

# Impatiemment Attendu

Robert Sedgewick  
Princeton University

A  
CONFERENCE  
ON  
THEORETICAL COMPUTER SCIENCE



August 15 – 17, 1977  
University of Waterloo  
Waterloo, Ontario  
Canada

August 15 – 17, 1977

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Data Movement in Odd-Even Merging  
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$$\zeta(2z, a) = \frac{1}{2} - a + z(2 \ln \Gamma(a) - \ln 2\pi) + O(z^2).$$

The expansion for  $\Gamma(z)$  is well known (see [1, eq. 6.1.33]), the next two expansions are elementary, and the expansion for the  $\zeta$ -function follows directly from [16, p. 271]. Multiplying these series together, we find the expansion at  $z=0$  of the integrand in (23):

$$\frac{1}{4z^2 \ln 2} + \frac{1}{4} \lg j - \frac{\gamma}{4 \ln 2} - \frac{1}{4} + \lg \frac{\Gamma(1/4)^2}{2\pi} + O(1). \quad (24)$$

This gives the residue at  $z=0$  (the coefficient of  $1/z$ ).

To this we must add the residue at the other poles of  $1/(4z-1)$ . The effect of these other terms is small (but not insignificant), and we shall encapsulate them in a single term,

$$\epsilon(j) \equiv \frac{2}{\ln 4} \sum_{k \neq 0} \Gamma\left(\frac{2k\pi i}{\ln 4}\right) j^{2k\pi i / \ln 4} \zeta\left(\frac{4k-1}{\ln 4}, \frac{1}{4}\right) = \sum_{k=1}^{\infty} (\xi_k \cos(k\pi \lg j) - \eta_k \sin(k\pi \lg j)),$$

where

$$\frac{2}{\ln 2} \Gamma\left(\frac{k\pi i}{\ln 2}\right) \zeta\left(\frac{2k\pi i}{\ln 2}, \frac{1}{4}\right) \equiv \xi_k + i\eta_k.$$

To finish the evaluation of our  $b_j$  and  $B_N$  we need to evaluate the  $\Gamma$  and  $\zeta$  functions at these points along the imaginary axis (see [6, secs. 6.3 and 6.4]). Table 3 shows the values of  $\xi_k$  and  $\eta_k$  for  $k=1, 2, 3$  computed in this way. The values get exceedingly small for larger  $k$ , as can be verified from the bounds (21) and (22).

Adding all the residues, we have, from (23):

$$b_j = \frac{1}{4} \lg j + \lg \frac{\Gamma(1/4)^2}{2\pi} - \frac{1}{4} - \frac{\gamma}{4 \ln 2} + \epsilon(j) + O(j^{-1/2}). \quad (25)$$

This leads to our final result.

**Theorem 3.** The average number of exchanges used by Batchier's odd-even merge for a random 2-ordered file of  $2N$  elements is

$$B_N = \frac{1}{4} N \lg N + \left( \lg \frac{\Gamma(1/4)^2}{2\pi} + \frac{1}{4} - \frac{\gamma+2}{4 \ln 2} + \delta(N) \right) N + O(\sqrt{N} \log N),$$

where  $\delta(N)$  is an oscillatory function which is

k	$\xi_k$	$\eta_k$
1	.003704670+	.002500177+
2	.000001560+	-.000000832-
3	.000000001-	.000000002+

$$\xi_k + i\eta_k = \frac{2}{\ln 2} \Gamma\left(\frac{k\pi i}{\ln 2}\right) \zeta\left(\frac{k\pi i}{\ln 2}, \frac{1}{4}\right)$$

$$\begin{aligned} \Gamma\left(\frac{1}{4}\right) &= 3.6256099082+ \\ \frac{1}{\ln 2} &= 1.4426950408+ \\ \gamma &= 0.5772156649+ \\ \pi &= 3.1415926535+ \end{aligned}$$

Table 3. Values of constants

constant if  $N$  is a power of two, with  $|\delta(N)| < .000680$ , and  $\delta(2^n) \approx .000317(-1)^n$ . (The constant

$$\alpha \equiv \lg \frac{\Gamma(1/4)^2}{2\pi} + \frac{1}{4} - \frac{\gamma+2}{4 \ln 2}$$

has the approximate value .385417224.)

**Proof.** From the discussion above, we need only substitute our result (25) for  $b_j$  into our eq. (14) for  $B_N$  and perform the summation. We have

$$B_N = \frac{1}{2} \frac{4^N}{\binom{2N}{N}} \sum_{1 \leq j < N} \frac{\binom{2j}{j}}{4^j} \left( \frac{1}{4} \lg j + \alpha + \frac{1}{2 \ln 2} + \epsilon(j) + O(j^{-1/2}) \right).$$

The  $\alpha + 1/(2 \ln 2)$  terms are easily taken care of; working backwards from (14), we see that they contribute  $(\alpha + 1/(2 \ln 2))N$  to  $B_N$ . (The sum may be evaluated directly as an interesting application of several identities in Knuth [13]; a simple induction could also be used.)

For the other terms, we can remove the binomial coefficients with Stirling's approximation, as in the derivation of (16). We have

$$\frac{\binom{2j}{j}}{4^j} = \frac{1}{\sqrt{\pi j}} + O(j^{-3/2}), \quad \frac{4^N}{\binom{2N}{N}} = \sqrt{\pi N} + O(N^{-1/2}).$$

Therefore the  $O(j^{-1/2})$  term sums to  $O(\sqrt{N} \log N)$ , and

$$\begin{aligned} \frac{1}{2} \frac{4^N}{\binom{2N}{N}} \sum_{1 \leq j < N} \frac{\binom{2j}{j}}{4^j} \frac{1}{4} \lg j &= \frac{\sqrt{N}}{8} \sum_{1 \leq j < N} \frac{\lg j}{\sqrt{j}} + O(\sqrt{N}) \\ &= \frac{\sqrt{N}}{8} \int_1^N \frac{\lg x}{\sqrt{x}} dx + O(\sqrt{N}) \\ &= \frac{1}{4} N \lg N - \frac{1}{2 \ln 2} N + O(\sqrt{N}). \end{aligned}$$

Here the second step follows from Euler-McLaurin summation (see, for example, [14, p. 110]) and the integral is evaluated by making the substitution  $x = y^2$ .

It remains to evaluate the oscillatory term

$$N\delta(N) = \frac{1}{2} \frac{4^N}{\binom{2N}{N}} \sum_{1 \leq j < N} \frac{\binom{2j}{j}}{4^j} \epsilon(j).$$

After substituting for  $\epsilon(j)$ , we proceed in the same way as we did for the  $\lg j$  term. The result of using Stirling's approximation on the binomial coefficients and Euler-McLaurin summation on the resulting sums is

$$\begin{aligned} \delta(N) &= \frac{1}{2\sqrt{N}} \sum_{k \geq 1} (\xi_k \int_1^N \frac{\cos k\pi \lg x}{\sqrt{x}} dx \\ &\quad - \eta_k \int_1^N \frac{\sin k\pi \lg x}{\sqrt{x}} dx) + O(\sqrt{N}). \end{aligned}$$

These integrals are elementary; the substitutions  $x = y^2$ , then  $t = 2\pi k \lg y$  transform them into standard integrals (for example, [1, Eqs. 4.3.136, 4.3.137]) with the eventual result

$$\begin{aligned} \delta(N) &= \sum_{k \geq 1} \frac{\sigma_k}{\sigma_k + 1} (\xi_k (\sigma_k \cos \pi k \lg N + \sin \pi k \lg N) \\ &\quad - \eta_k (\sigma_k \sin \pi k \lg N - \cos \pi k \lg N)) \end{aligned}$$

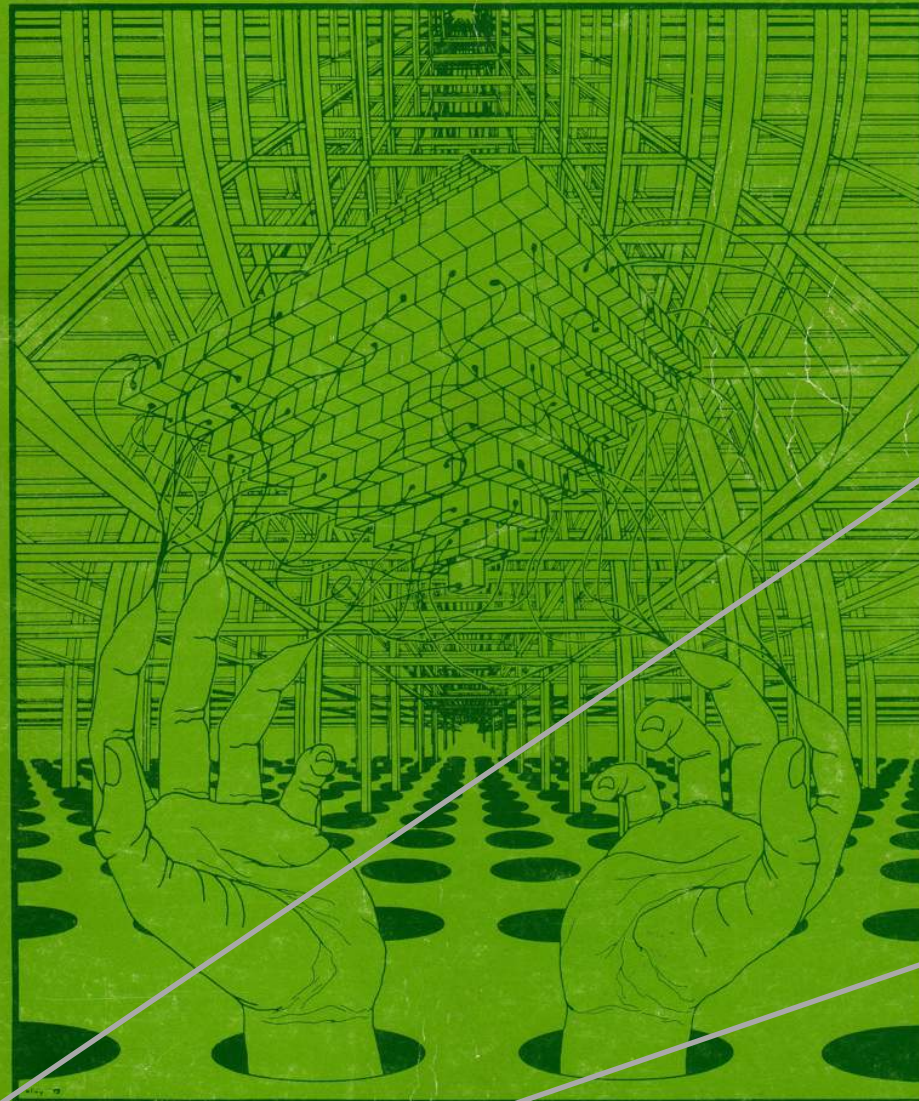
where  $\sigma_k$  is  $(\ln 2/2\pi k)$ . From this formula, we see that  $\delta(N)$  is constant when  $N$  is a power of two, and has an oscillatory nature between powers of two. With the aid of Table 3 we can easily compute the stated values.  $\square$

$$\frac{2}{\ln 2} \Gamma\left(\frac{k\pi i}{\ln 2}\right) \zeta\left(\frac{2k\pi i}{\ln 2}, \frac{1}{4}\right)$$



# 18th Annual Symposium on Foundations of Computer Science

*(Formerly called the Annual Symposium on Switching and Automata Theory)*



OCTOBER 31-NOVEMBER 2, 1977

OCTOBER 31-NOVEMBER 2, 1977

IEEE 77 CH1278-1 C

 IEEE COMPUTER SOCIETY

 INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS

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We write  $M_n = B_n + \frac{1}{n} \left( \frac{2n}{n} \right) I_n + o(B_n)$

with  $I_n = \int_0^{\infty} S_2((t/\sqrt{n})) e^{-t^2} \cdot H_3(t) dt$  (28)

Using Delange's formula (25),

$$U(x) = \int_0^x S_2((t/\sqrt{n})) dt = \frac{1}{2} x \log_2 x + x F(\log_2 x) + h(x),$$

we compute  $I_n$  by parts :

$$I_n = \frac{1}{\sqrt{n}} U(t/\sqrt{n}) e^{-t^2} H_3(t) \Big|_0^{\infty} - \frac{1}{\sqrt{n}} \int_0^{\infty} U(t/\sqrt{n}) e^{-t^2} H_4(t) dt.$$

The first term reduces to zero for at least two reasons, and :

$$I_n = \frac{1}{\sqrt{n}} \int_0^{\infty} \left[ \frac{1}{2} (t/\sqrt{n}) \log_2(t/\sqrt{n}) + (t/\sqrt{n}) F(\log_2(t/\sqrt{n})) + h(t/\sqrt{n}) \right] e^{-t^2} H_4(t) dt.$$

Splitting in four :

$$\begin{aligned} I_{n_1} &= -\frac{1}{2} \int_0^{\infty} t \log_2 t H_4(t) e^{-t^2} dt - \frac{1}{4} \log_2 n \int_0^{\infty} t H_4(t) e^{-t^2} dt \\ &- \int_0^{\infty} t H_4(t) F\left(\frac{1}{2} \log_2 n + \log_2 t\right) e^{-t^2} dt - \frac{1}{\sqrt{n}} \int_0^{\infty} h(t/\sqrt{n}) H_4(t) e^{-t^2} dt \\ &= -\frac{1}{2} I_1 - \frac{1}{4} \log_2 n I_2 - I_3 - \frac{1}{\sqrt{n}} I_4. \end{aligned}$$

Each integral is computed in turn.

$$\begin{aligned} I_1 &= \int_0^{\infty} t \log_2 t H_4(t) e^{-t^2} dt = -\frac{1}{\log 2} \int_0^{\infty} (\log t + 1) H_3(t) e^{-t^2} dt \\ &= -\frac{1}{\log 2} \int_0^{\infty} \log t (-8t^2 + 12t) e^{-t^2} dt - \frac{2}{\log 2} \\ &= -\frac{1}{\log 2} \int_0^{\infty} \log t^2 (-2t^2 + 3) e^{-t^2} dt - \frac{2}{\log 2} \\ &= -\frac{1}{\log 2} \int_0^{\infty} \log x (3-2x) e^{-x} dx - \frac{2}{\log 2} \\ &= -\frac{3}{\log 2} \int_0^{\infty} \log x e^{-x} dx + \frac{2}{\log 2} \int_0^{\infty} x \log x e^{-x} dx - \frac{2}{\log 2} \\ &= -\frac{1}{\log 2} \int_0^{\infty} \log x e^{-x} dx. \end{aligned}$$

This last integral is classical (see [Bu,33]) and its value is  $-\gamma$ ,  $\gamma \approx 0.577$  being Euler's constant.

Simple integrations by parts give  $I_2 = -2$ ; as for  $I_4$ , the function  $h(t/\sqrt{n})$  ranges between 0 and 1, thus  $I_4$  has some definite value and

$$\frac{1}{\sqrt{n}} I_4 = o(1).$$

There just remains  $I_3 = K(\log_2 n)$ , where

$$(29) \quad K(u) = \int_0^{\infty} t H_4(t) F(u + \log_2 t) e^{-t^2} dt$$

Since  $F$  is periodic with period 1 then so is  $K$ . The change of variable  $x = u + \log_2 t$  shows that  $K$  is indefinitely differentiable.

We have thus proved :

Theorem : The average number  $A_n = M_n/B_n$  satisfies

$$A_n = \log_4 n + D(\log_2 n) + o(1)$$

in which

$$D(u) = 1 - \frac{\gamma}{2 \log 2} + K(u),$$

$K(u)$  being the periodic function defined by (29).

Apart from the periodic term, one can, in principle, develop an asymptotic expansion as far as needed. H. Delange (private communication) has also computed the Fourier series of  $D$  :

$$D(t) = \sum_{k \in \mathbb{Z}} a_k e^{i2k\pi t}$$

and finds :

$$a_0 = \frac{1}{2} - \frac{\gamma + 2}{2 \log 2} + \log_2 \pi,$$

which is the mean value of  $D(t)$ , and for  $k \neq 0$ ,

$$a_k = \frac{2k\pi i - \log 2}{\log 2} \Gamma\left(\frac{k\pi i}{\log 2}\right) \zeta\left(\frac{2k\pi i}{\log 2}\right)$$

where  $\Gamma$  and  $\zeta$  are the classical gamma and Riemann's zeta functions.

The values of  $A_n$  have been machine computed for  $n$  ranging from 2 to 300 by means of formula (20). The results are plotted on figure 4, where the horizontal axis represents values of  $n$  in a logarithmic scale and the vertical axis represents values of  $A_n - \log_4 n$ .

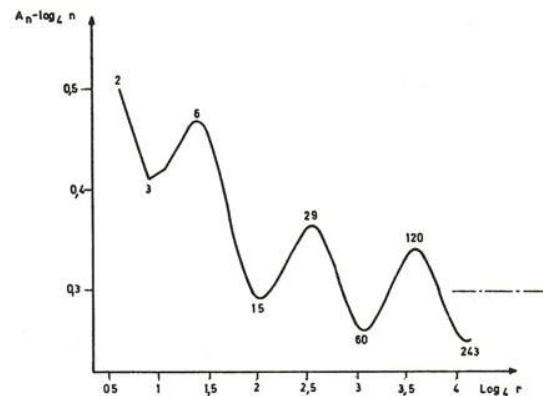


Figure 4

The diagram shows an excellent agreement of  $A_n$  to  $\log_4 n$  already for small values of  $n$ . It also shows the rapid convergence of  $M_n - \log_4 n$  to the term  $D(\log_2 n)$  centered around  $a_0 = 0.292$ , whose periodicity is apparent.

$$\frac{2k\pi i - \log 2}{\log 2} \Gamma\left(\frac{k\pi i}{\log 2}\right) \zeta\left(\frac{2k\pi i}{\log 2}\right)$$

# Genesis of “Analytic Combinatorics”

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Late 1970s / early 1980s: optimism and opportunity

Knuth volumes 1-3



Search for generality

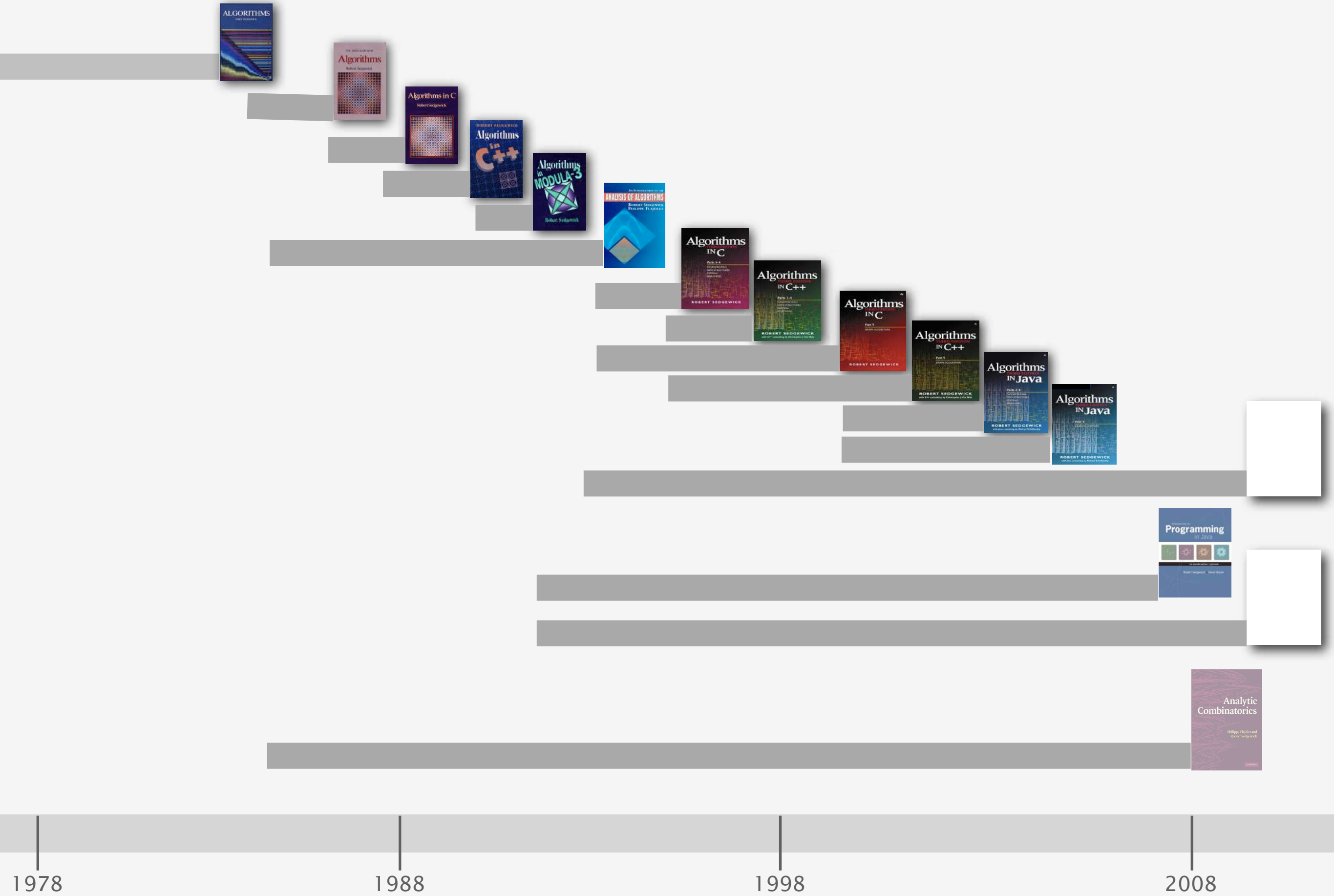
Algorithms for the masses

Teaching and research in AofA





# When will the book be available?





# An alert: “best-seller **impatiemment attendu**”

## God Plays Dice

A random walk through mathematics – mostly through the random part.

25 SEPTEMBER 2008

### Best-seller **impatiemment attendu**

From “Quelques contributions au carrefour de la géométrie, de la combinatoire et des probabilités”, by [Nicolas Pouyanne](#):

“Dans leur best-seller **impatiemment attendu** [28], P. Flajolet et R. Sedgewick offrent un développement approfondi de la méthode symbolique en analyse combinatoire, auquel on pourra se référer.”

That is, “In their impatiently awaited best-seller [28], P. Flajolet and R. Sedgewick offer a detailed development of the symbolic method in combinatorial analysis, to which one will be able to refer.”

I don't laugh often while reading mathematics, but this made me laugh, because I am among those impatiently awaiting this book. I've seen citations to Flajolet and Sedgewick's book *Analytic Combinatorics* in things written as long ago as 1998 or so; [Amazon.com says the book is due December 31, 2008](#), and the publisher, Cambridge University Press, says it will be published in December 2008. It can be downloaded from [Flajolet's web page](#).

Posted by Michael Lugo at [4:16 PM](#)

Labels: [analytic combinatorics](#)

3 comments:

[Intrinsicallyknotted](#) said...

So does this count as “proof by impending publication”? Not “proof by vigorous handwaving”, but very amusing!

[September 25, 2008 7:10 PM](#)

[Michael Lugo](#) said...

Not really. I feel like “proof by impending publication” refers to things that aren't publicly available; Flajolet and Sedgewick's book is on the web. (I'm not sure if it will stay that way after publication. In any case, I'll buy a copy, because I spend enough time flipping back and forth between my PDF of it and other windows that having a paper copy would be more practical.)

[September 25, 2008 7:15 PM](#)

[David](#) said...

I downloaded it, but, woohoo, 800+ pages. I think the boss would be



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Blog Archive

▼ 2008 (343)

November (33)

... in their **impatiently awaited best-seller**, P. Flajolet and R. Sedgewick offer a detailed development of the symbolic method in combinatorial analysis, to which one will be able to refer.

Philadelphia, Pennsylvania,  
United States

I'm a fourth-year PhD student in mathematics at the University of Pennsylvania. I study combinatorics and probability.

[View my complete profile](#)



# “Impatiently awaited” on the web

---

*Prince of Persia*



*plasma display*



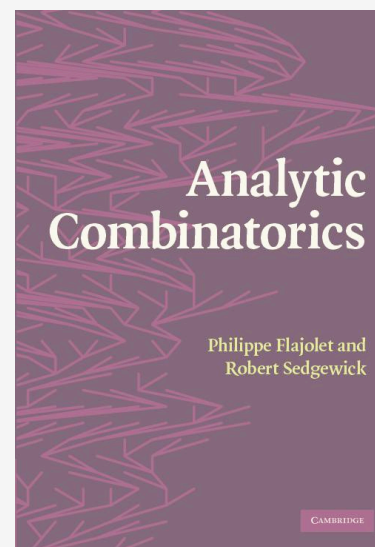
*iPod*



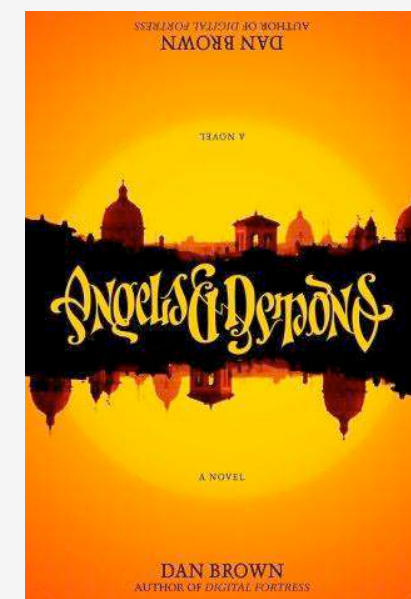
*student visa*



*Berkeley Unix 4.4*



*Dan Brown novel*



*Heroes episode*



# “Impatiently awaited” in literature

---

*Prince Andrew listened to the account of the opening of the Council of State, which he had so **impatiently awaited** and to which he had attached such importance*

Leo Tolstoy, War and Peace

*Custer and his cavalry contingent **impatiently awaited** marching orders.*

Kingsley Bray, Crazy Horse: A Lakota Life

*This done, he **impatiently awaited** the return of his companions.*

Alexandre Dumas, The Count of Monte Cristo

*A handsome young fellow like you does not obtain long leaves of absence from his mistress, and we were **impatiently awaited** at Paris, were we not ?*

Alexandre Dumas, The Three Musketeers

*He **impatiently awaited** her husband's departure.*

Guy de Maupassant, Bel Ami

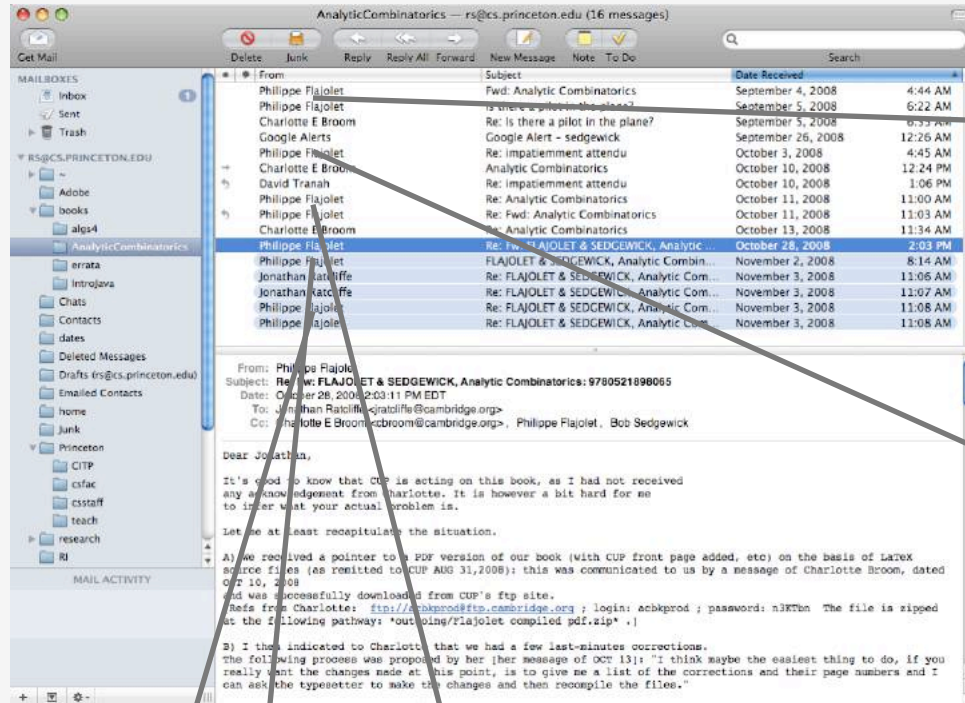
# Digression: Separated at birth??

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# Impatiemment attendu



**Last Sunday I sent the message that follows, then a reminder mid-week, but haven't received an ACK. Did you get it? Was it properly addressed?**

**Is there a pilot in the plane?**

**At full speed on the highway ... but in reverse gear?**

**Where are we?????????**

**Did some e-mail message get lost? Please at least ACK having received the message below, first sent OCT 28, 2008.**

# Patience

---

Dear Phillippe,

Apologies for not acknowledging, but I have been working around the clock to get this book to press which it did this morning. All of your corrections have been incorporated and we are on schedule for a stock date of **28th November**.

Best Wishes,

[ CUP staff member ]



A calendar for November 2008. The days of the week are listed at the top: Sun, Mon, Tue, Wed, Thu, Fri, Sat. The dates are arranged in a grid. The date 28 is highlighted with a red circle.

November 2008						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
26	27	28	29	30	31	1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

**From:** [david.tranah@gmail.com](mailto:david.tranah@gmail.com)

**Subject:** Re: Is there a pilot in the plane?

**Date:** **November 28, 2008** 1:12:56 PM EST

**To:** [Philippe.Flajolet@inria.fr](mailto:Philippe.Flajolet@inria.fr)

**Cc:** [rs@CS.Princeton.EDU](mailto:rs@CS.Princeton.EDU), [dtranah@cambridge.org](mailto:dtranah@cambridge.org)

**Dear Philippe and Bob**

**Just a quick note to say that I am looking forward to meeting you both again next week at the 60th birthday colloquium, and that we are on schedule to have books on display for SODA in NYC in January and at the annual meeting of the American Mathematical Society in Washington, also in January.**

**best wishes**

**david**

# It's on the web. Why publish a book?

---

The New York Times

OP-ED CONTRIBUTOR

## How to Publish Without Perishing

By JAMES GLEICK

Published: November 29, 2008

As a technology, the book is like a hammer. That is to say, it is perfect: a tool ideally suited to its task. Hammers can be tweaked and varied but will never go obsolete. Even when builders pound nails by the thousand with pneumatic nail guns, every household needs a hammer.

....Now even modest titles have been granted a gift of unlimited longevity.

What should an old-fashioned book publisher do with this gift? Forget about cost-cutting and the mass market. Don't aim for instant blockbuster successes. You won't win on quick distribution, and you won't win on price. Cyberspace has that covered.

Go back to an old-fashioned idea: that a book, printed in ink on durable paper, acid-free for longevity, is a thing of beauty. Make it as well as you can. People want to cherish it.





# Analytic Combinatorics

Philippe Flajolet and  
Robert Sedgewick

CAMBRIDGE