A New Formulation of Laser Step Diagonal Measurement – Two-dimensional Case –

Soichi Ibaraki *, Takafumi Hata, and Atsushi Matsubara

Department of Micro Engineering, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan

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Abstract

The laser step diagonal measurement modifies the diagonal displacement measurement by executing a diagonal as a sequence of single-axis motions. It has been claimed that the step-diagonal test enables the identification of all the volumetric error components, including linear errors, straightness and squareness errors, in three-dimensional space. In this paper, we show that the conventional formulation of the step diagonal measurement is valid only when implicit assumptions related to the configuration of laser and mirror setups are met, and that its inherent problem is that it is generally not possible to guarantee these conditions when volumetric errors of the machine are unknown. To address these issues, we propose a new formulation of the step diagonal measurement, in order to accurately identify volumetric errors even under the existence of setup errors. To simplify the discussion, this paper only considers the two-dimensional version of laser step diagonal measurement to estimate volumetric errors on the XY plane. The effectiveness of the proposed modified identification scheme is experimentally investigated by an application example of two-dimensional laser step diagonal measurement to a high-precision machine tool.

Key words: Step diagonal measurement, volumetric errors, machine tool, laser interferometer

* Corresponding author. Phone/Fax: +81-75-753-5227
Email address: `ibaraki@prec.kyoto-u.ac.jp` (Soichi Ibaraki).

1 Introduction

To meet increasing demands in the manufacturing of optical parts or electronic parts, high-precision and ultra-precision machine tools have been rapidly introduced into the market in recent years. Typically, a machine tool of the positioning resolution up to 1$\sim$0.1nm is called an ultra-precision machine tool, and a machine of the positioning resolution of 0.1 μm$\sim$10 nm and higher stiffness, higher speed, and larger workspace than ultra-precision machines is referred to as a high-precision machine tool. To ensure the motion accuracy over the entire three-dimensional workspace of such a machine tool, it is important to evaluate all the volumetric errors including 3 linear displacement errors, 6 straightness errors and 3 squareness errors [1]. For ultra-precision and high-precision machines of a nanometer-order positioning resolution, it is particularly important to evaluate straightness and squareness errors, since it is often the case in practice that straightness and squareness errors of such a machine are much larger than linear positioning errors. Currently, ASME B5 (TC52) and ISO 230 (TC39) are working on the standardization of the definition of volumetric accuracy [2].

For the measurement of linear displacement errors, laser interferometers of the resolution sufficient to measure high-precision and ultra-precision machines are widely available in today's market. The measurement of other volumetric errors such as straightness and squareness errors is more difficult and time-consuming. Typically, straightness and squareness errors are measured by using a high-precision displacement sensor and an artifact such as a straight edge or a square edge. For the measurement on high-precision or ultra-precision machines, the artifact whose geometric and dimensional accuracies are guaranteed to be higher than the accuracy of the measured machine is needed, which requires higher measurement cost. Furthermore, since the measurement is one-dimensional and its path is restricted to a line or a square, an operator must change the setup of a sensor and an artefact every time for the measurement of each different error component. Dual-beam laser systems or autocollimators to measure straightness and squareness errors are also available from many companies. They do not require an artefact such as a straight edge, but it is the same in that a different setup is needed to measure each different error component.

In the literature, many research efforts have been reported on the evaluation of volumetric errors. For example, laser trackers have been applied to evaluate volumetric errors on coordinate measuring machines [3,4]. A laser tracker is capable of measuring the three-dimensional position of the target, and thus can be directly applied to the evaluation of volumetric errors. In general, however, there are many error sources in a laser tracker, and their precise calibration is crucial to obtain the measurement accuracy sufficient to evaluate volumetric errors of high-precision machine tools. Furthermore, laser trackers are at this stage too expensive to be widely accepted to test machine tools in the industry. The laser interferometer with the auto-alignment of laser direction [5] has similar issues. Especially for CMMs, three-dimensional volumetric errors are often evaluated by using a ball plate (e.g. [6]). The cost to make the ball plate of the geometric accuracy precisely calibrated can be an issue in practical applications to machine tools. It is also a critical limitation that the measurement volume is strictly restricted to the size of the ball plate.

For quicker, lower-cost evaluation of volumetric errors of a machine tool, the standards ASME B5.54 [7] and ISO230-6 [8] define the diagonal measurement by using a laser interferometer. Its two-dimensional version is illustrated in Fig. 1. In the diagonal measurement, the machine moves along each of diagonals of the machine's workspace in turn, and the diagonal displacement is measured by using a laser interferometer. In the three-dimensional case, the machine moves along each of its four body diagonals. Although the diagonal measurement can be considered as a good quick check of volumetric errors, it is almost obvious that it cannot be used as a strict diagnosis of each volumetric error. As has been discussed in details by Chapman [9], under certain conditions, a machine can achieve a good result on diagonal tests, even though it has a poor volumetric accuracy. Furthermore, more importantly, it is impossible to distinguish the linear error, the straightness error, and the squareness error of each axis from the results of diagonal tests.

As an extension of the diagonal measurement, the step diagonal measurement, or the vector measurement, has been proposed by Wang [10,11]. In the step diagonal measurement, each axis is moved one at a time along the "zig-zag" path toward the body diagonal direction. Figure 2 illustrates the setup of the two-dimensional version of the step-diagonal measurement. Wang claimed that additional data enables the identification of all the volumetric errors, namely the linear error, the straightness error, and the squareness error of each axis, from step diagonal measurements.

The first objective of this paper is to discuss the validity of the volumetric error identification based on step diagonal measurements. In this paper, we show that the formulation of the step diagonal measurement presented by Wang [10] is valid only when implicit assumptions related to laser and mirror setups are met, and that its inherent problem is that it is generally not possible to guarantee these conditions when volumetric errors of the machine are unknown. As a result, setup errors potentially impose significant errors on the identification of volumetric errors. As a critical issue with the step diagonal measurement, Chapman [9] discussed that the misalignment of the mirror may cause a large estimation error. The discussion presented in this paper contains this issue, but we will present rather critical issues with Wang's formulation of the step diagonal measurement.

As remedies for these issues, this paper will propose a new formulation of the step diagonal measurement, such that each volumetric error can be identified from step diagonal measurements even when setup errors exist. To simplify the discussion, this paper only considers the two-dimensional version of step diagonal measurements. The extension to the three-dimensional case is straightforward, and will be studied in our future research. The validity of the discussion on issues in step diagonal measurements and the effectiveness of the proposed modified identification scheme will be investigated experimentally by showing an application example to a high-precision machine tool. The remainder of the paper is organized as follows: Section 2 first defines volumetric errors and the step diagonal measurement, and then briefly reviews the conventional formulation of step diagonal measurement proposed by Wang [10]. Section 3 discusses inherent issues with the conventional formulation of step diagonal measurement. A new formulation of step diagonal measurement is proposed in Section 4. Section 5 presents an experimental validation of the present formulation by showing its application example to a high-accuracy machine tool. Finally, the paper concludes with a brief summary as presented in Section 6.

2 Conventional Formulation of Laser Step Diagonal Measurement

2.1 Problem Statement

Figure 2 depicts the setup of 2D step-diagonal measurement. As the machine spindle, where a plane mirror is attached, moves along a "zig-zag" path, the moving distance along the face diagonal is measured by using a laser interferometer. Suppose that the laser is aligned to the diagonal AD, as is illustrated in Fig. 2. Suppose that this direction is represented by the unit vector $l_{pp} = \begin{bmatrix} l_{x,pp}, \ l_{y,pp} \end{bmatrix}$ (this setup is referred to as pp measurement hereafter).

Suppose that reference location of the target mirror in X and Y directions, $\tilde{P}_x(k)$ and $\tilde{P}_y(k) \in \mathbb{R}^2$, are respectively given by:

$$\tilde{P}_x(k) = \begin{bmatrix} a \cdot k \\ 0 \end{bmatrix}, \quad \tilde{P}_y(k) = \begin{bmatrix} 0 \\ a \cdot k \end{bmatrix} \quad (k = 0, \cdots, N) \tag{1}$$

where a is constant. To simply the discussion, this paper assumes that the aspect ratio of each block is one.

Define $E_x(x(k))$ and $E_y(x(k))$ ($k = 1, \cdots, N$) as the positioning error in X- and Y-directions, respectively, when the machine moves toward the X direction from the reference position $\tilde{P}_x(k-1)$ to $\tilde{P}_x(k)$. In other words, the actual position of the target mirror, $P_x(k) \in \mathbb{R}^2$, is given by:

$$P_x(k) = P_x(k-1) + \begin{bmatrix} a + E_x(x(k)) \\ E_y(x(k)) \end{bmatrix} = \tilde{P}_x(k) + \sum_{i=1}^{k} \begin{bmatrix} E_x(x(i)) \\ E_y(x(i)) \end{bmatrix} \tag{2}$$

For the motion toward Y direction, $P_y(k)$, $E_x(y(k))$ and $E_y(y(k))$ ($k = 1, \cdots, N$) are defined similarly. In this paper, total $4N$ error components, $E_x(x(k))$, $E_y(x(k))$, $E_x(y(k))$ and $E_y(y(k))$ ($k = 1, \cdots, N$), are called volumetric errors. The objective of step diagonal measurements is to identify each of these volumetric errors.

In the pp measurement, when the target moves toward the X direction from the reference position $\tilde{P}_x(k-1)$ to $\tilde{P}_x(k)$, its diagonal displacement from the start point (point A), denoted by $R_{x(k),pp}$, is measured by using a laser interferometer. Similarly, when the target moves toward the Y direction from the reference position $\tilde{P}_y(k-1)$ to $\tilde{P}_y(k)$, its diagonal displacement from the start point is denoted by $R_{y(k),pp}$.

A similar measurement is conducted as the laser is aligned along the diagonal BC in Fig. 2 (this setup is referred to as np measurement). Diagonal displacements, $R_{x(k),np}$ and $R_{y(k),np}$ ($k = 1, \cdots, N$), are defined similarly.

2.2 Conventional Formulation of Laser Step Diagonal Measurement

This section briefly reviews the conventional formulation of the identification of volumetric errors based on the step diagonal measurement presented by Wang [10].

To simplify the discussion, the single block case (i.e. $N = 1$) is first considered. As depicted in Fig. 3, under the assumption that the mirror is aligned perpendicular to the laser direction, the diagonal displacement, $R_{x,pp}$, for the motion toward the X direction from the point A to B is given as follows:

$$R_{x,pp} = \begin{bmatrix} l_{x,pp} & l_{y,pp} \end{bmatrix} \cdot \begin{bmatrix} a + E_x(x) \\ E_y(x) \end{bmatrix} \tag{3}$$

By combining similar formulations for $R_{y,pp}$, $R_{x,np}$ and $R_{y,np}$, we have:

$$\begin{bmatrix} l_{x,pp} & l_{y,pp} & 0 & 0 \\ 0 & 0 & l_{x,pp} & l_{y,pp} \\ -l_{x,np} & -l_{y,np} & 0 & 0 \\ 0 & 0 & l_{x,np} & l_{y,np} \end{bmatrix} \begin{bmatrix} a + E_x(x) \\ E_y(x) \\ E_x(y) \\ a + E_y(y) \end{bmatrix} = \begin{bmatrix} R_{x,pp} \\ R_{y,pp} \\ R_{x,np} \\ R_{y,np} \end{bmatrix} \tag{4}$$

Assume nominal laser directions, i.e.:

$$\begin{bmatrix} l_{x,pp} & l_{y,pp} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad \begin{bmatrix} l_{x,np} & l_{y,np} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 \end{bmatrix} \tag{5}$$

Then, volumetric errors, $E_x(x)$, $E_y(x)$, $E_x(y)$ and $E_y(y)$, can be estimated from $R_{x,pp}$, $R_{y,pp}$, $R_{x,np}$ and $R_{y,np}$ by solving Eq. (4).

In the multiple blocks case (i.e. $N \geq 2$), Eq. (4) can be extended as follows:

$$\begin{bmatrix} l_{x,pp} & l_{y,pp} & 0 & 0 \\ 0 & 0 & l_{x,pp} & l_{y,pp} \\ -l_{x,np} & -l_{y,np} & 0 & 0 \\ 0 & 0 & l_{x,np} & l_{y,np} \end{bmatrix} \begin{bmatrix} a + E_x(x(k)) \\ E_y(x(k)) \\ E_x(y(k)) \\ a + E_y(y(k)) \end{bmatrix} = \begin{bmatrix} R_{x(k),pp} \\ R_{y(k),pp} \\ R_{x(N-k+1),np} \\ R_{y(k),np} \end{bmatrix} \quad (k = 1, \cdots, N) \tag{6}$$

By assuming nominal laser directions, volumetric errors at each block can be identified similarly as in the single block case.

3 Issues in Conventional Formulation of Laser Step Diagonal Measurement

To understand an inherent issue with the identification of volumetric errors based on Eq. (6), notice that Eq. (6) is valid only when the following conditions are satisfied:

(1) Laser beam directions must be precisely aligned to nominal directions, Eq. (5).
(2) The flat mirror must be precisely aligned perpendicular to the laser beam direction.
(3) The angular errors of the machine are negligibly small.

An inherent problem with the conventional formulation (the identification of volumetric errors by solving Eq. (6)) is that when the conditions above are not met, it potentially imposes a significant error on the estimates of volumetric errors. Furthermore, since the direction of the laser beam and the flat mirror can be only aligned based on the motion of the machine to be measured, no matter how careful an operator sets up laser beam and mirror directions, it is simply not possible to guarantee the satisfaction of the conditions (1) and (2), when volumetric errors of the machine are unknown. Therefore, we claim that, in general cases where setup errors cannot be completely eliminated, it is not possible to accurately identify all the volumetric errors by using the conventional formulation of laser step diagonal measurement. The detailed discussion will follow.

3.1 Misalignment of laser beam directions

Except for a special case, the laser beam direction in pp and np measurements can be only aligned *based on the motion of the machine to be measured.* That is, in a typical setup, the laser beam direction is aligned such that it becomes parallel to the machine's diagonal. For example, when the machine moves from A to D in Fig. 3, the laser direction is adjusted such that the deviation of the location of the laser spot on the mirror is minimized (this alignment can be done more precisely by using a quad-detector). Here, if the machine has volumetric errors and they are unknown, it is not possible to align the laser beam perfectly to the nominal direction. An illustrative example is shown in Fig. 4. This example assumes that $E_y(y) > 0$, $E_x(x) = E_y(x) = E_x(y) = 0$. By carefully setting up laser directions, the laser direction can be ideally aligned to the machine's diagonal. However, since the machine's diagonals are not perpendicular to each other due to volumetric errors, laser directions in pp- and np- measurements cannot be perpendicular to each other, no matter how careful an operator sets up laser directions.

Wang [10] pointed out that a small misalignment error of laser direction does not affect much measured diagonal distances, i.e. $R_{pp} := R_{x,pp} + R_{y,pp}$, and $R_{np} := R_{x,np} + R_{y,np}$. However, it causes much larger error on each of $R_{x,pp}$, $R_{y,pp}$, $R_{x,np}$, and $R_{y,np}$. For example, as illustrated in Fig. 5, assume that the laser direction is misaligned by 0.01 mm from the nominal direction for the step size of 10 mm. From our experiences, this is within typical level of misalignment in practical measurement. Assume that the machine has no positioning error when it moves from A to B then to C. Also assume that the mirror is aligned perfectly perpendicular to the laser direction. The misalignment in the laser direction reduces the diagonal distance, R_{pp}, by only 0.004 μm. On the other hand, the same misalignment reduces $R_{x,pp}$ by 5.0 μm and increases $R_{y,pp}$ by 5.0 μm. This is a significant error compared to typical machine tool's positioning error.

The quantitative sensitivity of the laser misalignment on the estimated volumetric errors will be discussed in Section 3.4.

3.2 Misalignment of mirror directions

Similarly, the direction of the flat mirror can be only aligned *based on the motion of the machine to be measured.* Typically, the mirror direction is adjusted as illustrated in Fig. 6. For example, in the pp measurement, when the machine moves from B to C, the mirror direction is adjusted such that the measured diagonal distance becomes approximately equal at both ends of the mirror. This adjustment aligns the mirror ideally parallel to the diagonal direction (BC), but it does not ensure the perpendicularity of laser and mirror directions. Fig. 6 illustrates the case where $E_y(y) > 0$, $E_x(x) = E_y(x) = E_x(y) = 0$. In such a case for example, it is clear that the mirror direction cannot be perpendicular to the laser direction, no matter how careful an operator sets up the mirror. Furthermore, notice that if the mirror is perfectly aligned to the diagonal direction by this adjustment, it simply makes $R_{x,pp} = R_{y,pp}$ and $R_{x,np} = R_{y,np}$. It means that the step diagonal measurement only gives half of diagonal distances (half of AD or BC), and thus the error identification based on Eq. (4) will obviously fail.

It is to be noted that when the machine's diagonals are perpendicular to each other, it is possible to align laser and mirror directions perfectly. Figure 7 shows examples of such a "special" case. For example, when the machine has the same linear positioning error in both X and Y directions as illustrated in Fig. 7(a), or when the machine has only the squareness error as illustrated in Fig. 7(b), then the machine's diagonals are still approximately perpendicular to each other, and thus misalignment errors of laser and mirror directions can be eliminated if an operator aligns them very carefully. If the machine to be measured "happens to" meet this condition, the conventional formulation presented in Section 2.2 can potentially estimate volumetric errors. In general cases where the machine's accuracy is not known in priori, however, it is not possible to guarantee the conditions (1) and (2) in Section 3.1.

3.3 Machine's angular errors

On typical machining centers, it is often the case that the straightness error of a feed drive is caused by the deformation of guideways, and thus that the straightness error is accompanied with angular errors. Notice that, in the two-dimensional case, we only considers the yaw error. The angular error changes the relative direction of the mirror with respect to the laser beam direction as the machine moves A→B→D in Fig. 3. As a result, angular errors also affect diagonal displacements, $R_{x,pp}$ and $R_{y,pp}$. The formulation (4) ignores this effect, under the assumption that angular errors are negligibly small compared to positioning errors. When the machine's angular errors are not negligibly small, they may cause significant estimation errors. Soons [12] and Yang et al. [13] gave the formulation of the effect of angular errors on identification accuracies for the three-dimensional laser step diagonal measurement, and thus it is not repeated here.

For example, when the angle of the mirror changes by $\gamma(x)$ due to the machine's angular error as the machine moves from A to B in Fig. 3, its effect on the diagonal displacement is approximately given by $\frac{\sqrt{2}}{2}a\gamma(x)$. On typical latest machining centers, this is often significantly small compared to the effect from volumetric errors. Considering this, and considering that the aforementioned previous papers have discussed the formulation of the effect of angular errors in details, this paper ignores the effect of angular errors of the machine in the estimation of volumetric errors. It should be emphasized that the assumption of the angular errors of the machine to be negligibly small shall be mandatory requirement for both the conventional formulation and the proposed formulation to be presented in Section 4.

3.4 Sensitivity analysis of setup errors

To quantitatively evaluate the effect of aforementioned setup errors on the estimation accuracy of laser step diagonal measurements and to further clarify critical issues with the conventional formulation, this section presents the sensitivity analysis of setup errors on the estimates of volumetric errors. In particular, the misalignment error of laser direction discussed in Section 3.1 and the misalignment error of mirror direction discussed in Section 3.2 are considered (these errors are referred to as setup errors hereafter).

As has been discussed in Section 3.2, notice that setup errors potentially impose significant effect on the measured diagonal displacement when the mirror center is not on the laser axis, while its effect becomes negligibly small when the mirror center is on the laser axis. For example, in the pp measurement as shown in Fig. 3, when the mirror center is at the point B, both of laser and mirror misalignment errors may cause significant error on $R_{x,pp}$, as has been discussed in Sections 3.1 and 3.2. However, their effect on the diagonal distance, R_{pp}, is negligibly small. In the k-th block, suppose that "nominal" step diagonal distances are denoted by $\tilde{R}_{x(k),pp}$ and $\tilde{R}_{y(k),pp}$, when the laser direction is perfectly aligned to the nominal direction and the mirror is aligned perfectly perpendicular to the laser direction. The effect of the misalignment of laser and mirror directions can be given in the following form:

$$R_{x(k),pp} = \tilde{R}_{x(k),pp} + \delta R_{x,pp}, \qquad R_{y(k),pp} = \tilde{R}_{y(k),pp} + \delta R_{y,pp}, \tag{7}$$

where $\delta R_{x,pp}$ and $\delta R_{y,pp}$ are diagonal displacement errors caused by setup errors. Under the assumption that the machine's angular error is negligibly small, we can approximate that the effect of setup errors, $\delta R_{x,pp}$ and $\delta R_{y,pp}$, are the same for all the blocks. Furthermore, as discussed above, the following approximation holds:

$$\delta R_{x,pp} + \delta R_{y,pp} \approx 0 \tag{8}$$

For the np measurement, $\tilde{R}_{x(k),np}$, $\tilde{R}_{y(k),np}$, $\delta R_{x,np}$, and $\delta R_{y,np}$ are analogously defined.

By solving Eq.(6) for estimated volumetric errors in the k-th block, $\hat{E}_x(x(k))$, $\hat{E}_y(x(k))$, $\hat{E}_x(y(k))$, and $\hat{E}_y(y(k))$, with measured diagonal measurements, $R_{x(k),pp}$, $R_{y(k),pp}$, $R_{x(k),np}$, and $R_{y(k),np}$, we have:

$$\hat{E}_x(x(k)) = \frac{\sqrt{2}}{2}\left\{R_{x(k),pp} + R_{x(N-k+1),np} - \delta R_{x,pp} - \delta R_{x,np}\right\} - a \tag{9}$$

$$\hat{E}_y(x(k)) = \frac{\sqrt{2}}{2}\left\{R_{x(k),pp} - R_{x(N-k+1),np} - \delta R_{x,pp} + \delta R_{x,np}\right\} \tag{10}$$

$$\hat{E}_x(y(k)) = \frac{\sqrt{2}}{2}\left\{R_{y(k),pp} - R_{y(k),np} + \delta R_{x,pp} - \delta R_{x,np}\right\} \tag{11}$$

$$\hat{E}_y(y(k)) = \frac{\sqrt{2}}{2}\left\{R_{y(k),pp} + R_{y(k),np} + \delta R_{x,pp} + \delta R_{x,np}\right\} - a \tag{12}$$

The nominal laser beam directions (5) are assumed here. Notice that the approximation (8) (and also $\delta R_{x,np} + \delta R_{y,np} \approx 0$) is used.

Equations (9) and (12) indicate that the estimated linear positioning errors, $\hat{E}_x(x(k))$ and $\hat{E}_y(y(k))$, are subject to the influence of setup errors by $\mp\frac{\sqrt{2}}{2}\{\delta R_{x,pp} + \delta R_{x,np}\}$. As is clear from the discussion in Sections 3.1 and 3.2, it is practically not possible to guarantee that this effect is sufficiently small compared to the machine's positioning error. It is also clear from Eqs. (9)∼(12) that it is not possible to identify $\delta R_{x,pp}$ and $\delta R_{x,np}$ from laser step diagonal measurements.

As is shown in Eqs. (10) and (11), $\hat{E}_y(x(k))$ and $\hat{E}_x(y(k))$ are also subject to the influence of setup errors by $\mp\frac{\sqrt{2}}{2}\{\delta R_{x,pp} - \delta R_{x,np}\}$. However, notice that the reference direction to define normal error components can be arbitrarily determined. For example, the reference direction can be set such that $E_y(x(1)) = 0$. Under this assumption, the effect of setup errors can be estimated from Eq. (10) by:

$$\delta R_{x,pp} - \delta R_{x,np} = R_{x(1),pp} - R_{x(N),np} \tag{13}$$

Therefore, normal error components, $\hat{E}_y(x(k))$ and $\hat{E}_x(y(k))$, can be estimated even under the existence of setup errors. This will be further discussed in Section 4.

To further illustrate that setup errors may potentially results in a significant identification error, numerical simulations are presented. In the simulations, it is assumed that the machine has given volumetric errors in the single block as shown in Table 1 (Three sets of given errors, (a), (b) and (c), are tested). Laser directions are assumed to be aligned perfectly to the machine's diagonal directions (therefore, due to the machine's volumetric errors, they may not be aligned to nominal directions). For the simplicity of simulation, the mirror is assumed to be perfectly aligned perpendicular to the laser beam direction (notice that the effect of the mirror misalignment can be discussed in exactly the same manner as the laser misalignment). Estimated volumetric errors are computed by using the conventional formulation (4). Note that the reference direction is set such that $E_y(x)$ becomes zero.

Table 1 compares given and estimated volumetric errors. In the case (a), the estimates of linear positioning errors, $E_x(x)$ and $E_y(y)$, contain a large estimation error, while the cases (b) and (c) have almost no error. This can be understood by observing that the cases (b) and (c) correspond to the cases shown in Fig. 7(a) and (b), respectively. In case (a), estimation errors are as large as half of the given error.

4 New formulation of laser step diagonal measurement

The discussion in Sections 3.1 and 3.2 shows that it is not possible to completely eliminate the misalignment of laser and mirror directions when the machine's accuracy is unknown. In this section, we present a new formulation of laser step diagonal measurement such that setup errors do not impose any effect on estimated volumetric errors, and thus the machine's volumetric errors can be accurately estimated even when there exist significant setup errors in both laser and mirror alignments.

As is clear from Eqs. (9)$\sim$(12), it is not possible to identify setup errors, namely $\delta R_{x,pp}$ and $\delta R_{x,np}$, from measured diagonal distances, $R_{x(k),pp}$, $R_{y(k),pp}$, $R_{x(k),np}$ and $R_{y(k),np}$. In order to cancel $\delta R_{x,pp}$ and $\delta R_{x,np}$, a remedy is to directly measure linear error components, $E_x(x(k))$ and $E_y(y(k))$. When $E_x(x(k))$ and $E_y(y(k))$ (k=1,$\cdots$,N) are known, from first and second rows of Eq. (6), the estimated $E_y(x(k))$ and $E_x(y(k))$ can be given by:

$$\begin{aligned} \hat{E}_y(x(k)) &= \sqrt{2} \cdot R_{x(k),pp} - (a + E_x(x(k))) \\ \hat{E}_x(y(k)) &= \sqrt{2} \cdot R_{y(k),pp} - (a + E_y(y(k))) \end{aligned} \tag{14}$$

where k=1,$\cdots$,N. Notice that in this case, only pp measurement is necessary to identify normal error components, $\hat{E}_y(x(k))$ and $\hat{E}_x(y(k))$.

Suppose that the measured diagonal displacements, $R_{x(k),pp}$ and $R_{y(k),pp}$, are subject to setup errors as is given in Eq. (7). Then, analogously to Eqs. (9)$\sim$(12), the estimates become:

$$\begin{aligned} \hat{E}_y(x(k)) &= \sqrt{2} \left(R_{x(k),pp} - \delta R_{x,pp}\right) - (a + E_x(x(k))) \\ \hat{E}_x(y(k)) &= \sqrt{2} \left(R_{y(k),pp} + \delta R_{x,pp}\right) - (a + E_y(y(k))) \end{aligned} \tag{15}$$

Analogously to the discussion in Section 3.4, the influence of setup errors, $\delta R_{x,pp}$, can be identified in this formulation, since the reference direction can be arbitrarily set. Therefore, in this formulation, the influence of setup errors can be cancelled, and thus normal error components, $\hat{E}_y(x(k))$ and $\hat{E}_x(y(k))$, can be identified even then there exists significant setup errors.

Remark 1: Notice that $\hat{E}_y(x(k))$ and $\hat{E}_x(y(k))$ can be also estimated from the np measurement:

$$\begin{aligned}\hat{E}_y(x(k)) &= -\sqrt{2}\cdot R_{x(N-k+1),np} + (a + E_x(x(k)))\\ \hat{E}_x(y(k)) &= -\sqrt{2}\cdot R_{y(k),np} + (a + E_y(y(k)))\end{aligned} \tag{16}$$

Adding Eqs. (14) and (16), we have:

$$\begin{aligned}\hat{E}_y(x(k)) &= \frac{\sqrt{2}}{2}\left\{R_{x(k),pp} - R_{x(N-k+1),np}\right\}\\ \hat{E}_x(y(k)) &= \frac{\sqrt{2}}{2}\left\{R_{y(k),pp} - R_{y(k),np}\right\}\end{aligned} \tag{17}$$

which are the same as the solutions for the conventional formulation given in Eqs. (10) and (11). This observation indicates that, as for normal error components, $\hat{E}_y(x(k))$ and $\hat{E}_x(y(k))$, the proposed formulation (14) and the conventional formulation (6) are essentially the same. Setup errors only affect the estimates of linear error components, $\hat{E}_x(x(k))$ and $\hat{E}_y(y(k))$, which validates the discussion in Section 3.4. In other words, when $\hat{E}_x(x(k))$ and $\hat{E}_y(y(k))$ are replaced with the measured values, the conventional formulation (6) can give a good estimate for $\hat{E}_y(x(k))$ and $\hat{E}_x(y(k))$.

It should be noted that this holds only in the two-dimensional case of step diagonal measurement. In the three-dimensional version of step diagonal measurement, the conventional formulation (the extension of Eq. (6) to the three-dimensional case) cannot give accurate estimates under the existence of setup errors, even when all the estimated linear errors are replaced with the measured values. The extension of the proposed formulation to the three-dimensional case will be presented in our near-future publication.

Remark 2: Since the proposed formulation requires the measurement of linear error components, $E_x(x(k))$ and $E_y(y(k))$ (k=1,$\cdots$,N), in addition to step diagonal measurements, it certainly requires longer measurement time, which may obscures an advantage of step diagonal measurements. However, when compared to the conventional measurement scheme where an artifact such as a straight edge or a squareness edge is used, we claim that the step diagonal measurement can still shorten the total measurement time. In the two-dimensional case, the conventional measurement requires two setups to measure linear positioning errors, two setups to measure straightness errors, and one setup to measure the squareness error. The proposed scheme requires two setups for step diagonal measurement, and two setups for linear measurements.

We claim that a significant practical value of laser step diagonal measurement is in that it can evaluate straightness and squareness errors by only using a linear laser interferometer. Compared to the conventional artifact-based measurement, the step diagonal measurement requires lower cost, applying a laser interferometer to the evaluation of all the errors, including linear, straightness, and squareness errors, without requiring a artifact of high geometric accuracy. It has also an advantage in the measurement of volumetric errors over a large measurement range, since it does not require a large artifact.

5 Experimental validation

The problems with the conventional formulation of laser step diagonal measurement, and the effectiveness of its proposed formulation, are experimentally validated by an application example to a three-axis vertical-type high-precision milling machine.

The machine has three orthogonal linear axes, which are all driven by a linear motor with a aerostatic guideway. Its positioning resolution is 10 nm in all the axes. The strokes of X and Y axes are both 100mm. For laser measurements, a laser doppler displacement meter, MCV-500 by Optodyne, Inc. is used. Laser beam directions are aligned by using a quad-detector, LD42 by Optodyne, Inc. The step diagonal measurements are done with the step size $a = 10$ mm, over the entire range of 60 mm $\times$ 60 mm (i.e. 6 blocks in X and Y directions). Figure 8 shows the experimental setup of laser step diagonal measurement.

First, volumetric errors are estimated by the conventional formulation using step diagonal measurements only. Figure 9 shows measured diagonal displacement profiles with respect to nominal diagonal distances in pp and np directions. In particular, the profile in the np direction has a sawteeth shaped variation of significant amplitude. This is mostly caused by the misalignment of the mirror and/or the laser direction as has been discussed in Section 3. Figure 10(a)(b) shows estimated linear positioning errors in X and Y directions with respect to each reference point, $\hat{\epsilon}_x(x(k))$ and $\hat{\epsilon}_y(y(k))$, computed by using the conventional formulation of step diagonal measurement described in Section 2.2. Note that estimated positioning errors, $\hat{\epsilon}_x(x(k))$ and $\hat{\epsilon}_y(y(k))$, are respectively given by the accumulation of $\hat{E}_x(x(k))$ and $\hat{E}_y(y(k))$, namely (see also Eq. (2)):

$$\begin{bmatrix} \hat{\epsilon}_x(x(k)) \\ \hat{\epsilon}_y(x(k)) \end{bmatrix} := \hat{P}_x(k) - \tilde{P}_x(k) = \sum_{i=1}^{k} \begin{bmatrix} \hat{E}_x(x(i)) \\ \hat{E}_y(x(i)) \end{bmatrix}$$
$$\begin{bmatrix} \hat{\epsilon}_x(y(k)) \\ \hat{\epsilon}_y(y(k)) \end{bmatrix} := \hat{P}_y(k) - \tilde{P}_y(k) = \sum_{i=1}^{k} \begin{bmatrix} \hat{E}_x(y(i)) \\ \hat{E}_y(y(i)) \end{bmatrix} \tag{18}$$

Their measured values, $\epsilon_x(x(k))$ and $\epsilon_y(y(k))$, obtained by using the same laser interferometer aligned directly toward X- and Y-directions, are also shown in Figure 10. For each of step diagonal measurement and linear measurement, the same measurement is repeated by three times. Figure 10 plots the mean of estimated and measured errors by the marks "∘", as well as their variation at each measurement point by horizontal parallel lines ("="). The mean positioning error measured by the laser interferometer is +1.14 μm over 60 mm in the X direction, and +0.69 μm over 60 mm in the Y direction. As is clearly seen from the figure, the conventional formulation of laser step diagonal measurement results in a large estimation error in both X and Y directions. The estimate of mean positioning error is -0.74 μm over 60 mm in the X direction, and +2.27 μm over 60 mm in the Y direction. As has been discussed in Section 3, these errors are mostly caused by the misalignment of laser directions and mirror directions.

Then, errors in the normal direction, $E_y(x)$ and $E_y(y)$, are estimated based on the proposed formulation of step diagonal measurements presented in Section 4, by using measured displacement profiles in pp-, np-, X-, and Y-directions. Estimated profiles of the accumulated positioning error in the normal direction, $\hat{\epsilon}_y(x(k))$ and $\hat{\epsilon}_x(y(k))$, are plotted in Fig. 11(a)(b). To validate their estimation accuracy, positioning errors in the normal direction, $\epsilon_y(x(k))$ and $\epsilon_x(y(k))$, are measured using a cross grid encoder (KGM), KGM182 by Heidenhain. Note that in all the cases, errors in the normal directions are defined such that the mean of normal errors with the motion toward the X direction, i.e. $\epsilon_y(x(k))$ (or $\hat{\epsilon}_y(x(k))$), becomes zero. See the remark below for the measurement accuracy of the KGM.

Table 2 summarizes measured and estimated straightness and squareness errors. Here, the straightness error is defined by the maximum variation of mean values of normal errors ($\epsilon_y(x(k))$ and $\epsilon_x(y(k))$) from their least-square mean line. The squareness error is defined by the gradient of the least-square mean line of $\epsilon_x(y(k))$ with respect to that of $\epsilon_y(x(k))$.

Figure 11 and Table 2 show a good match between measured and estimated volumetric errors obtained by step diagonal measurements. The maximum estimation error, i.e. the maximum difference between measured and estimated errors obtained based on the proposed formulation, is less than 0.1 μm for $\epsilon_x(y(k))$, and 0.21 μm for $\epsilon_y(x(k))$ ($k = 1,\cdots,6$). As is summarized in Table 2, on this machine, straightness errors of X and Y axes are both about 0.1 μm, according to the measurement by using the KGM. Compared to straightness errors, this machine has relatively larger squareness errors. The proposed formulation of laser step diagonal measurement succeeds to estimate the squareness error with an estimation error of less than 0.1 μm.

Remark: According to the maker's calibration chart, the measurement uncertainly of the laser doppler displacement meter used in this experiment is calibrated as ± 0.18 ppm. The experiment was conducted in a room with the temperature control for $\pm$ 1°C. The laser measurement was conducted under the compensation of air and material temperatures. However, since other environmental variation was not accurately evaluated, further analysis of the measurement uncertainty of laser displacement measurement is difficult. As is shown in Fig. 10, the variation in three measurements of linear positioning error was at maximum 0.17 μm over 60 mm.

According to the maker's calibration chart, the measurement uncertainty of the KGM used in this experiment is calibrated as: $U_{95\%} = 0.02\mu\mathrm{m} + 0.36 \cdot 10^{-6} \cdot L$, where L is the measurement interval length. Note that this is the uncertainty in the measurement of linear positioning errors. The measurement uncertainty of the positioning error in the direction normal to the feed direction is not calibrated in the maker's calibration charts, since the uncertainty calibration of the two-dimensional measurement is difficult due to the unavailability of the comparator. It is also to be noted that the above uncertainty is evaluated only along two lines (lines in X and Y directions passing the center of the grid plate); the uncertainty over the entire grid plate is not evaluated. In [14], the authors applied a self-calibration scheme to estimate the two-dimensional measurement accuracy of the same KGM. The estimated measurement error in the direction normal to the feed direction (along a line in the Y direction) was within $\pm 0.12\mu$m over 80mm.

6 Conclusion

The conventional formulation of the step diagonal measurement proposed by Wang [10] is valid only when the following implicit conditions are met: (1) laser beam directions are precisely aligned to nominal directions, and (2) the flat mirror is precisely aligned perpendicular to the laser beam direction. An inherent problem with the conventional formulation of the step diagonal measurement is that it is generally not possible to meet these conditions by the adjustment of the setup, when volumetric errors of the machine are unknown. This paper first presented the quantitative analysis of the effect of setup errors on estimated volumetric errors. It was shown that setup errors may impose a significant effect on the estimates of linear positioning errors ($E_x(x)$ and $E_y(y)$), while their effect on estimated positioning errors in the normal direction ($E_y(x)$ and $E_x(y)$) is sufficiently small. Therefore, in general cases where setup errors cannot be completely eliminated, it is not possible to accurately identify all the volumetric errors by using the conventional formulation of laser step diagonal measurement.

The new formulation proposed in this paper suggests that linear positioning errors must be independently measured, and then normal error components (namely, straightness and squareness error components) can be identified by using step diagonal measurements even under the existence of setup errors. As an application example, the proposed scheme was applied to estimate two-dimensional volumetric errors on a high-precision milling machine of the positioning resolution of 10 nm. Experimental results indicated that the squareness error of X and Y axes (1.21 μm over 60mm measured by the KGM) was estimated with an estimation error less than 0.1 μm.

The extension of the present formulation of laser step diagonal measurement to the three-dimensional case will be studied in our future research.

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References

[1] Wang, C. A Definition of Volumetric Error. *Manufacturing Engineering* 2005;134;1.

[2] Svoboda, O., Bach P., Liotto, G., Wang, C. Definitions and correlations of 3D volumetric positioning errors of CNC machining centers. *The IMTS 2004 Manufacturing Conference*, 2004.

[3] Umetsu, K., Furutnani, R., Osawa, S., Takatsuji, T., Kurosawa, T., Geometric calibration of a coordinate measuring machine using a laser tracking system, *Meas. Sci. Technol.*, 2005;16;2466-2472.

[4] Schwenke, H., Franke, M., Hannaford, J., Error mapping of CMMs and machine tools by a single tracking interferometer, *Annals CIRP*, 2005;54;1;475-478.

[5] Chen, J. S. Koua, T. W., Chiou, S. H. Geometric error calibration of multi-axis machines using an auto-alignment laser interferometer. *Precision Engineering*, 1999;23;4;243-252.

[6] Bringmann, B., Kung, A., Measuring artefact for true 3D machine testing and calibration, *Annals CIRP*, 225;54;1;471-474.

[7] American Society of Mechanical Engineers (ASME). B5.54:1992 Methods for performance evaluation of computer numerically controlled machining centers. *An American National Standard*, 1992.

[8] International Standards Organization (ISO). ISO 230-6:2002 Test code for machine tools, Part 6: Determination of positioning accuracy on body and face diagonals (diagonal displacement tests), *an International Standard*, 2002.

[9] Chapman, M. Limitations of laser diagonal measurements. *Precision Engineering*, 2003;27;401-406.

[10] Wang, C. Laser vector measurement technique for the determination and compensation of volumetric positioning errors. Part 1: Basic theory, *Review of Scientific Instruments*, 2000;71;10;3933-3937.

[11] Wang, C. Vector measurement for coordinate measuring machine. U.S. Patent No. 6519043, 2003.

[12] Soons, J. A. Analysis of the step-diagonal test. *Laser Metrology and Machine Performance VII*. 2005;126-137.

[13] Yang, J., Ren, Y., Wang, C., Liotto, G., Theoretical derivations of 4 body diagonal displacement errors in 4 machine configurations, *Laser Metrology and Machine Performance VII.* 2005;126-137.

[14] Ibaraki, S., Goto, W., Matsubara, A., Ochi, T., Hamamura, M., Self-calibration of cross grid encoder, *Journal of the Japan Society for Precision Engineering*, 2006;728;1032-1037. (in Japanese)

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Table 1
Comparison of given and estimated volumetric errors by the conventional formulation of step diagonal measurements.

	Case (a)		Case (b)		Case (c)	
	given	estimated	given	estimated	given	estimated
$E_x(x)$	0	-0.05	0.1	0.1	0	-0.0002
$E_y(x)$	0	0	0	0	0	0
$E_x(y)$	0	0	0	0	0.1	0.1000
$E_y(y)$	0.1	0.15	0.1	0.1	0	0.0002

unit: mm

Table 2

Measured and estimated straightness errors (in X and Y) and squareness errors.

	Measured by KGM	Estimated by step digonal measurements
Straightness error in X	0.05 μm	0.07 μm
Straightness error in Y	0.11 μm	0.24 μm
Squareness error in XY	-1.21 μm	-1.23 μm

$*$ All the errors are over the range of 60 mm.

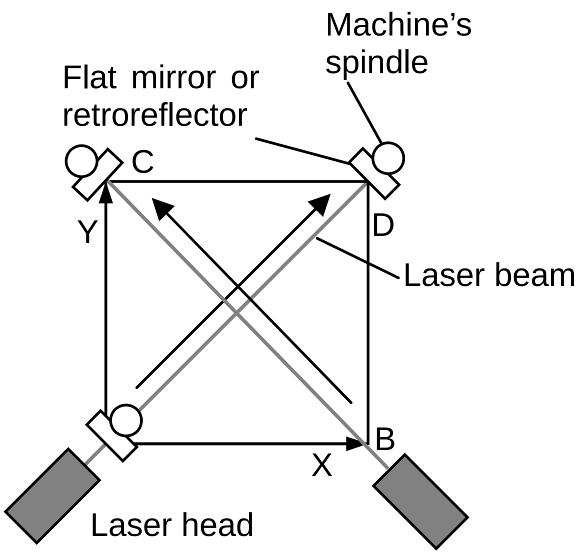


Fig. 1. The schematics of two-dimensional diagonal measurement.

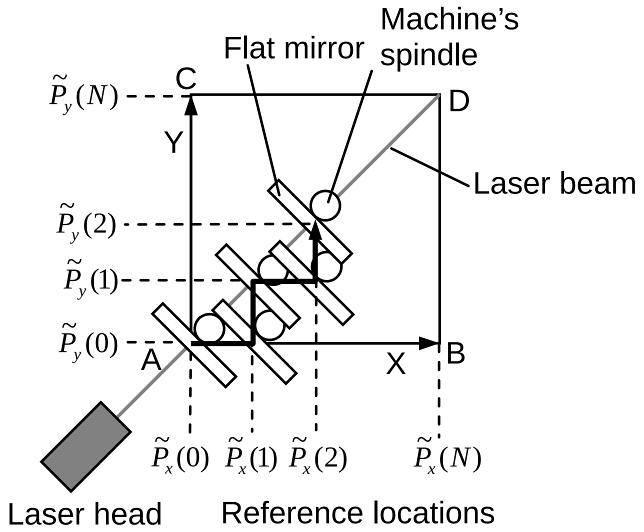


Fig. 2. The schematics of two-dimensional step-diagonal measurement (pp measurement).

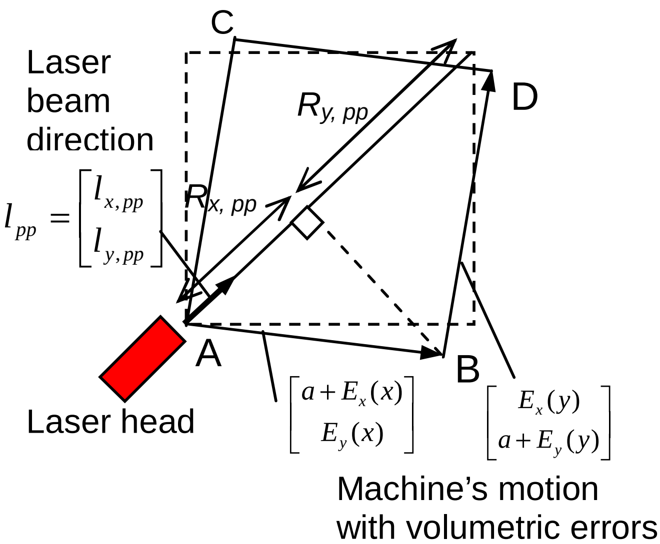


Fig. 3. Volumetric errors and diagonal displacements (single block case).

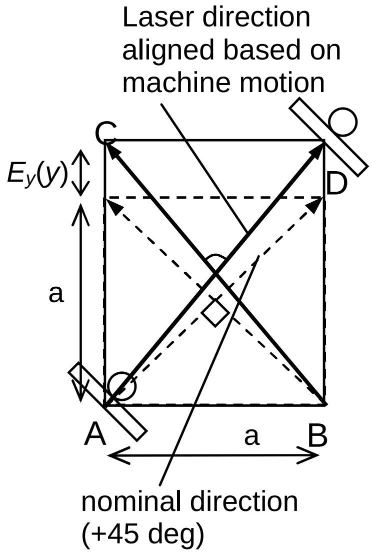


Fig. 4. Misalignment of laser beam direction caused by machine's volumetric error.

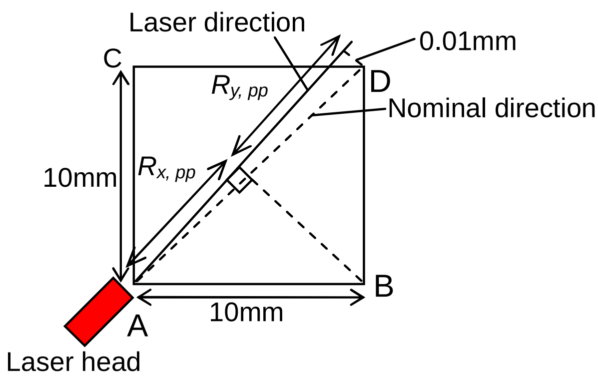


Fig. 5. Effect of misalignment of laser beam direction on measured diagonal distances.

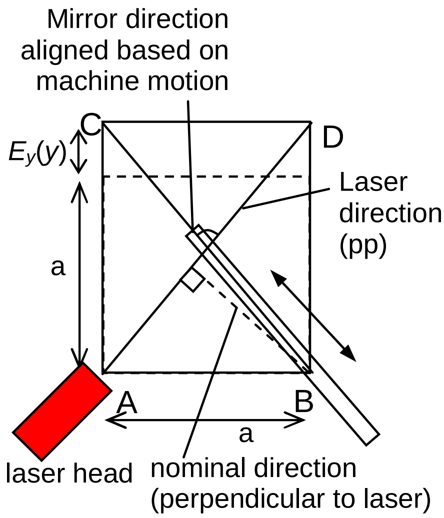
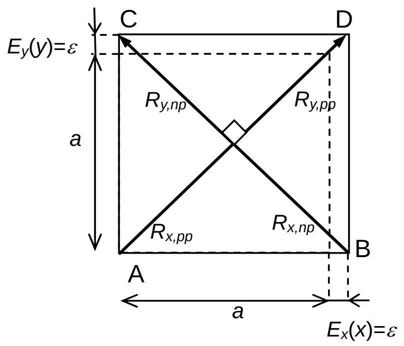
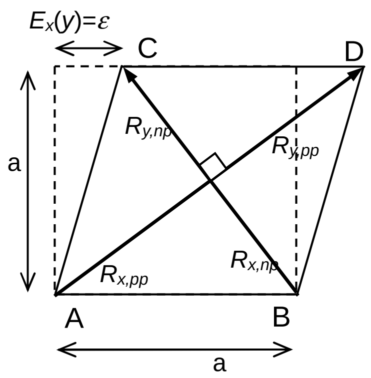


Fig. 6. Misalignment of mirror direction caused by machine's volumetric error.



(a) Example 1: $E_x(x) = E_y(y) = \epsilon$, $E_y(x) = E_x(y) = 0$



(b) Example 2: $E_x(y) = \epsilon$, $E_x(x) = E_y(y) = E_y(x) = 0$

Fig. 7. Example of cases where setup errors can be eliminated on the conventional formulation.

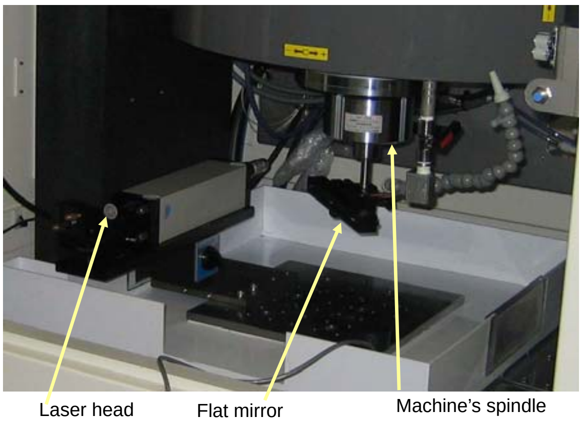


Fig. 8. Experimental setup.

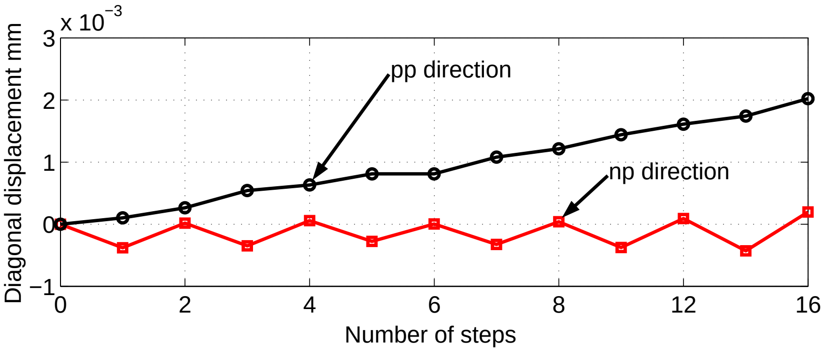
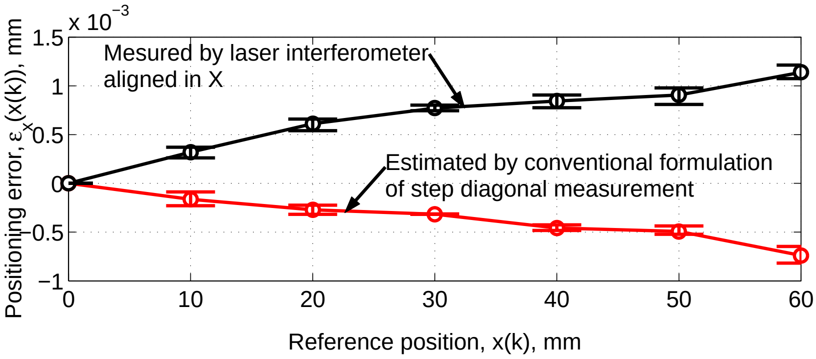
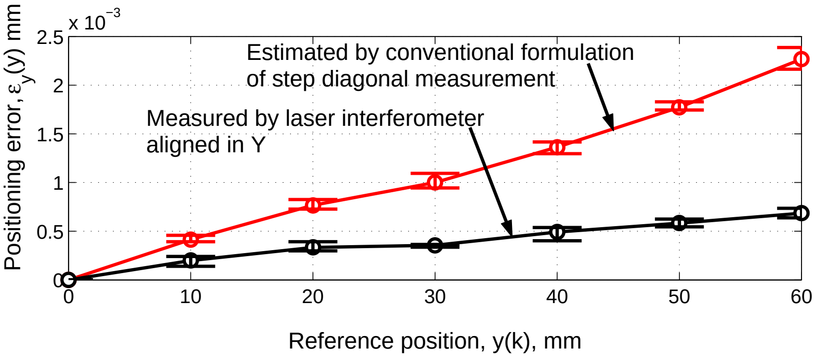


Fig. 9. Measured diagonal displacements in pp and np directions.

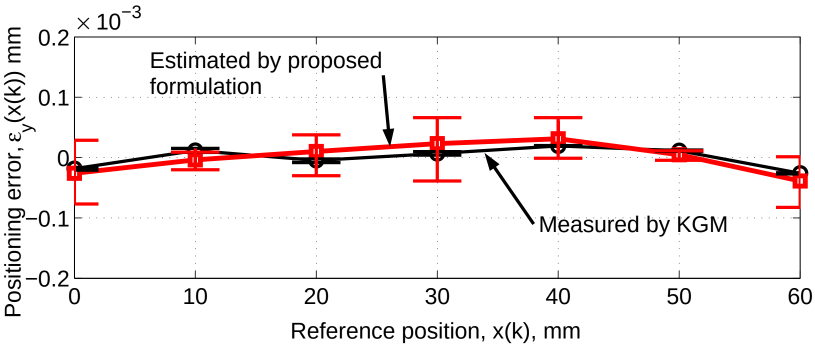


(a) Linear positioning error in X direction, $\epsilon_x(x(k))$

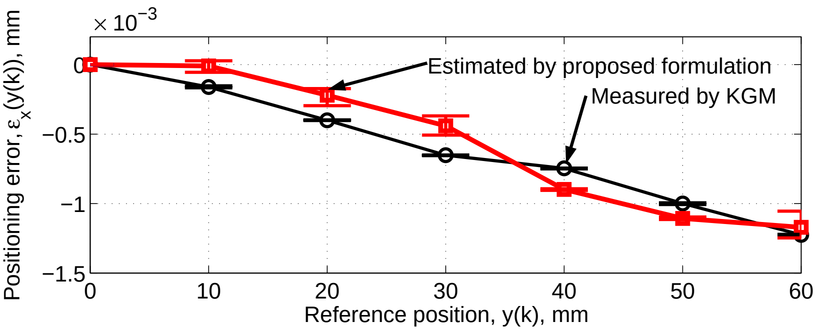


(b) Linear positioning error in Y direction, $\epsilon_y(y(k))$

Fig. 10. Comparison of linear positioning errors, $\epsilon_x(x(k))$ and $\epsilon_y(y(k))$, measured by a laser interferometer and estimated by the conventional formulation of laser step diagonal measurement.



(a) The positioning error in Y direction with the motion toward X direction, $\epsilon_y(x(k))$



(b) The positioning error in X direction with the motion toward Y direction, $\epsilon_x(y(k))$

Fig. 11. Comparison of positioning errors in the normal direction, $\epsilon_y(x(k))$ and $\epsilon_x(y(k))$, measured by the KGM and estimated by laser step diagonal measurements.