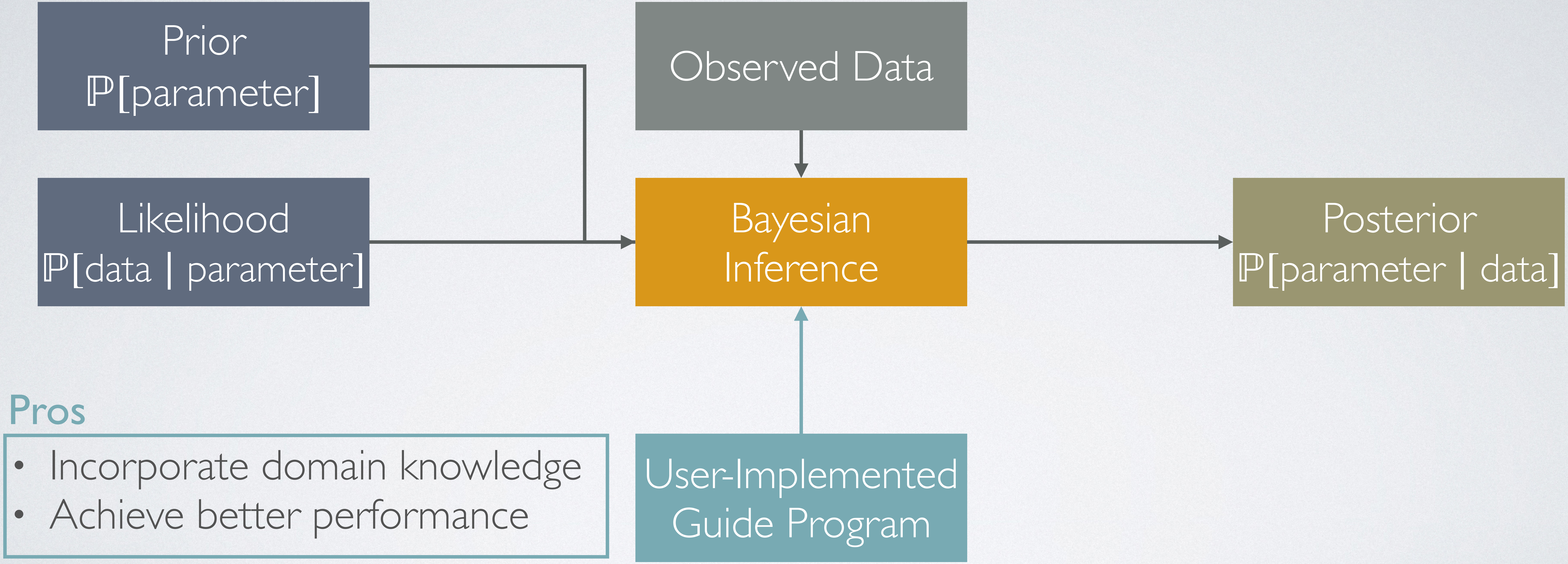

Prior
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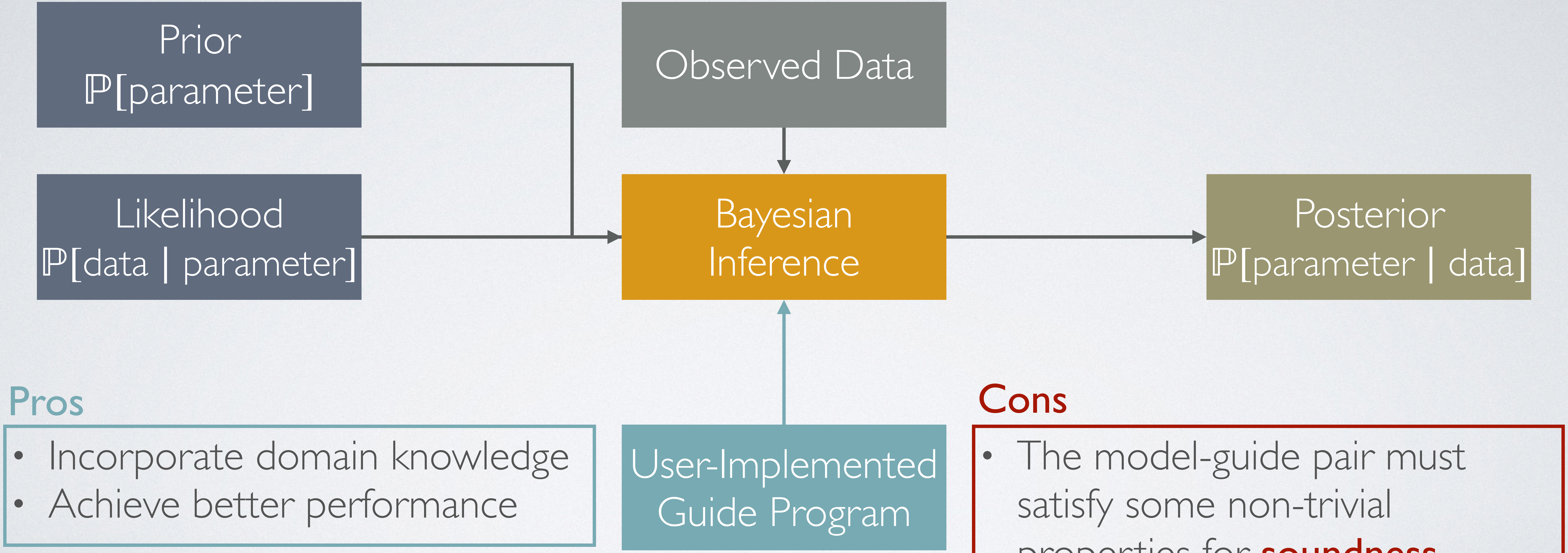
User-Implemented Guide Program

Pros

- Incorporate domain knowledge
- Achieve better performance

GUIDE-BASED INFERENCE



Pros

- Incorporate domain knowledge
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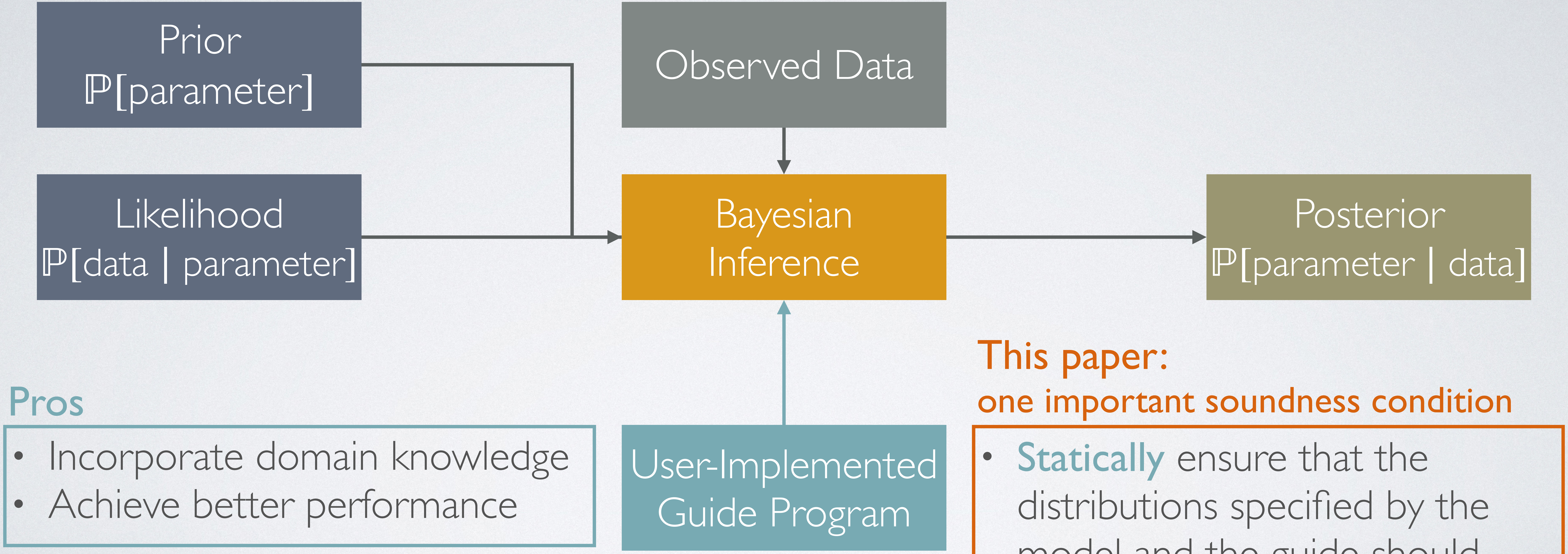
Cons

- The model-guide pair must satisfy some non-trivial properties for **soundness**

GUIDE-BASED INFERENCE



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User-Implemented Guide Program

Pros

- Incorporate domain knowledge
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This paper:

one important soundness condition

- **Statically** ensure that the distributions specified by the model and the guide should **have the same support**

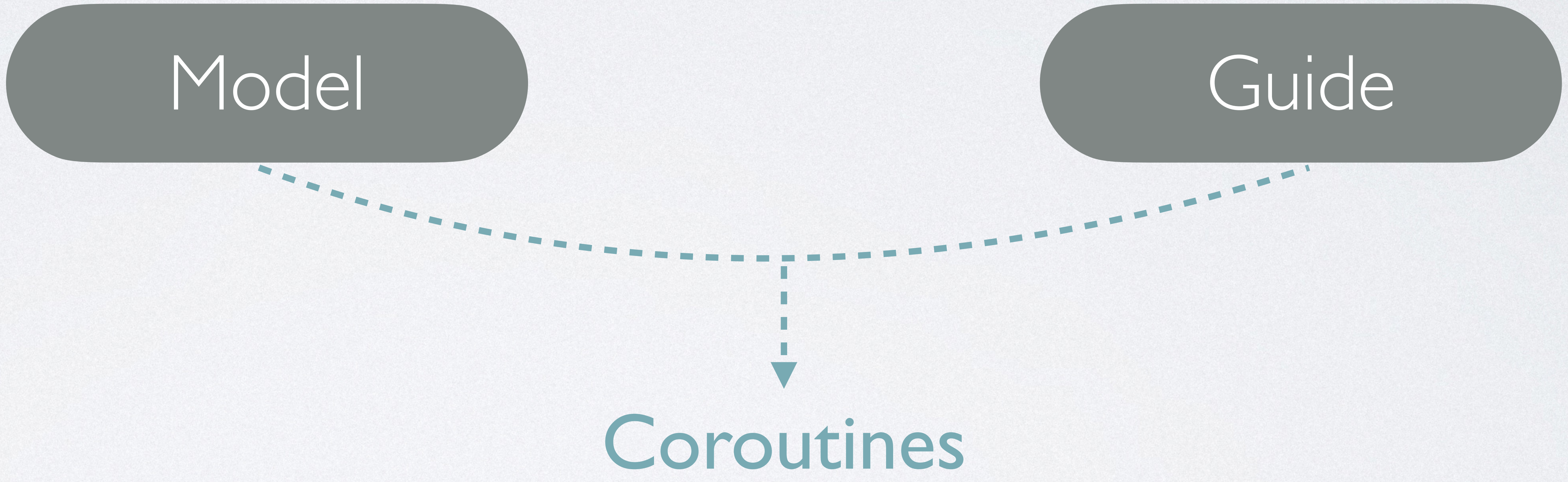
Our Approach

Our Approach

Model

Guide

Our Approach

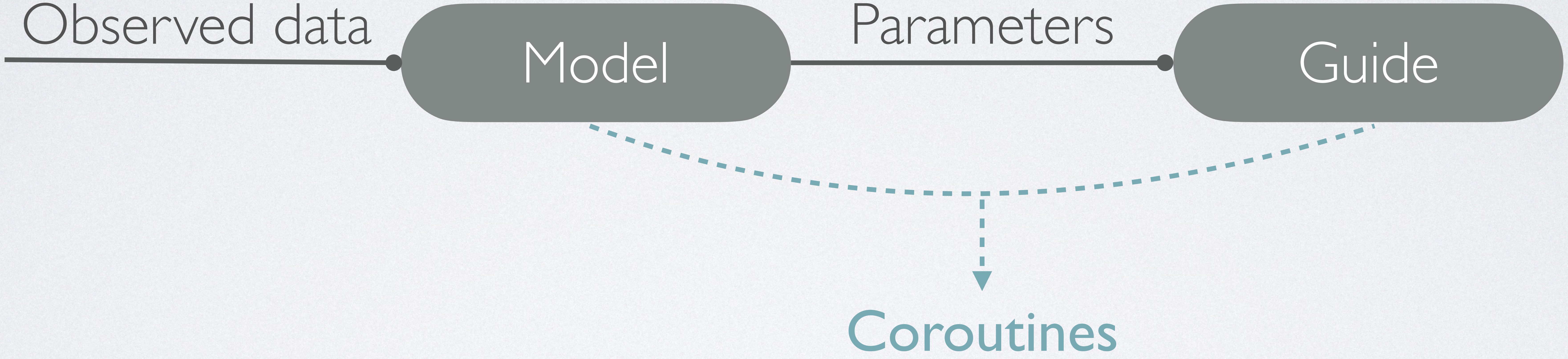


Model

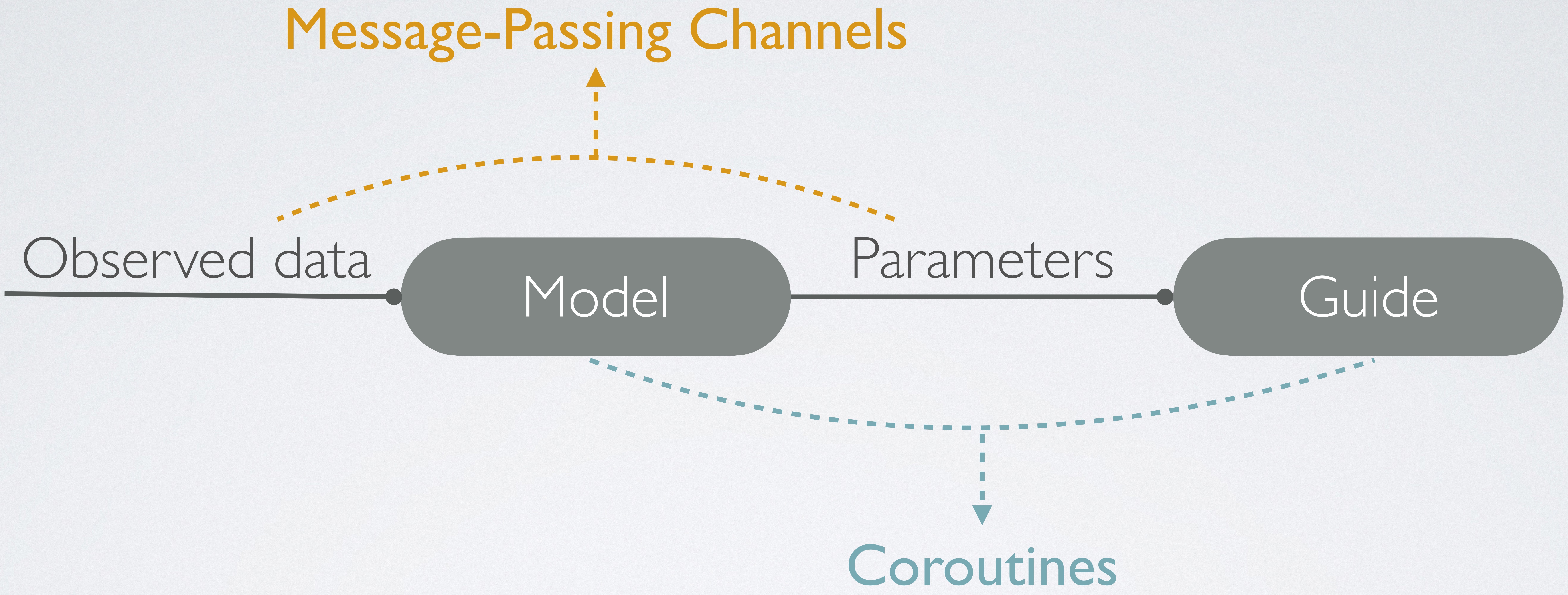
Guide

Coroutines

Our Approach



Our Approach



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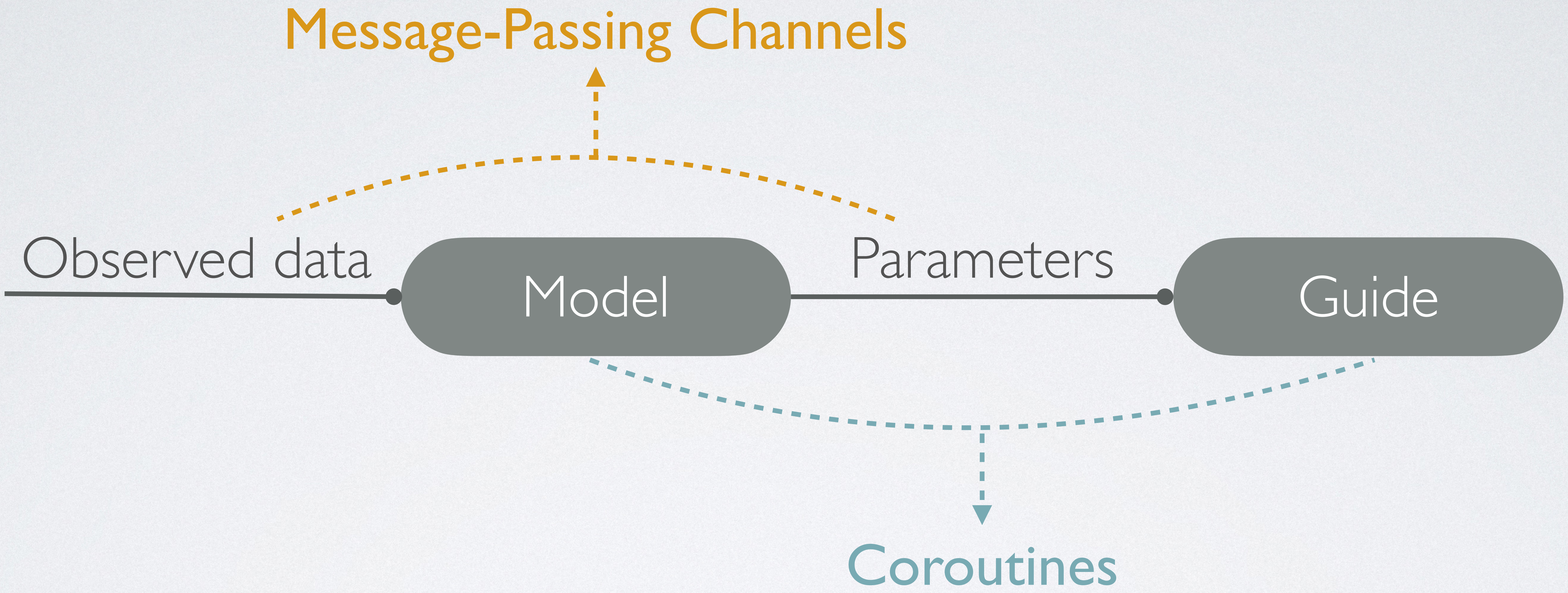

Key idea: Use communication to exchange random samples and control-flow branches, and impose type-based constraints on communication to guarantee inference soundness

Sample-site compatibility:
Corresponding sample sites should have the same support

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Corresponding sample sites should have the same support

Model

```
param1 <- sample_recv{param}(Gamma(2, 1))
```

Guide

```
param1 <- sample_send{param}(Gamma(1, 1))
```

Sample-site compatibility: Corresponding sample sites should have the same support

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Control-flow compatibility:

Corresponding program paths should sample the same set of model parameters

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Corresponding program paths should sample the same set of model parameters

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if_send{param} param1 < 2 then … else …
```

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if_recv{param} then … else …
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GUIDE TYPES

Key idea: We take inspiration from session types and develop a type system to prescribe communication protocols

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```
proc sound_guide() {
  param1 <- sample_send{param}(Gamma(1, 1));
  if_recv{param} {
    return
  } else {
    param2 <- sample_send{param}(Uniform(0, 1));
    return
  }
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Guide Types

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The guide type for channel `param`:

$$\mathbb{R}_+ \wedge (\mathbf{1} \,\&\, (\mathbb{R}_{(0,1)} \wedge \mathbf{1}))$$

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A nonnegative sample

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Receive a branch selection

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A [0,1]-valued sample

MORE IN THE PAPER

- How our system supports recursion, control-flow divergence, and type reconstruction
- Full formalism of the coroutine-based semantics and the system of guide types
- Proof of type safety and inference soundness
- A prototype implementation and experiments on expressibility and performance
- Comparison with prior work by Lew et al. [1] and Lee et al. [2]

[1] A. K. Lew, M. F. Cusumano-Towner, B. Sherman, M. Carbin, and V. K. Mansinghka. *Trace Types and Denotational Semantics for Sound Programmable Inference in Probabilistic Languages*. POPL'20.
[2] W. Lee, H. Yu, X. Rival, and H. Yang. *Towards Verified Stochastic Variational Inference for Probabilistic Programs*. POPL'20.