Creating “Algorithms”

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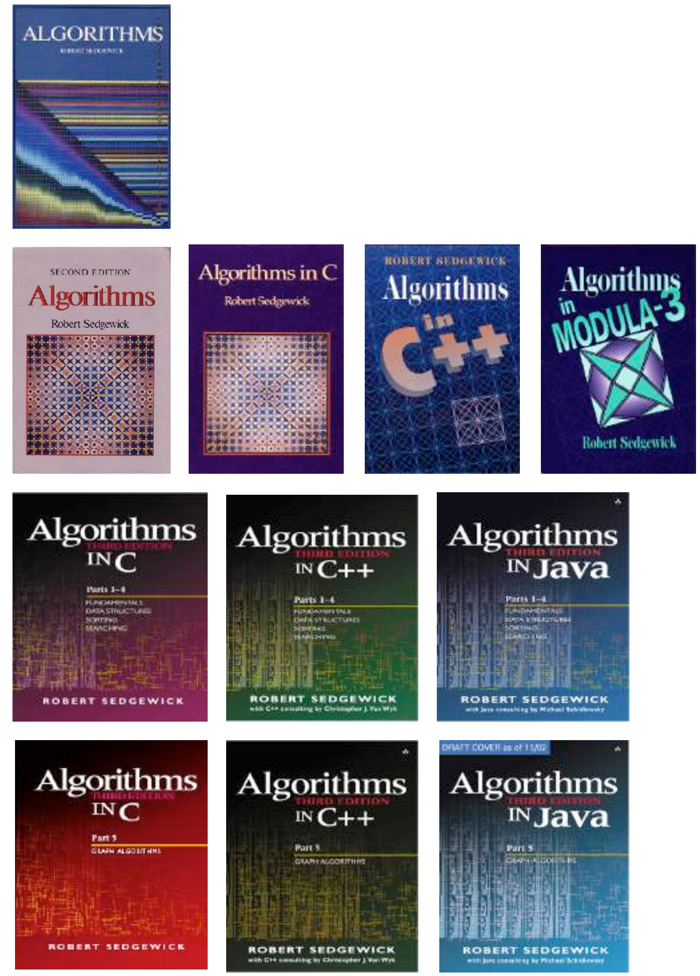
Brief history of books

Algorithms ~550 pages	1982	Pascal
Second Edition ~650 pages	1988	Pascal
	1990	C
	1992	C++
	1993	Modula-3
Third Edition 1-4 basic/ADTs/sort/search ~700 pages	1997	C
	1998	C++
	2002	Java
Third Edition 5 graph algorithms ~500 pages	2001	C
	2001	C++
	2003	Java



Translations: Japanese, French, German, Spanish, Italian, Polish, Russian

20 years, 11 books, 17+ translations, 400,000+ copies in print

Ground rules for book authors

1. You are on your own
2. Deadlines exist
3. Content over form
4. Focus on the task at hand
5. Tell the truth about what you know
6. Revise, revise, revise

First edition 1977-1982

Goals:

Algorithms for the masses
Use real code, not pseudocode
Exploit computerized typesetting technology

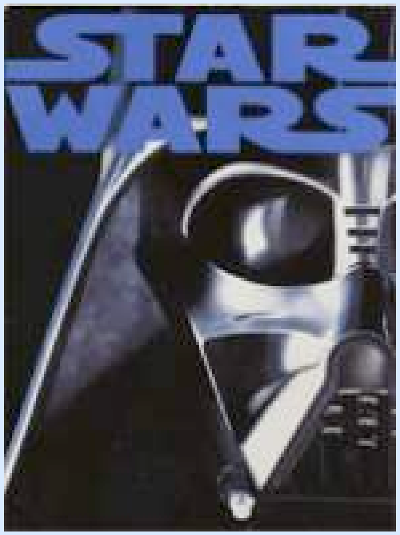
Problems:

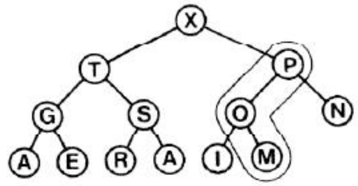
Real code hard to find for many algorithms
Laser printers unavailable outside research labs
low resolution
software to create figures?

Approach:

emacs + TeX for text
pen-and-ink for figures

1977 historical context





The code for this method is straightforward. In the following implementation, *insert* adds a new item to a[N], then calls *upheap*(N) to fix the heap condition violation at N:

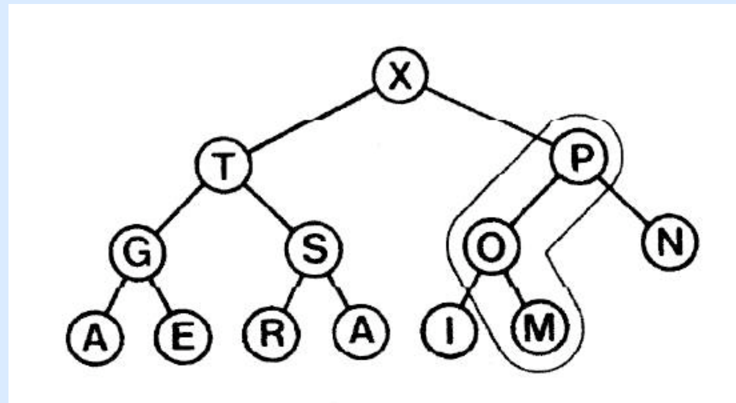
```
procedure upheap(k: integer);
  var v: integer;
  begin
  v:=a[k]; a[0]:=maxint;
  while a[k div 2]<=v do
    begin a[k]:=a[k div 2]; k:=k div 2 end;
  a[k]:=v;
  end;
procedure insert(v: integer);
  begin
  N:=N+1; a[N]:=v;
  upheap(N)
  end;
```

As with insertion sort, it is not necessary to do a full exchange within the loop, because v is always involved in the exchanges. A sentinel key must be put in a[0] to stop the loop for the case that v is greater than all the keys in the heap.

The *replace* operation involves replacing the key at the root with a new key, then moving down the heap from top to bottom to restore the heap condition. For example, if the X in the heap above is to be replaced with C, the first step is to store C at the root. This violates the heap condition, but the violation can be fixed by exchanging C with T, the larger of the two sons of the root. This creates a violation at the next level, which can be fixed

First edition features:

- **phototypeset final copy**
- **real Pascal code**
- **pen-and-ink drawings**



not enough *G's* in Paris

loom

Goal:

All the book's code should be **real** code.

Problems:

- Pascal compiler expects code in .p file
- TeX formatter expects code in .tex file
- Not all the code goes into the book
- Code has to be formatted
- Continually need to fix bugs and test fixes

Solution:

- Add comments in .p files to id and name code fragments
- Add "include" lines to source that refer to names
- **loom**: shell script to build .tex file

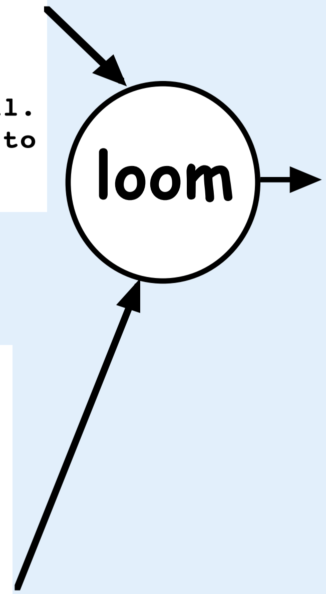
loom example (1st edition)

Text (.loom file)

```
\Sh{Example program}
One of the simplest algorithms for
this task works as follows: first
do this, then do that. This code
does this and that:
\prog{
%include example.p code
}
\noindent
This algorithm is sometimes useful.
Its running time is proportional to
...
```

Program (.p file)

```
program example(input,output);
var a: array[1..100] of integer;
     N,i: integer;
{include code}
procedure solve;
   var i,j,t: integer;
   begin
   ...
   end;
{end include code}
begin
...
solve;
...
end.
```



Typescript (.tex file)

```
\Sh{Example program}
One of the simplest algorithms for
this task works as follows: first
do this, then do that. This code
does this and that:
\prog{
procedure solve;
   var i,j,t: integer;
   begin
   ...
   end;
}
\noindent
This algorithm is sometimes useful.
Its running time is proportional to
...
```

Second edition 1986-87

Goals:

Make content more widely accessible
Eliminate pen-and-ink
Add visual representations of data structures

Problem:

Figures are numerous and intricate

Opportunities:

LaserWriter + PostScript
Algorithm animation research

Approach:

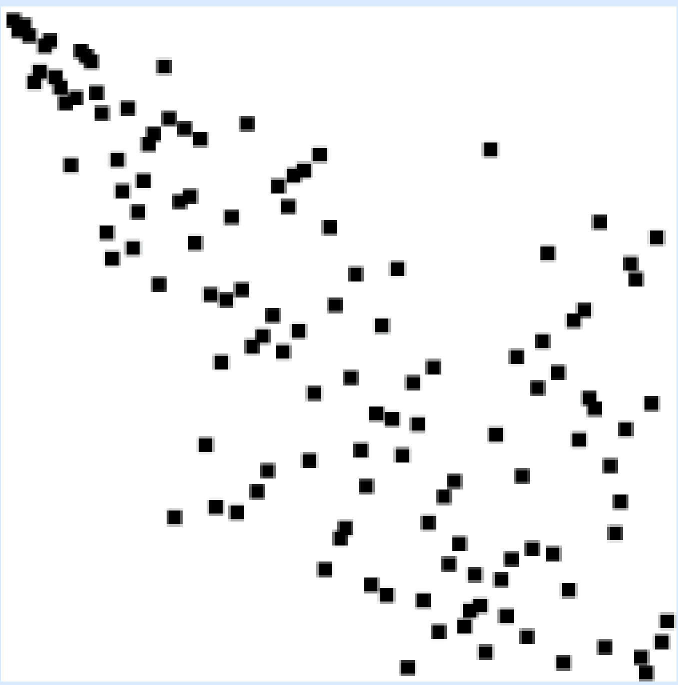
Add introductory material; move math algs to end
dsdraw: package for drawing data structures
fig: use loom to include program output in figs

dsdraw

PostScript code to draw data structures

- basic graphics
- automatic layout of snapshots

Ex: points in the plane

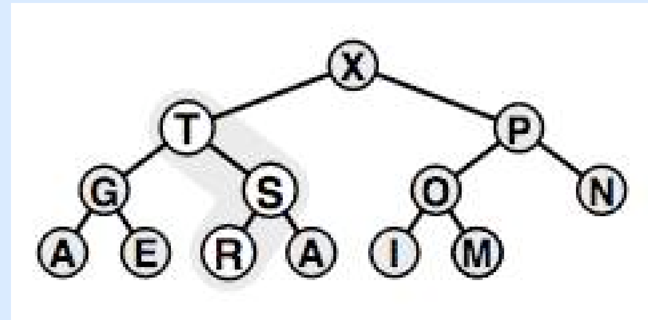


```
/points
   % Points in the plane
   % Stack: array containing the points
([label,x,y] for each node).
   %   (Example: [[(C) 1 3] [(B) 2 5] [(D) 3 5]
[(A) 3 1]])
   % Optional fourth argument can change nodestyle
   % Put a dummy point [N M] to fool (size) (?)
{/option exch def
   option (size) eq
    {dup
      /xmax 0 def /ymax 0 def
      {aload length 4 eq {pop} if
           dup ymax gt {/ymax exch def}{pop} ifelse
           dup xmax gt {/xmax exch def}{pop} ifelse
       pop} forall
       xmax ymax} if
   option (plot) eq
     {{aload length 3 eq {nodestyle} if drawnode}
forall} if
 } def
```

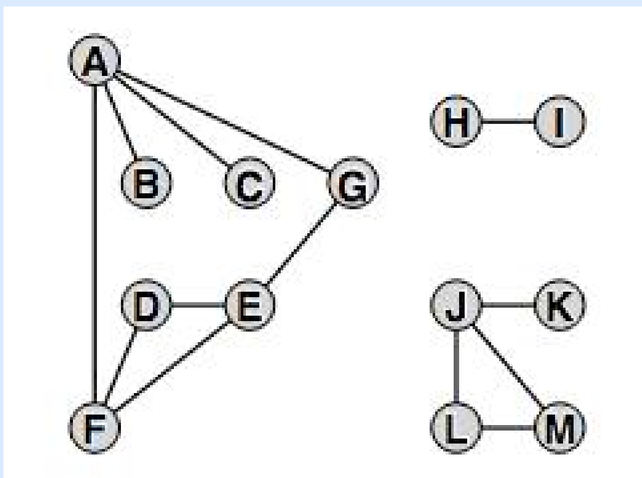
dsdraw: basic data structure drawings

permutation
array of ints
2D array
points

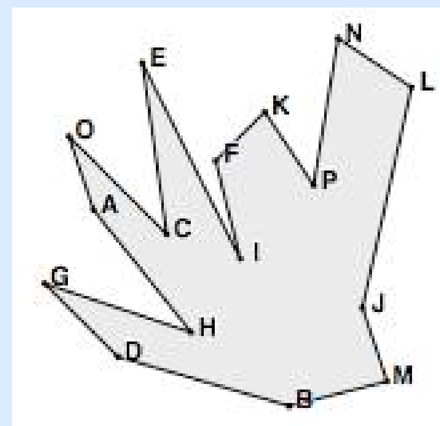
completetree
tree
polygon
graph




```
[[[(X)]
  [(T)  A][(P)]
  [(G)][(S)  A][(O)][(N)]
  [(A)][(E)][(R)  A][(A)][(I)][(M)]]]
(completetree)
```




```
[[[[(A)  1  7][(B)  2  5][(C)  4  5][(D)  2  3][(E)  4  3]
     [(F)  1  1][(G)  6  5][(H)  8  6][(I)  10  6][(J)  8  3]
     [(K)  10  3][(L)  8  1][(M)  10  1]]
   [[()  1  7][()  1  2][()  1  3][()  12  13][()  10   13]
     [()  10  12][()  10  11][()  5  4][()  6  4][()  8  9]
     [()  6  5][()  1  6][()  7  5]]]]
(graph)
```




```
[[...]]
(polygon)
```

fig

Goal:

Use programs to produce figures

Problem:

figures are PostScript programs

Opportunities:

loom

Solution:

instrument Pascal code to produce .ps code

use loom to include program output in .ps files

(filter out instrumentation)

include refs to .ps files in .tex files

fig example (2nd edition)

Text (.loom file)

```
\Sh{Example program}
One of the simplest algorithms for
this task works as follows: first
do this, then do that. This code
does this and that:
\prog{
%include example.p code | grep -v IE
}
\noindent
This algorithm is sometimes useful.
This figure shows how it works:
\fig{... psfile: fig1.ps ...}
...
```

loom

Typescript (.tex file)

```
\Sh{Example program}
One of the simplest algorithms for
this task works as follows: first
do this, then do that. This code
does this and that:
\prog{
procedure solve;
   var i,j,t: integer;
   begin
   ...
   end;
}
\noindent
This algorithm is sometimes useful.
This figure shows how it works:
\fig{... psfile: fig1.ps ...}
...
```

Program (.p file)

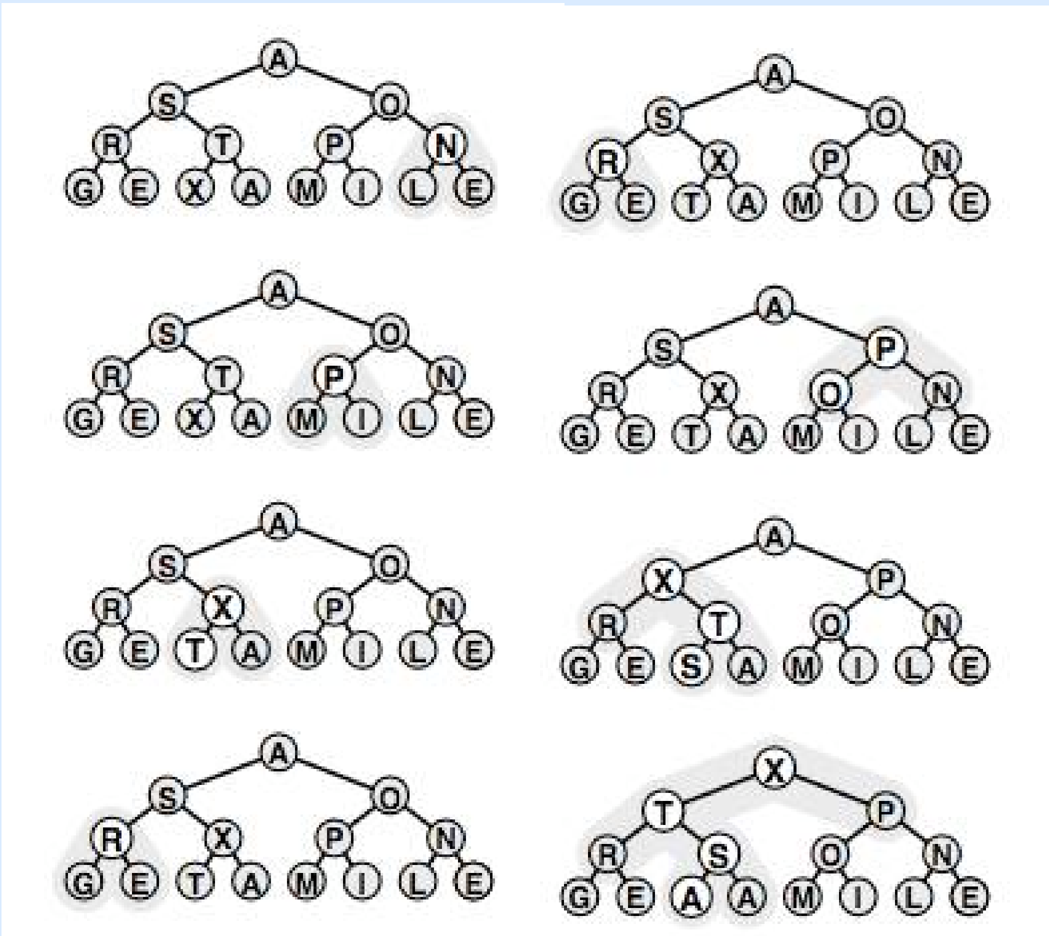
```
...
{include code}
procedure solve;
   var i,j,t: integer;
   begin
   ...
{IE} for i:= 1 to r do
{IE }        write(a[i]:4);
   ...
   end;
{end include code}
...
```

Figure (.ps file)

```
[
[127 125 126 124 115 117 122 123 ... ]
]
(permutation) showdata
```

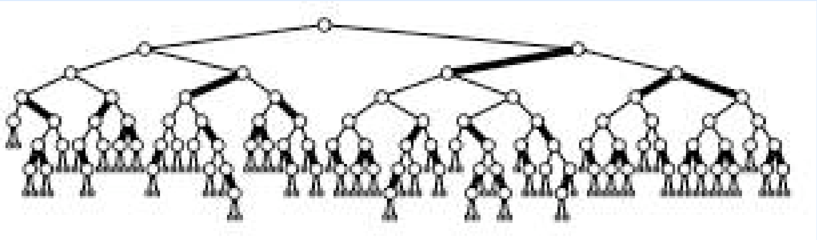
Note: can use loom here, too!

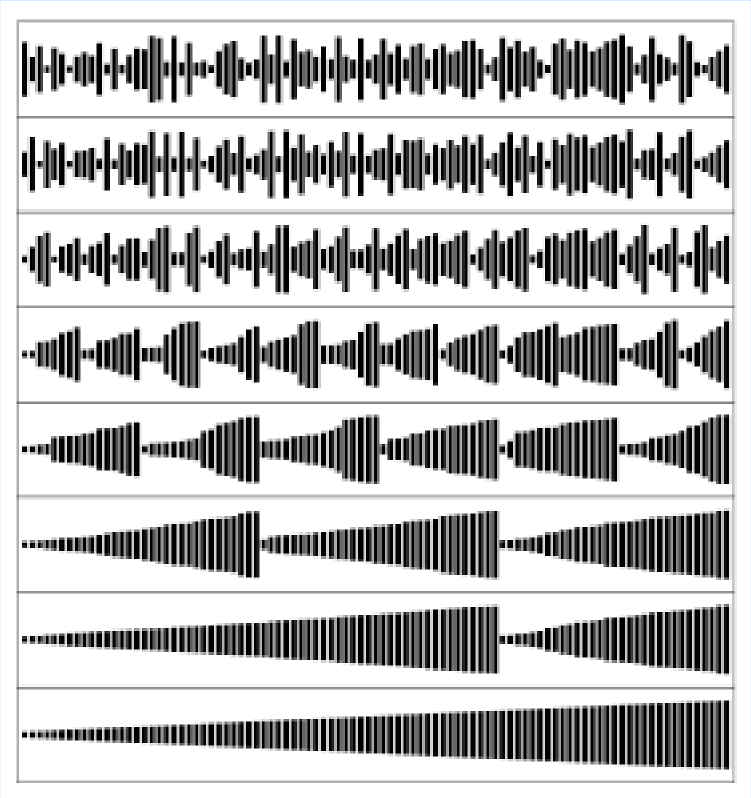
dsdraw: automatic layout of snapshots

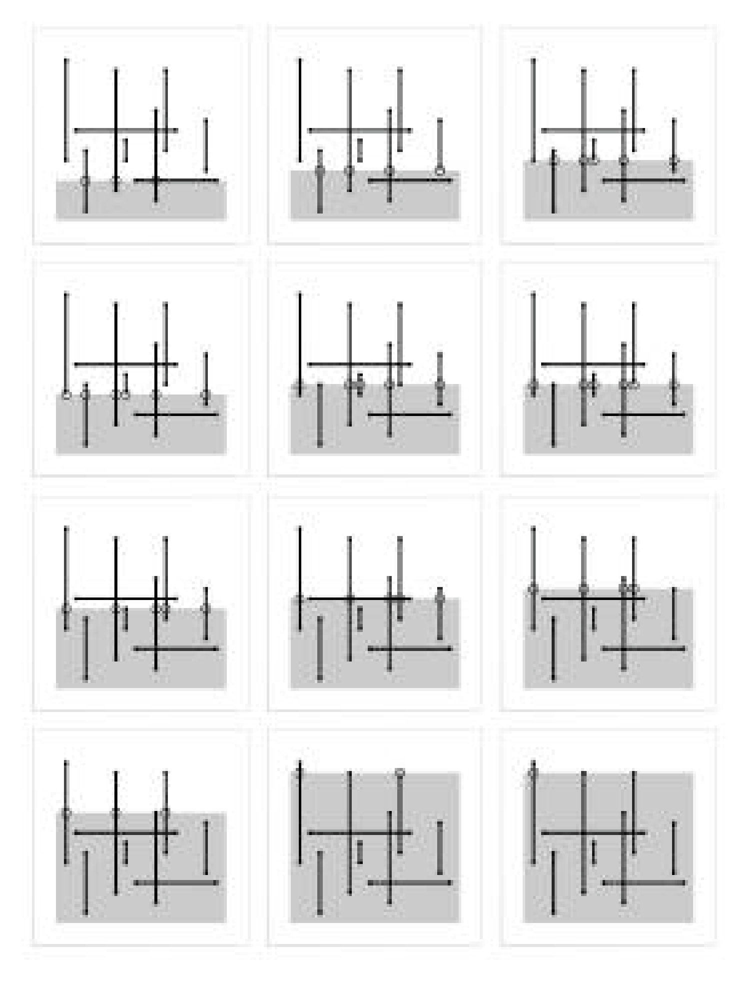



```
[
[[(A)][(S)][(O)][(R)][(T)]
    [(I)][(N)  A][(G)][(E)][(X)]
    [(A)][(M)][(P)][(L)  B][(E)  B]]
[[(A)][(S)][(O)][(R)][(T)]
    [(P)  A][(N)][(G)][(E)][(X)]
    [(A)][(M)  B][(I)  A][(L)][(E)]]
[[(A)][(S)][(O)][(R)][(X)  A]
    [(P)][(N)][(G)][(E)][(T)  A]
    [(A)  B][(M)][(I)][(L)][(E)]]
[[(A)][(S)][(O)][(R)  A][(X)]
    [(P)][(N)][(G)  B][(E)  B][(T)]
     [(A)][(M)][(I)][(L)][(E)]]
[[(A)][(S)][(P)  A][(R)][(X)]
    [(O)  A][(N)  B][(G)][(E)][(T)]
     [(A)][(M)][(I)][(L)][(E)]]
[[(A)][(X)  A][(P)][(R)  B][(T)  A]
    [(O)][(N)][(G)][(E)][(S)  A]
    [(A)  B][(M)][(I)][(L)][(E)]]
[[(X)  A][(T)  A][(P)  B][(R)  B]
    [(S)  A][(O)][(N)][(G)][(E)]
    [(A)  A][(A)  B][(M)][(I)][(L)]
   [(E)]]
]

(completetree)
```

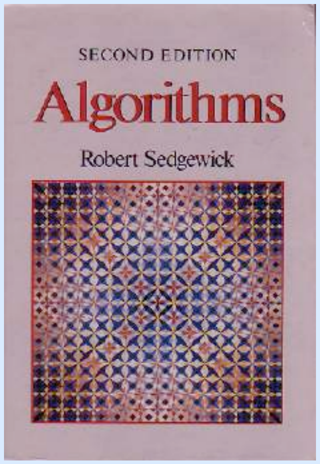
Beyond manual drafting







Second edition features



Algorithms for the masses

Uses real code, not pseudocode

Fully exploits technology

Original goals realized, PLUS

Innovative, detailed visualizations

Done?

Other languages (1990-1993)

Mandate:

Spread the word in other programming languages

Challenges:

Which languages? (Answer: C, C++, and Modula-3)

Who translates?

Early versions of new languages are unstable

Solution:

Copy-and-edit to implement programs in new language

Use conditionals in typescript for language-dependent text

Problems:

(figs were produced by Pascal programs)

difficult to take advantage of language features

typescript is a mess; layout is painful

Third edition 1993-

Goals:

Full coverage, not summary
Take visualizations to next level
Analyses with empirical verification

Challenges:

Typescript filled with conditionals
Program code filled with instrumentation
figs made with Pascal code
Many algorithms not well-understood

Approach:

START OVER, one language at a time
Status: 9 books, 6 done

Algorithms { Parts 1-4 / Part 5 / Parts 6-8 } in { C / C+ / Java }

Starting over (third edition)

Layout:

- Structured text, figures, exercises, programs, tables
- Multiple story flows (figs with captions in margins)

Figures:

- Direct PostScript implementations
- Visualize “large” examples
- Explanatory captions

Programs:

- Full implementations to support empirical studies
- Emphasize ADTs in all languages
- Use consultants to champion language features

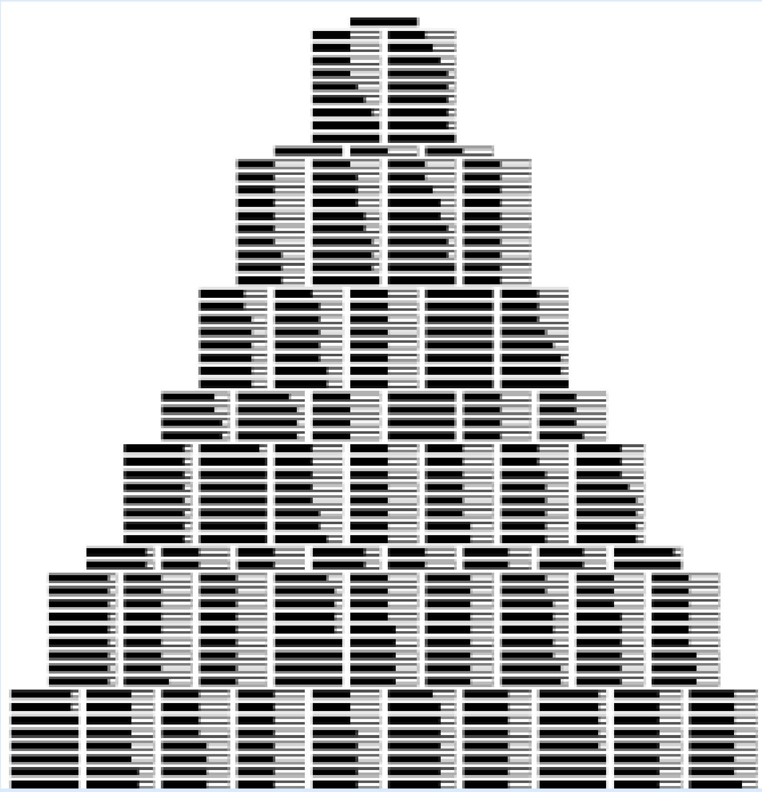
Exercises:

- All questions addressed

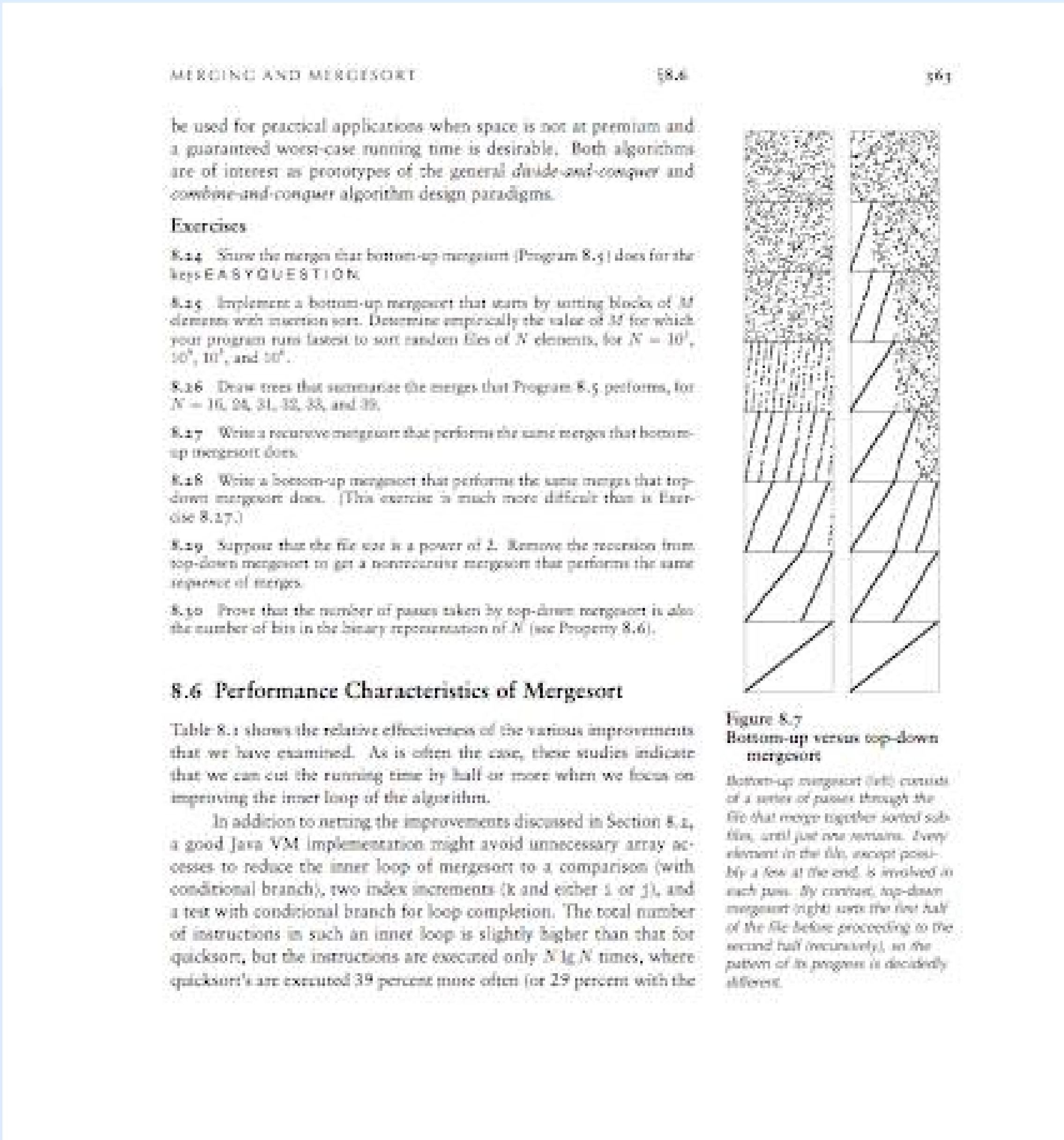
Tables:

- Summarize full empirical studies

PostScript as algorithm visualization tool



```
/insert
  {
    /X rand 1000 idiv N mod def
    /N N 1 add def
    /sum 0 def
    /a [
    0 1 a length 1 sub
     {
       a exch get /nd exch def
       X sum ge X sum nd add lt and
         {
           nd 1 add M 1 add ge
             { M 1 add 2 div dup
               /S S 1 add def }
             { nd 1 add } ifelse
         } { nd } ifelse
       /sum sum nd add def
     } for
    ] def
  } def

/doit
  {
    /a [ M ] def showline
    Nmax { insert showline } repeat
  } def
```



MERGING AND MERGESORT §8.6 363

be used for practical applications when space is not at premium and a guaranteed worst-case running time is desirable. Both algorithms are of interest as prototypes of the general *divide-and-conquer* and *combine-and-conquer* algorithm design paradigms.

Exercises

8.24 Show the merges that bottom-up mergesort (Program 8.5) does for the keys E A S Y Q U E S T I O N.

8.25 Implement a bottom-up mergesort that starts by sorting blocks of M elements with insertion sort. Determine empirically the value of M for which your program runs fastest to sort random files of N elements, for $N = 10^3$, 10^4, 10^5, and 10^6.

8.26 Draw trees that summarize the merges that Program 8.5 performs, for $N = 16, 24, 31, 32, 33$, and 39.

8.27 Write a recursive mergesort that performs the same merges that bottom-up mergesort does.

8.28 Write a bottom-up mergesort that performs the same merges that top-down mergesort does. (This exercise is much more difficult than is Exercise 8.27.)

8.29 Suppose that the file size is a power of 2. Remove the recursion from top-down mergesort to get a nonrecursive mergesort that performs the same sequence of merges.

8.30 Prove that the number of passes taken by top-down mergesort is also the number of bits in the binary representation of N (see Property 8.6).

8.6 Performance Characteristics of Mergesort

Table 8.1 shows the relative effectiveness of the various improvements that we have examined. As is often the case, these studies indicate that we can cut the running time by half or more when we focus on improving the inner loop of the algorithm.

In addition to netting the improvements discussed in Section 8.2, a good Java VM implementation might avoid unnecessary array accesses to reduce the inner loop of mergesort to a comparison (with conditional branch), two index increments (k and either i or j), and a test with conditional branch for loop completion. The total number of instructions in such an inner loop is slightly higher than that for quicksort, but the instructions are executed only $N \lg N$ times, where quicksort's are executed 39 percent more often (or 29 percent with the

Figure 8.7
Bottom-up versus top-down mergesort

Bottom-up mergesort (left) consists of a series of passes through the file that merge together sorted subfiles, until just one remains. Every element in the file, except possibly a few at the end, is involved in each pass. By contrast, top-down mergesort (right) sorts the first half of the file before proceeding to the second half (recursively), so the pattern of its progress is decidedly different.

Third edition features

- programs
  - C, C++, Java
- figures
  - dsdrawn
  - direct
- tables
  - empirical
  - summaries
- exercises
  - (1000s)
- properties
  - (theorems)
- layout design
- links**

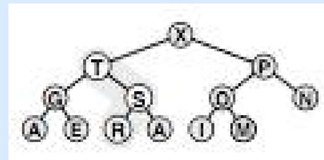
** not enough (stay tuned)

Creating “Algorithms”

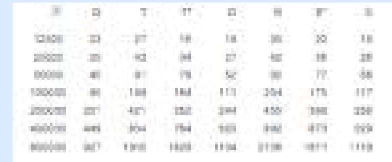
text sections

```
G R A P H   P R O P E R T I E S   A N D   T Y P E S

Many computational applications naturally involve not just a set of
items, but also a set of connections between pairs of those items.
The relationships implied by these connections lead immediately to a
host of natural questions: Is there a way to get from one item to another
by following the connections?  How many other items can be reached
from a given item? What is the best way to get from this item to this
other item?

To model such situations, we use abstract objects called
graphs. In this chapter, we examine basic properties of graphs in
detail, setting the stage for us to study a variety of algorithms that
are useful for answering questions of the type just posed.  These
algorithms make effective use of many of the computational tools that
we considered in Parts 1--4. They also serve as the basis for
attacking problems in important applications whose solution we could
not even contemplate without good algorithmic technology.
...
```

figures



tables

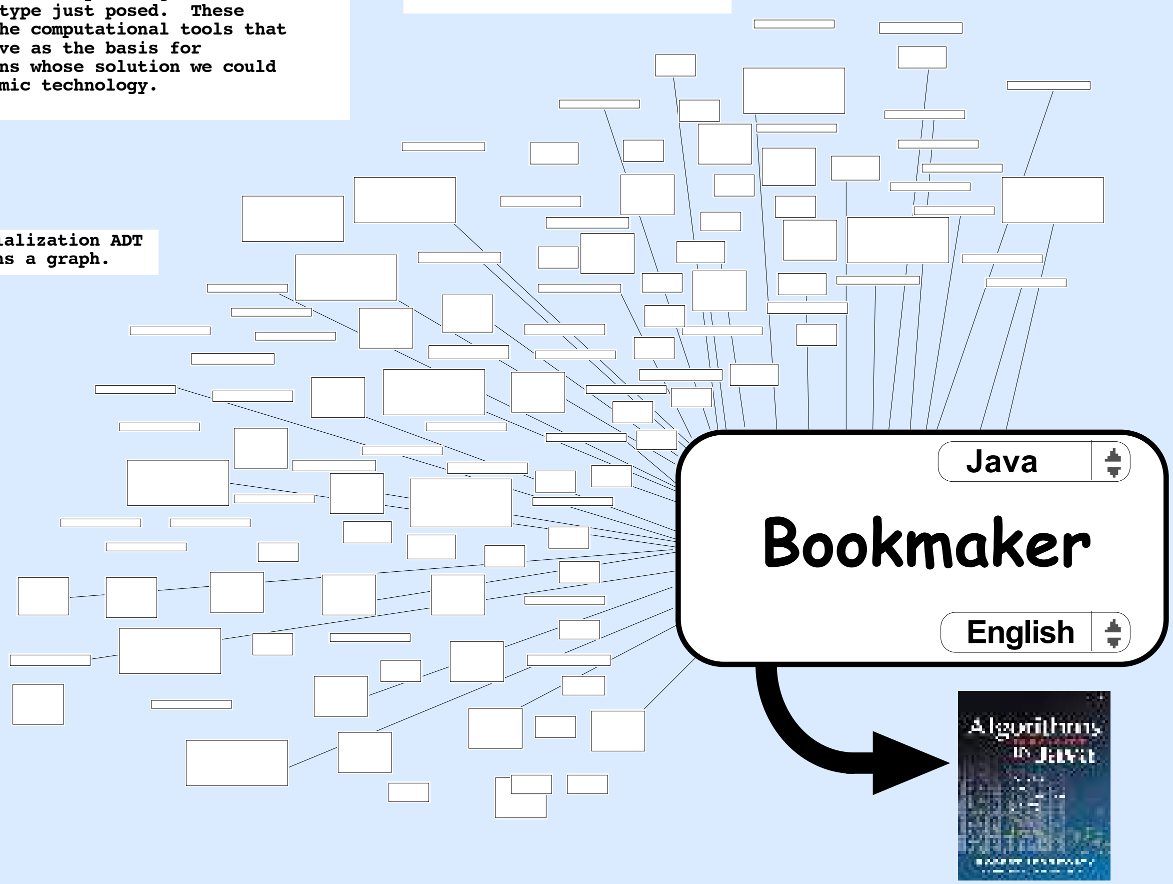


exercises

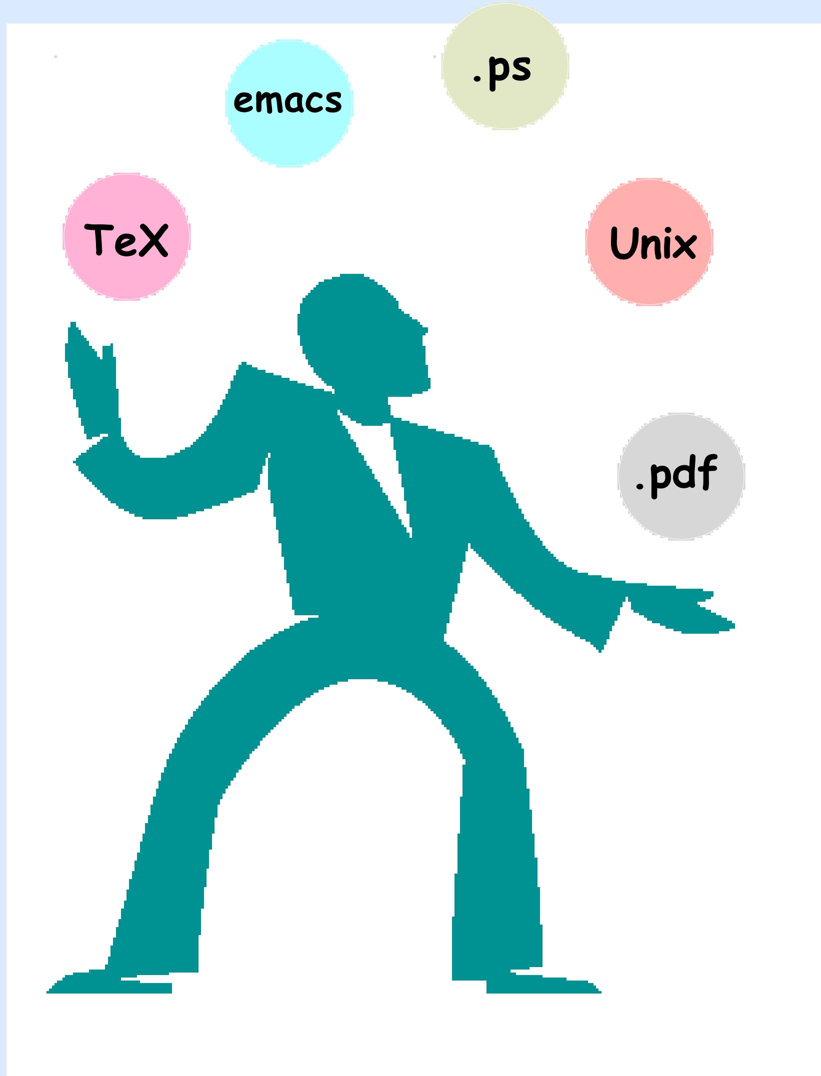
```
Write a representation-independent graph-initialization ADT
function that, given an array of edges, returns a graph.
```

programs

```
program euclid(input,output);
var x,y: integer;
function gcd(u,v:integer): integer;
  begin
  if v=0 then gcd:=u
         else gcd:=gcd(v, u mod v)
  end;
begin
while not eof do
  begin
  readln(x,y);
  if x<0 then x:=-x;
  if y<0 then y:=-y;
  writeln(x,y,gcd(x,y));
  end;
end.
```



Bookmaker (the lonely author)

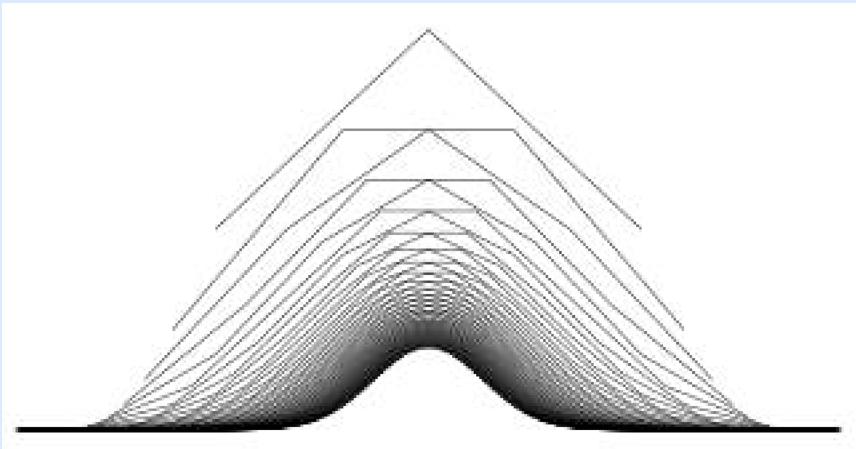


juggler image from Northern Lights Software

Facts and figures

	pages	programs	figures	tables	exercises	files
Algorithms	550	140	150	0	400	600
Second edition (typical)	650	200	350	0	400	6,000
Third edition 1-5 (typical)	1200	250	500	75	2,000	25,000
Third edition 1-8 (est.)	2000	400	800	120	3,500	40,000

digression: PostScript as math visualization tool



```
/doit
{ /M exch def /Nmax exch def
  /A [0 M 1 sub M div 1 M div 0] def
  3 1 Nmax
    { /N exch def
      [ 0
        1 1 N
         { /k exch def
           A k 1 sub get M div A k get M 1 sub mul M div add
         } for
      0 ]  /A exch def
      A drawcurve
    } for
} def
```

Fourth edition 2003-??

Goals:

- Do answers to exercises
- Stabilize content
- Create interactive and dynamic eBook supplements

Problems:

- Tens of thousands of files
- Thousands of exercises
- Different typescripts for C, C++, Java
- Deep hacks throughout figs (need new dsdraw)
- Ancient typesetting engine

Approach:

- Back to single typescript??
- Layout language??
- Scripting language??

Needs for fourth edition

1. Structured-document authoring and editing tool
   - simple system- and machine- independent editor
   - manage nonlinear organization of fragments
   - TeX-like plugin for equations
   - application-independent primary source format
   - cross-reference/indexing across all types of fragments

2. Programming tools
   - Source language with flexible ADT and IO mechanisms
   - Postscript

3. Flexible document-creation engine
   - semiautomatic layout
   - programming language
   - smart filters with link/embed/unlink/unembed

Inventing the Future

Q: Where is the "Algs" e-/dynamic-/interactive- book?

A: (1984): Done. Balsa (with M. Brown).

1985 choice: content over form

Triumph of content leads to (reasonable) demand for:

- Answers to exercises
- Online lecture notes
- Customizable versions
- Dynamic figures
- Interactive testing/drill
- ...

Inventing the Future

Q: Where is the "Algs" e-/dynamic-/interactive- book?

A: (2002): Where are the tools that an individual author could use to make one??