PRICING LOS ANGELES FREEWAYS

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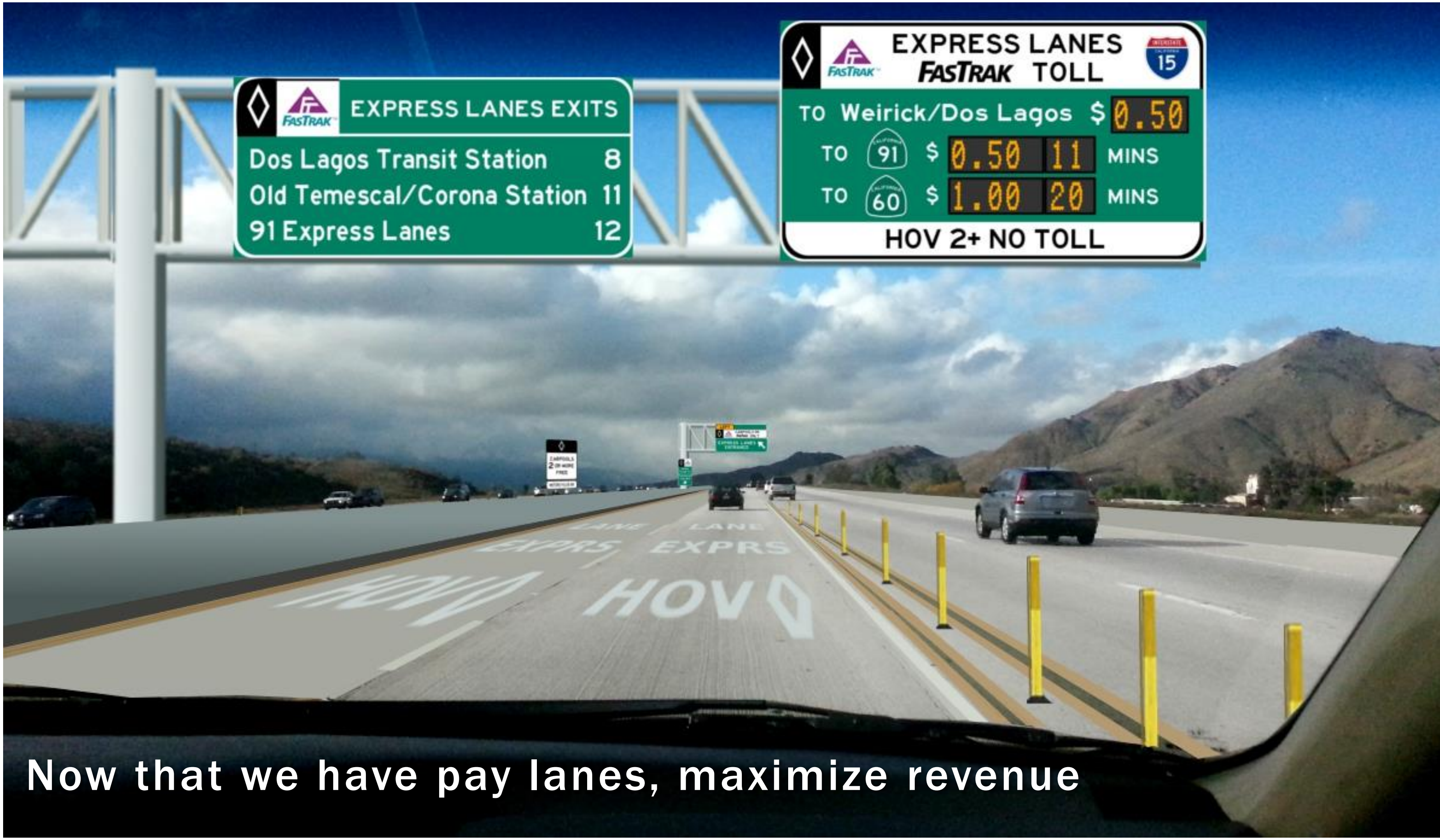
TRAFFIC IS A PROBLEM

SOLUTION: PAY LANES

Repurpose existing carpool lanes

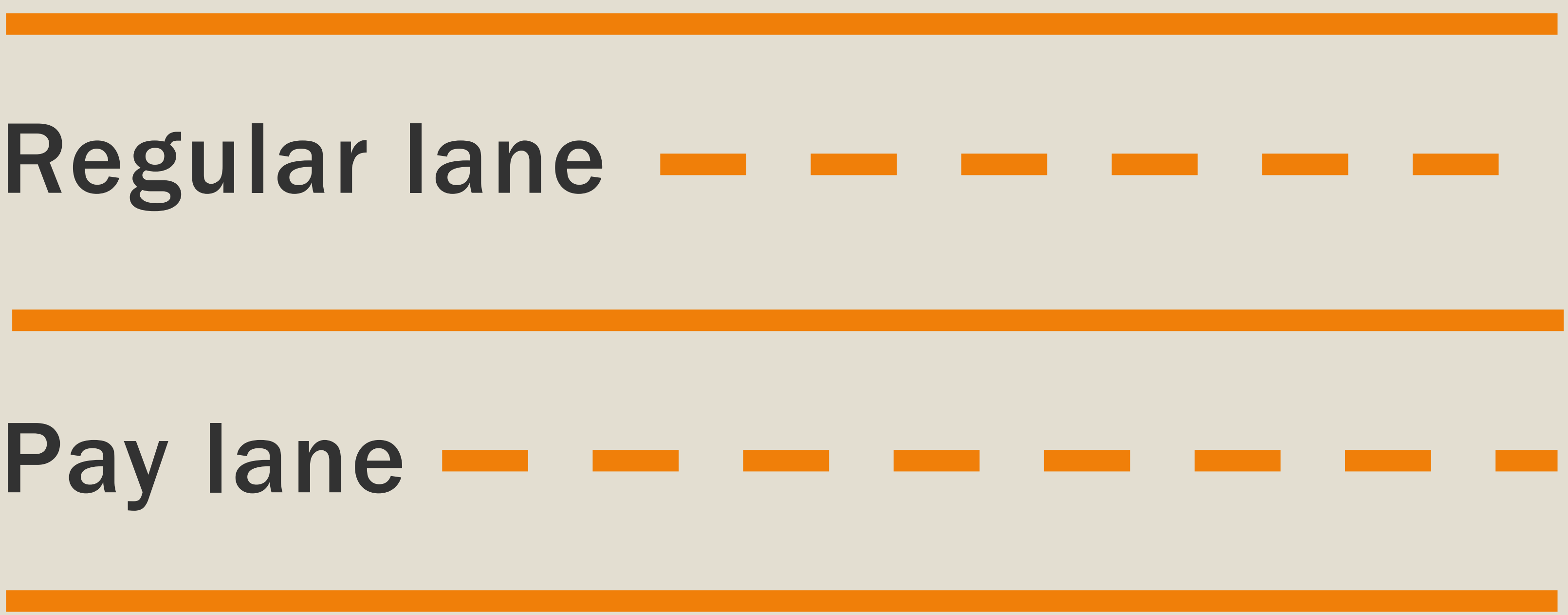
Generate revenue and reduce congestion!

GOVERNMENT OBJECTIVE



HOW WE MODEL PAY LANES

Simply the freeway into two lanes:



HOW WE MODEL PAY LANES

Overall traffic volume is constant but use of pay lane is variable

We model 3 different time periods:

- AM Rush Hour (7:00 – 9:00 AM)
- PM Rush Hour (5:00 – 8:00 PM)
- Graveyard Shift (2:00 – 4:00 AM)

HOW WE MODEL PAY LANES

In each 10-minute interval, the government sets the price of the pay lane

Then, the government observes demand/revenue

HOW WE MODEL PAY LANES

Lookup Table
(Lookup Table, Online)

Linearized Logistic Belief
(Linearized Logistic Belief, Online)

OFFLINE / ONLINE LEARNING

Clearly, the problem is online

We collect revenue with each sample

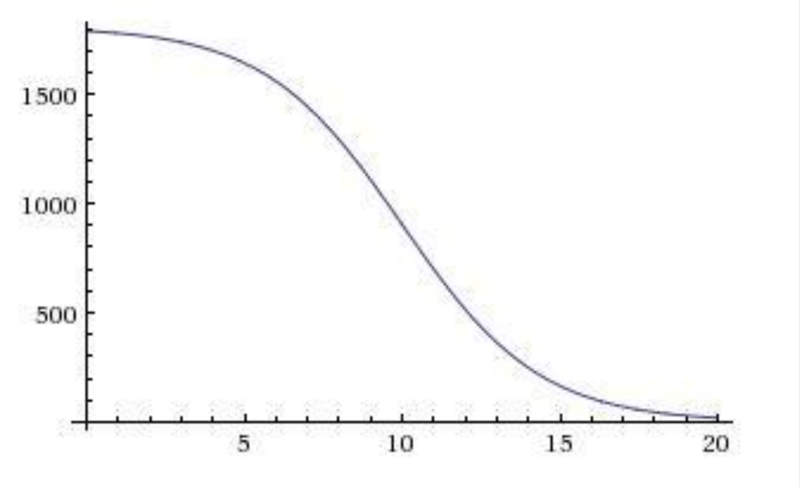
But there is value in an offline simulation!

Offline: $\zeta = max_P^N(P \times D(P))$

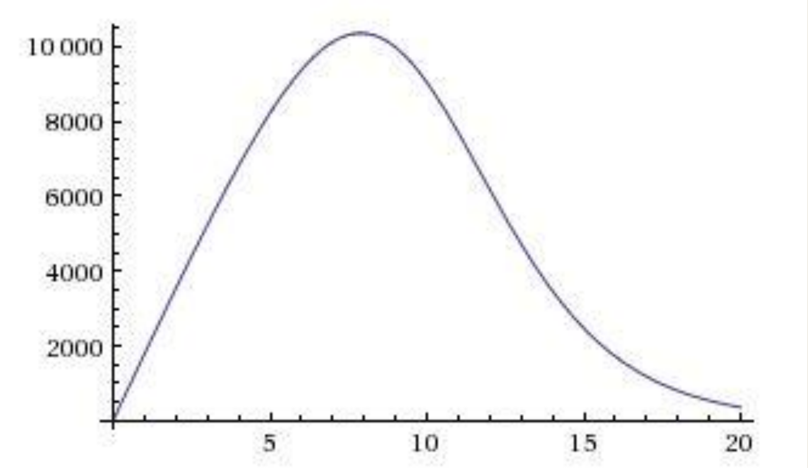
Online: $\zeta = \sum_{n=0}^{N} \gamma^n \times P \times D(P)$

DEMAND / REVENUE FUNCTION

$$D(P) = \frac{M}{1+e^{-\mu_1+\mu_2 P}}$$



$$R(P) = P \times D(P)$$



LOOKUP TABLE

Discretize price in \$0.10 increments (\$0-\$20)

Prior: $\theta_x^0 = -\frac{\beta}{200}x + \beta$

Observation: $R^n = R(P) + \epsilon^n \ , \ \epsilon^n \sim N(0, \sigma^2)$

Covariance matrix:

$Cov^0(R(P), R(P')) = \sigma^2 e^{-\alpha|P-P'|}$, where $Var^0(R(P)) = \sigma^2$

LOOKUP TABLE

- **UPDATING OUR BELIEF MODEL** (from Book, 2.2.3)

$$\theta^{n+1}(x) = \theta^n + \frac{W^{n+1} - \theta_x^n}{\lambda^W + \Sigma_{xx}^n} \Sigma^n e_x,$$

$$\Sigma^{n+1}(x) = \Sigma^n - \frac{\Sigma^n e_x (e_x)^T \Sigma^n}{\lambda^W + \Sigma_{xx}^n}.$$

LOOKUP TABLE

Test the following policies:

Knowledge Gradient w/ Correlated Beliefs

Interval Estimation (Offline)

Upper Confidence Bound 1 (Online)

Pure Exploitation

LINEARIZED LOGISTIC BELIEF MODEL

Linearizing a logistic belief model

$$D(P) = \frac{M}{1+e^{-\mu_1+\mu_2 P}}$$

$$D^n = D(P) + \epsilon^n$$

$$\overline{D^n} = -\mu_1 + \mu_2 P = ln\left(\frac{M-(D^n-\epsilon^n)}{(D^n-\epsilon^n)}\right)$$

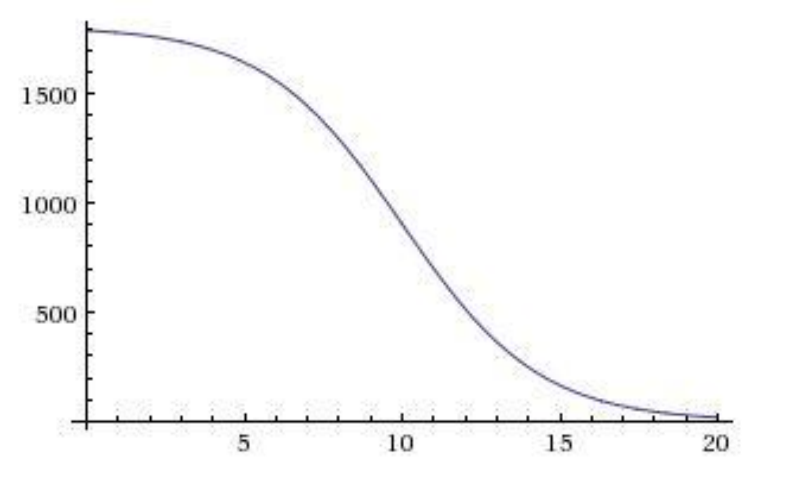
<u>Proof</u>

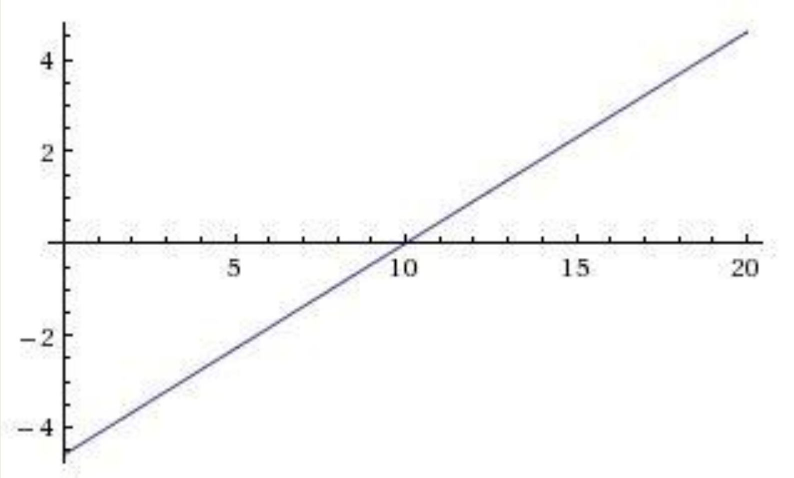
$$D^n = \frac{M}{1+e^{-\mu_1+\mu_2 P}} + \epsilon^n$$

$$(D^n - \epsilon^n) + (D^n - \epsilon^n) \times e^{-\mu_1+\mu_2 P} = M$$

$$e^{-\mu_1+\mu_2 P} = \frac{M-(D^n-\epsilon^n)}{(D^n-\epsilon^n)}$$

$$-\mu_1 + \mu_2 P = ln\left(\frac{M-(D^n-\epsilon^n)}{(D^n-\epsilon^n)}\right)$$

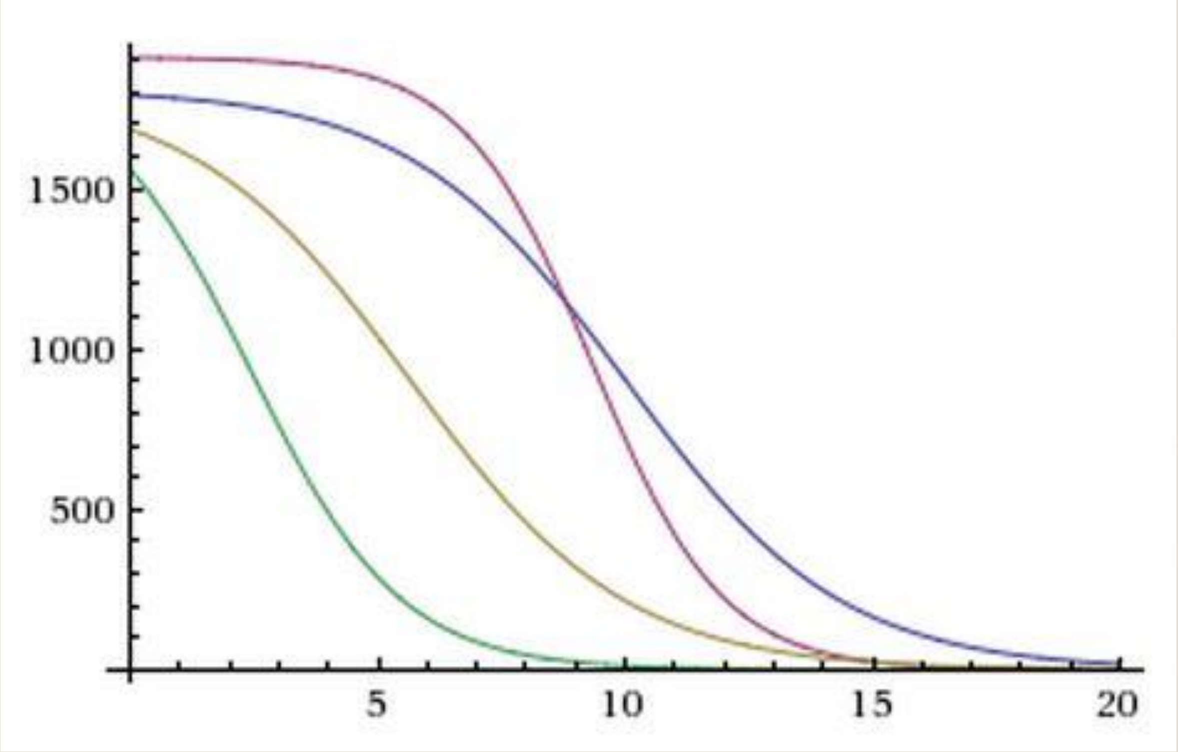




LINEARIZED LOGISTIC BELIEF MODEL

DIFFERENCES FROM LOOKUP TABLE

- Continuous Price
- Prior and Covariance Creation



LINEARIZED LOGISTIC BELIEF MODEL

- **UPDATING OUR DEMAND MODEL** (from Book, 8.2)

$$\theta^n = \theta^{n-1} + \frac{1}{\gamma^n} B^{n-1} x^n \varepsilon^n,$$

$$\gamma^n = 1 + (x^n)^T B^{n-1} x^n.$$

$$B^n = [(X^n)^T X^n]^{-1}$$

$$B^n = B^{n-1} - \frac{1}{\gamma^n} (B^{n-1} x^n (x^n)^T B^{n-1}).$$

LINEARIZED LOGISTIC BELIEF MODEL

Test the following policies:

- Knowledge Gradient w/ Correlated Beliefs
- Interval Estimation (Offline)
- Upper Confidence Bound 1 (Online)
- Pure Exploitation

ANTICIPATED CHALLENGE

Using revenue instead of demand in learning policies for linear belief model (and coding thereof)

EXTENSIONS OF OUR MODEL

- Maximum Overall Traffic not constant
- Add congestion factor into objective function:

$$\zeta = P \times D(P) + \int_{M-D(P)}^{M} \psi(x)dx ,$$

- Additional variables – weather conditions, events, bidirectional traffic, etc.

IMPLICATIONS