

SHELLSORT (Survey Talk)

Shellsort and Shellsort networks

Ancient results

Old results

Average-case analysis

Variants of Shellsort

New ideas

Shellsort

```
shellsort(itemType a[], int l, int r)
{
    int incs[16] =
        { 1391376, 463792, 198768, 86961, 33936,
          13776, 4592, 1968, 861, 336, 112, 48,
          21, 7, 3, 1 };
    int i, j, h, v;

    for ( k = 0; k < 16; k++)
        for (h = incs[k], i = l+h; i <= r; i++)
            {
                v = a[i]; j = i;
                while (j > h && a[j-h] > v)
                    { a[j] = a[j-h]; j -= h; }
                a[j] = v;
            }
}
```

Running time depends on increment sequence

Solved problem:

- * running time is

Open problems:

- * "best" increment sequences for practical N
- * average-case analysis for any interesting sequence
- * $N \log N$ variants
- * variants corresponding to $\log N$ depth networks

Pratt Bounds (1971)

UPPER BOUND

Use following increments

1	2	4	8	16	32	64	128	256
.	3	6	12	24	48	96	192	384
.		9	18	36	72	144	288	576
.			27	54	108	216	432	864
.				81	162	324	648	1296
.					243	486	962	1924
.						729	1458	2916
.							2187	4374
.								6561

Total running time is

Applies to networks

Too slow in practice

LOWER BOUND

If increment sequence is "almost geometric"
then total running time must be

Sedgewick Upper Bound (1982)

Use the following increments

1 8 23 77 281 1073 4193 16577 . . .

Increment sequence not "almost geometric"

Connection to "Frobenius problem"

Smaller of two bounds

first bound

second bound

Use first bound for small increments

Use second bound for large increments

Total running time is

Frobenius Problem

A country wishes to issue k different stamps

- * Number of values that cannot be achieved?
- * Largest value that cannot be achieved?

Examples

Two stamps, relatively prime (Curran-Sharp, 1884)

Three stamps (Selmer, 1977)

Chazelle Upper Bound

Generalize Pratt "network" construction

Example

1	7	49	343	2401	16807	117649
.	8	56	392	2744	19208	134456
.		64	448	3136	21952	153664
.			512	3584	25088	175616
.				4096	28672	200704
.					32768	229376
.						262144

Total running time is

Choose parameter optimally

(restrict to logarithmic number of passes)

Too slow in practice

Sedgewick-Incerpi Upper Bound

Start with a "basis" of relatively prime numbers

1 3 7 16 41

Build a sequence with every number the product of
 a basis number and
 a number earlier in the sequence

1	1*3	1*3* 7	1*3* 7*16	1*3* 7*16* 41
.	1*7	1*3*16	1*3* 7*41	1*3* 7*16*101
.		1*7*16	1*3*16*41	1*3* 7*41*101
.			1*7*16*41	1*3*16*41*101
.				1*7*16*41*101

1	3	21	336	13776
.	7	48	861	33936
.		112	1968	86961
.			4592	198768
.				463792

Asymptotically optimal (same as Chazelle)

Fast in practice

Poonen Lower Bound

Using M increments on a file of size N requires
at least

comparisons in the worst case, for some $c > 0$.

Applies to any algorithm that

- * uses a number of passes
 compare-exchanging items
 at a fixed increment
- * does at least comparisons on each pass
- * does not disturb k -ordering once achieved

Complexity "gap"

thousand	10	3	9	78
million	20	4	22	482
billion	30	6	43	1933
trillion	40	9	78	6233

UPPER BOUND

passes:
total cost:

LOWER BOUND

passes:
total cost:

AVERAGE CASE

No results for any interesting sequences
Simulations show
 average case close to worst case
 for sequences designed to worst case

Average case (two or three increments)

Analysis of $(h, 1)$ Shellsort (Knuth)

Analysis of $(h, k, 1)$ Shellsort (Yao)

Asymptotic result for three increments?

Shakersort (Incerpi, Sedgewick, 1984)

Shellsort "network"

Do one "cocktail shaker" pass
(not full insertion sort)
for each increment

Choose increments close to

Always seems to sort (!)

Poonen's bound applies; can't always sort

Can serve as basis for probabilistic
sorting network with $N \log N$ comparators

Variants

try more sophisticated increment sequences
do multiple shakes for each increment
add 1-shakes at end if necessary

DISADVANTAGE

network is "depth" N

Bricksort (Sedgewick, Lemke, 1995)

Shellsort "network"

Do one "brick" pass
(not full shaker pass)
for each increment

Choose increments close to

Always seems to sort (!!)

Poonen's bound applies?

Can serve as basis for probabilistic
sorting network of "depth" $\log N$

Variants

try more sophisticated increment sequences
do multiple brick passes for each increment
add 1-passes at end if necessary

Average-case analysis??

Analysis of Bricksort