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# River Rating Complexity

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## Abstract

Accuracy of streamflow data depends on the veracity of the rating model used to derive a continuous time series of discharge from the surrogate variables that can readily be collected autonomously at a streamgage. Ratings are typically represented as a simple monotonic increasing function (simple rating), meaning the discharge is a function of stage alone, however this is never truly the case unless the flow is completely uniform at all stages and in transitions from one stage to the next. For example, at some streamflow-monitoring sites the discharge on the rising limb of the hydrograph is discernably larger than the discharge at the same stage on the falling limb of the hydrograph. This is the so-called "loop rating curve" (loop rating). In many cases, these loops are quite small and variation between rising- and falling-limb discharge measurements made at the same stage are well within the accuracy of the measurements. However, certain hydraulic conditions can produce a loop that is large enough to preclude use of a monotonic rating. A detailed data campaign for the Mississippi River at St. Louis, Missouri during a multi-peaked flood over a 56-day period in 2015 demonstrates the rating complexity at this location. The shifting-control method used to deal with complexity at this site matched all measurements within 8%.

## 1. Introduction

The relation between streamflow discharge (discharge) and water surface elevation (stage) is termed a rating curve (rating). Ratings are used for a variety of reasons in water resources investigations, but a predominant use of ratings is at streamgages, where autonomously-collected stage is converted to discharge by use of a rating. No widely accepted method for direct discrete continuous measurement of streamflow is available. In the absence of direct discrete continuous measurements of streamflow (discharge measurements), streamflow typically is determined through surrogate measures of one or more variables such as stage, water surface slope, rate of change in stage, or index velocity at a streamgage. The derivation of streamflow through these surrogate variables utilizes various models that will be termed a "rating". The rating is developed and calibrated using discharge measurements, collected onsite by field staff.

The simplest rating relates discharge to stage of the river (simple rating). From a pure hydrodynamics perspective (ignoring channel-bed mobility), all rivers and streams have some form of hysteresis (loop effect) in the relation between stage and discharge, even in prismatic channels, because of unsteady flow as a flood wave passes (Fig. 1).

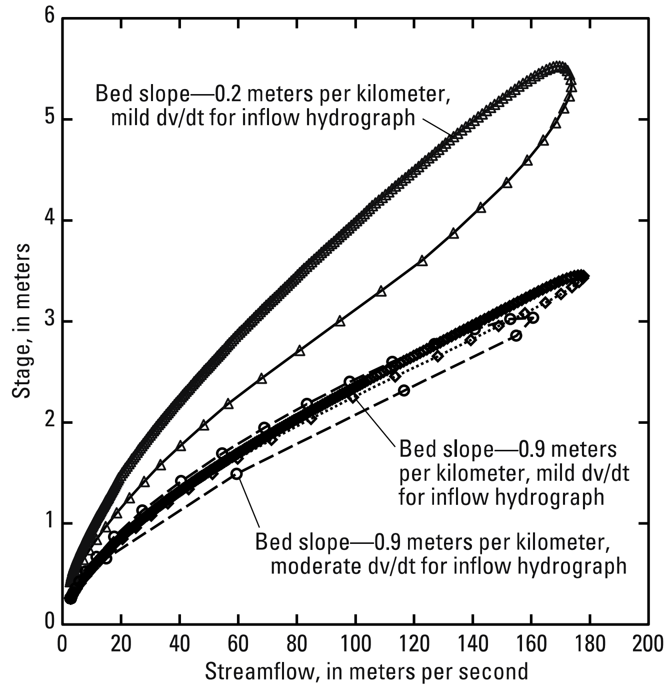


**Figure 1.** Theoretical determination of the relation between stage and discharge for a 30.5 m wide rectangular prismatic channel using a one-dimensional unsteady fully dynamic open-channel hydraulic model with varying bed slopes and rates of unsteadiness for the inflow hydrograph at the upstream end.

Many times the hysteresis is small enough that it is hidden within the error of the measurements and, thus, simple ratings work well.

Simple ratings do not work well for streamgages on low-gradient streams, streams with variable backwater, streams with large amounts of channel or overbank storage, streams with highly unsteady flow (rapid rises via flood wave movement), or streams with highly mobile beds. In these cases, a complex rating is required. A complex rating relates discharge to stage and other variables because of the lack of a unique, univariate relation between stage and discharge. Complex rating methods range from simply adding a second independent variable in the process of computing streamflow to computer models that solve conservation-of-momentum and conservation-of-mass partial differential equations.

The present paper examines the issue of rating complexity, methods of detection, and methods for dealing with complexity. The paper will present results to-date from an ongoing U.S. Geological Survey (USGS) study into complex rating detection and complex rating method application at streamgages in the United States, with results from one data campaign designed to look at rating complexity at the Mississippi River at St. Louis, Missouri (USGS Station 07010000). The paper will concentrate on complexity due to hydrodynamics, with only brief discussion on complexity due to bed mobility.

## 2. Study Motivation

When rating complexity is present at a particular site, USGS has dealt with the complexity in a variety of ways. Historically, the mean daily discharge was the main data product from a USGS streamgage, with errors from rating complexity lessened due to the averaging process going from sub-daily discharges (15- or 60-minute data) to mean daily discharges, particularly on smaller watersheds. However, in recent years, the growth of water resources science and modeling, coupled with the increased active management of water resources using real-time streamflow data has propelled the sub-daily streamflow data (typically 15 minute to 60 minute data) to the forefront as a high-demand product. USGS has been conducting analysis and studies to ensure that adequate processes are in place both to detect rating complexity and use proper methods that accurately account for this complexity in computation of the continuous streamflow data time series.

# 3. River Rating Theory

## 3.1 Hydrodynamic Effects

## 1.1 Hydrodynamic Effects

Streamgages are typically built in locations such that low flows have "section control" whereby a feature at a single point in the channel reach, such as a rock riffle or artificially constructed weir, forces the flow through critical depth, creating a unique relation between stage and discharge at a streamgage located immediately upstream. In essence, the downstream section feature "controls" the relation, thus the name. The rating for low flow is a simple monotonic relation between stage and discharge.

For medium to high streamflow, the influence of the section control is lost (submerged) and channel control is typically in effect. Channel control consists of all the physical features of the downstream channel reach (such as geometry, bed slope, expansion/contraction, sinuosity, and bed/bank roughness) that determine the stage of the river for a particular discharge (Rantz, 1982). Channel control can be conceptualized and formulated using a flow-resistance relation for open channels; flow resistance in an open channel can be described by the Manning equation (Yen, 1992):

$$V = \frac{k}{n} R^{2/3} S_f^{1/2} \qquad (1)$$

where the constant $k$ is 1.49 for English units and 1 for SI units, $n$ is the Manning resistance coefficient, $R$ is the hydraulic radius, and $S_f$ is the friction slope. The hydraulic radius is defined as

$$R = \frac{A}{P} \qquad (2)$$

where A is the cross-section area and $P$ is the wetted perimeter. It should be noted that Chezy's[1] equation is equally appropriate for describing the flow resistance (French, 1985), because the Manning equation has its roots in the Chezy equation (Yen, 1992).

[1] Chezy's equation $V = C\sqrt{RS_f}$ where C is Chezy's roughness coefficient and Sf is the friction slope

Because the streamflow is the product of the mean velocity and the cross-sectional area, A, we can express equation 1 in terms of streamflow as

$$Q = \frac{k}{n} A R^{2/3} S_f^{1/2} \tag{3}$$

For steady, uniform flow the friction slope (Sf) is equal to the bed slope (S0) (Chow, 1959) and equation 3 becomes

$$Q = \frac{k}{n} A R^{2/3} S_0^{1/2} \tag{4}$$

Equation 4 represents the assumption of steady, uniform flow. For a rectangular channel, where B is the width and Y is the depth, equation 4 becomes

$$Q = \frac{k}{n} B Y^{5/3} S_0^{1/2} \tag{5}$$

Equation 5 demonstrates (in a rectangular channel) for steady-uniform flow in a channel reach of unchanging geometry, roughness, and bed slope, a unique relation between stage and streamflow occurs.

Steady, uniform flow rarely exists in natural streams and rivers because of the dynamic and spatial variability of open-channel flow (unsteadiness, convective acceleration, and differential hydrostatic pressure forces). When flow is predominantly one dimensional, the friction slope can be expressed through a rearrangement of the one-dimensional unsteady equation of motion (Henderson, 1966) as

$$S_f = S_0 - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t} \tag{6}$$

where y is flow depth, x is distance in the longitudinal direction of flow, v is the velocity in the longitudinal direction, g is the gravitational constant, and t is time. Equation 6 readily shows the effects of differential hydrostatic pressure force ( $\partial y / \partial x$ ), convective acceleration ( $v/g \; \partial v / \partial x$ ), and unsteadiness ( $1/g \; \partial v / \partial t$ ) on the friction slope. Additionally, the conservation of mass in one dimensional flow with no side-channel inflow can be stated as (Henderson, 1966)

$$\frac{\partial Q}{\partial x} + T\frac{\partial y}{\partial t} = 0 \tag{7}$$

with all variables in equation 7 having been previously defined. The main aspects influencing hydrodynamic complexity: channel non-uniformity, flow unsteadiness, and hydrostatic pressure differences, are accounted for when equation 6 and 7 are combined in numerical models. Figure 1 shows the results of using an unsteady one-dimensional flow model that solves equations 6 and 7 for a flood hydrograph introduced into a 30.5-meter-wide rectangular channel. The rating shown in figure 1 is for a location 1.6 km downstream of the inflow hydrograph. The obvious hysteresis is the so-called “loop rating” (Kennedy, 1984), or “loop-rating curve” (Henderson, 1966).

Figure 2 explains the loop effects created by two different hydrologic phenomena, namely channel storage during the flood event (Fig. 2A) and the storage and subsequent return of water in the floodplain during transition into and out of overbank flows (Fig. 2B). Channel storage phenomena can often be captured by equations 6 and 7, as can the interplay between a stream and the floodplain for most instances (although some locations may require two-dimensional and even three-dimensional models to fully capture the interplay with the floodplain).

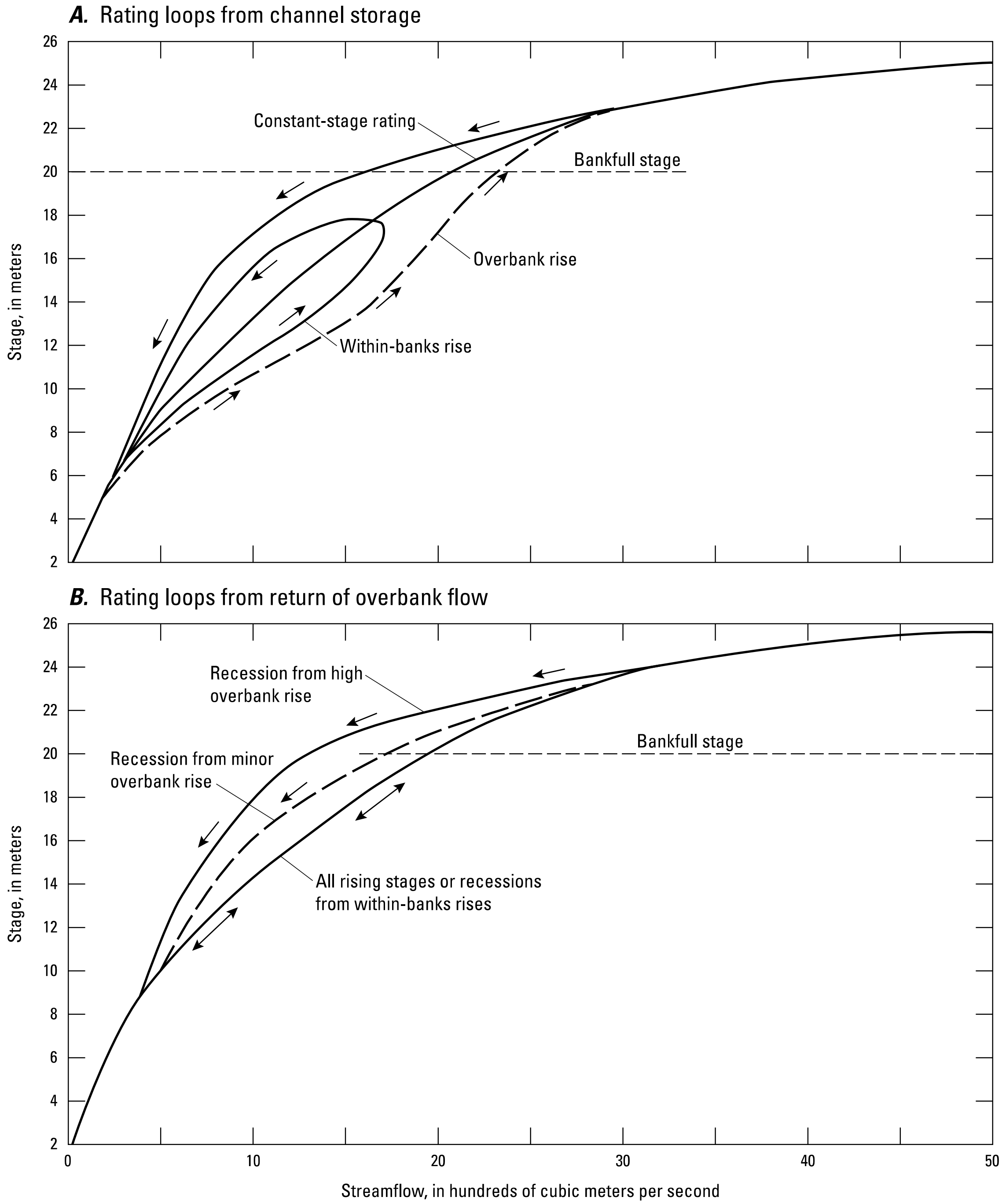


**Figure 2.** Typical loop ratings from single-storm events (adapted from Kennedy, 1984

Unsteady flows caused by rainfall events move as a “wave” down the river. In those cases where the streamflow can be accurately determined from a simple rating at all flows, the wave is said to move as a “kinematic wave” (Henderson, 1966) – Sf is approximated by S0 and conditions meet the steady-uniform flow assumption. If the wave does not fit the kinematic-wave model, or is subject to other factors (such as backwater) that preclude use of a simple rating, equations 6 and 7 form the basis for various complex rating methods.

### 3.2 Mobile Bed Effects

For alluvial channels with bedforms (sand-bed streams), the flow resistance often is partitioned into form resistance and grain resistance, with that partition represented in a variety of ways. One method is to partition it through the friction slope, Sf (Yen, 1992). As flow conditions or water temperatures change, the bedforms and thus the form resistance change (Simons and Richardson, 1966; Shen and others, 1978). One can see from equation 3, the influence the friction slope has on the discharge. Rating complexity due to mobile beds is not the focus of this paper.

## 4. Characteristics of Rivers with Complex Ratings

To examine the river characteristics that influence rating complexity from the hydrodynamics, substitution of components of the friction slope, equation 6, into the flow resistance relation, equation 3, results in

$$Q = \frac{k}{n} A R^{2/3} \sqrt{S_0 - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t}} \tag{8}$$

Neglecting the potential for changes in roughness, it is useful to look at the last three terms in equation 8 as they relate to the bed slope. If the last three terms are very small relative to the bed slope, then for all practical purposes, they can be neglected and the discharge can be computed as an approximate uniform flow. Thus, one can see that for lower gradient streams, the last three terms become more important than for higher gradient streams, with lower gradient streams often having more hydrodynamic complexity in the form of hysteresis. Figure 1 demonstrates the relative importance of channel gradient in the amount of hysteresis present in the 30.5 m wide rectangular channel; with the lower gradient channel have substantially more hysteresis than the higher gradient.

Figure 1 also demonstrates the importance of the last term, local acceleration due to unsteady flow, in equation 8. The original flood hydrograph is called “mild dv/dt” as a descriptor for the degree of unsteadiness as the flow increases at the inflow point. In the experiment, the amount of time from the zero discharge to the peak discharge of the inflow hydrograph was decreased (keeping the peak flow of the inflow hydrograph the same) of the original flood hydrograph. The slope of the rising section of the inflow hydrograph is increased and was designated “moderate dv/dt” in figure 1 for the high gradient (0.9 m/km) channel. The resultant increase in hysteresis is noted when compared to mild dv/dt for the high gradient channel.

## 5. Detecting Complexity

As part of the USGS efforts in continuous improvement of our data collection activities, automated scripts have been developed to mine the USGS National Water Information System database (http://waterdata.usgs.gov/nwis) of discrete discharge measurements to look for the potential of rating complexity. In addition to looking for rating complexity, these scripts also determine if sufficient data exists to assess the presence of rating complexity (for example, if sufficient measurements have been collected on both the rising and falling side of the hydrograph).

For each discharge measurement at a streamgage, two important quantities to compute are the change in gage height per unit time (dH/dt) and the percent difference of the measurement from the base rating (PDIFF)) computed as

$$PDIFF = \left( {(Q_m - Q_r)} \big/ {Q_r} \right) * 100 \qquad (9)$$

where $Q_m$ is the measured discharge and $Q_r$ is the rated discharge associated with the observed stage, $H_c$, during the discharge measurement.

Examination of dH/dt for all measurements at a streamgage allows assessment of any bias towards rising or falling limb hydrograph discharge measurements. Any bias allows USGS hydrographers to plan field work during floods to ensure measurements are made on both rising and falling limbs.

If a plot of dH/dt vs PDIFF is constructed for all measurements at a streamgage, each measurement will plot in one of the four quadrants centered on 0,0 of the graph as both variables will range from negative to positive. A site with perfect hysteresis in the rating, will have most all of the data plot in either Quadrant 1 (upper right) or Quadrant 3 (lower left) of a graph where dH/dt is on the horizontal axis (X) and PDIFF is on the vertical axis (Y), resulting in a positive slope to a best fit line through the data. USGS scripts perform linear regression of the dH/dt and PDIFF and assign a potential hysteresis severity value to each streamgage based on the correlation coefficient and slope of the regression line. Additionally, the scripts provide graphs of dH/dt versus PDIFF (figure 3) and the rating curve overlain with each discharge measurements delineated with symbols indicating if the measurement was made on the rising or falling limb, as discussed in section 7 of this paper.

## 6. Dealing with Hydrodynamic Complexity

A variety of complex rating methods exists to deal with situations where simple ratings will not work. The choice of method is dependent on the data available and the cause of the complexity. The choice often cannot be made prior to the collection and analysis of discharge measurements. More discharge measurements are required to characterize complex ratings than for simple ratings.

While fully-dynamic numerical models are sometimes used, many methods to deal with complexity were developed in the years prior to modern computational capabilities that now simplify the application of equations 6 and 7. For example, when variable backwater substantially impacts a site (effectively becoming a downstream control), two streamgages would be installed some distance apart and the water-surface slope would be used as an additional independent variable for the rating. Other rating methods treat the looped rating as a simple storage phenomena (Kennedy, 1984) or relate the predicted increase in water surface slope to the estimated celerity of the flood wave (Rantz, 1982). These methods, termed rate of change of stage methods, are still in use today (2016).

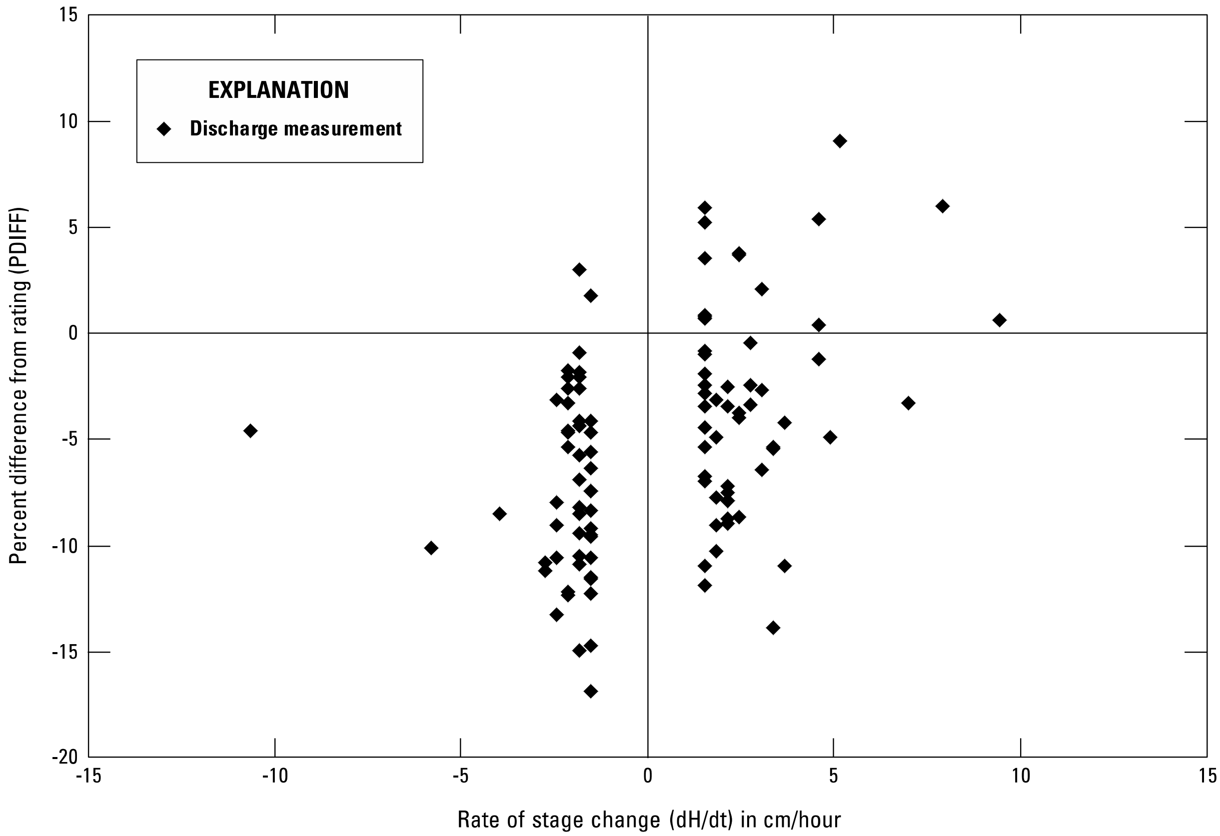


**Figure 3.** PDIFF versus dH/dt for measurements since 1990 on the Mississippi River at St. Louis, Missouri (07010000) where the hydrograph was either rising or falling stage was rising or falling during the measurement.

Even though we can mathematically characterize the physics of rating hydrodynamics, other factors present in natural systems can add to the complexity, creating the need for additional analysis and consideration. Mobile-bed channels, particularly sand channels, present challenges because of the dramatic changes in cross section geometry and concurrent changes in channel-bed roughness (form roughness) caused by the evolution of bedforms over the range of flows (Simons and Richardson, 1962). These unstable channels require special consideration in rating analysis (Kennedy, 1984).

## 6.1 Slope Rating Method

For sites with variable backwater, the friction slope (and water-surface slope) varies for a given stage. As evidenced from equation 3, varying friction slope means that multiple values of streamflow would be computed for the same stage. The causes of this variable backwater are most often due to variable stage at a downstream confluence or reservoir/lake (Rantz, 1982). Rating complexity due to variable backwater can often be addressed by use of a stage-fall-streamflow rating, or more commonly referred to as a slope rating. Slope ratings are determined empirically for discrete observations of (1) streamflow, (2) stage at a base streamgage (the streamgage at which the streamflow data are being computed, typically the upstream streamgage), and (3) the fall of the water surface between the base streamgage and an auxiliary streamgage.

To develop a slope rating, numerous discharge measurements are made with concurrent stage and fall values recorded for each measurement. The following equation (Rantz, 1982)

$$Q_m / Q_r = \left( \frac{f_m}{f_r} \right)^E \tag{10}$$

is the major component of the slope rating method, where Qm and Qr have been previously defined, fm (measured fall) is the actual fall associated with the measured streamflow at stage Hc (previously defined), fr (rated fall) is the expected fall under "normal" conditions at stage equal to Hc, and E has a value typically ranging from 0.4 to 0.6 based on the fit of the data. Equation 10 has its basis in equation 8, whereby the friction slope is approximated by the water surface slope, with the other portions of the friction slope ignored. From discharge measurements, a base stage-discharge rating, a stage-fall relation, and a factor curve effectively characterizing E are all calibrated. From these relations and equation 10, the discharge (Qm) is computed from the observations of stage and measured fall.

## 6.2 Rate of Change of Stage Method

Rate of change of stage methods were developed as simplified methods to deal with rating loops. A few of these methods include 1) storage method (Kennedy, 1984), 2) Jones method (Jones, 1915), 3) Lewis method (Corbett, 1943), 4) Wiggins method (Rantz, 1982), and 5) Boyer method (Rantz, 1982; Kennedy, 1984). These methods utilize the development of a simple rating, with an adjustment factor determined from a factor curve. Each method employs a different factor curve, depending on the primary loop-causing phenomena the method was developed to address. For example, the storage method was developed for those situations where storage in the channel is the main cause of the loop (Fig. 2A). The Boyer method is used to address situations where the loop effect cannot be addressed by simple storage alone (Kennedy, 1984). The Jones, Lewis, and Wiggins methods are similar to the Boyer, but not as widely used because of modifications present in the Boyer method that make it easier to use (Rantz, 1982).

The Boyer method is the predominant rate of change of stage method used within the USGS and is briefly shown here. The Boyer equation is

$$\frac{Q_m}{Q_r} = \sqrt{1 + \left( \frac{J}{US_c} \right)} \tag{11}$$

where $Q_m$ and $Q_r$ have been previously defined, $J$ is the rate of change of stage, $U$ is the celerity of the flood wave, and $S_c$ is the friction slope for steady-flow conditions measured at the same stage as $Q_m$. Jones (1915) has an explanation of how equation 11 was developed based on the water surface slope differences between the slope when the flow is at steady state and when the flood wave has arrived in the upstream end of the reach and just arriving (with no change in stage yet) on the downstream end of the reach.

Development of a Boyer method complex rating for a streamgage requires numerous discharge measurements, with concurrent stage and rate of change of stage values recorded for each measurement. A simple stage-discharge rating and stage versus 1/USc factor curve are developed through several iterations. Discharge is computed from the observed stage and J.

### 6.3 Index-Velocity Method

An increasingly common method to derive a continuous time series of streamflow, particularly at streamgages with rating complexity, is the index velocity method (Levesque and Oberg, 2012). Streamflow can be computed at a channel cross section as the product of the mean velocity and the cross-sectional area. The cross-sectional area can be readily determined through bathymetric and traditional land surveys. Using these data, a stage-area rating can be developed. Autonomous, continuous, direct determination of mean channel velocity is very difficult and typically not cost efficient. However, continuous measurement of velocity (index velocity) of a certain portion of a river can be made and used as an index to estimate the mean channel velocity. Discharge measurements and concurrent readings of the index velocity are made at a range of expected mean velocities. Once sufficient discharge measurements have been made, an index-velocity to mean-velocity rating (index-velocity rating) is developed. Discharge is computed using the stage-area rating and index-velocity rating.

### 6.4 Shifting-Control Method for Dealing with Rating Complexities

At certain streamgages, rather than utilize the complex rating methods mentioned earlier (including fully dynamic computational models), the shifting-control method is used to deal with rating complexity. This method involves making sufficient measurements during each flood event to shift the base rating (often multiple times during a flood) to mimic the complexity of the stage-discharge relation. Use of this method can be driven by various factors including inaccuracy of other complex rating method or strategic importance of the streamgage to other agencies warranting extra discharge measurements.

## 7. River Rating Case Study: Mississippi River at St. Louis, Missouri

Detailed data collection campaigns have been (and continue) to be conducted at selected streamgages whereby discrete flow measurements have been made over the entire rise and fall of the hydrograph to examine the complexity of the relation between stage and discharge. These campaigns range from hours to days in duration. During flooding in 2015, 39 discrete discharge measurements were made at the Mississippi River at St. Louis, Missouri (USGS Station 07010000) over a 56-day period (June 10 to July 29, 2015) (Fig. 4). A streamgage has been operated at this location since 1866, with daily discharges computed since 1928 and under the operation of the USGS since March, 1933.

Multiple peaks often occur on large river floods because of the timing of flow contributions from tributaries and the occurrence of additional rainfall during the long period of the flooding. Four separate peaks (independent peaks defined where the intervening trough is equal to or less than 75% of the adjacent larger peak) occurred during the flooding from June 10 to July 29, 2015, with a small trough leading up to the flood peak on June 30, 2015.

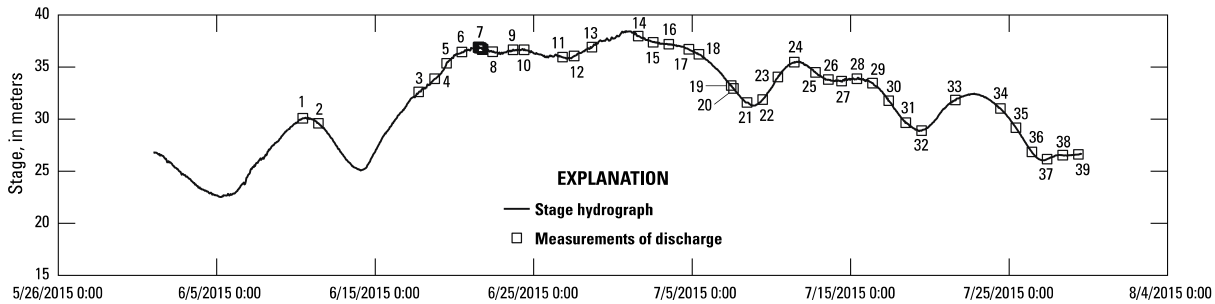


**Figure 4.** Mississippi River at St. Louis, Missouri (07010000) stage hydrograph and measurements of discharge.

Through time, changes in channel geometry and roughness (and other hydrodynamic factors) result in changes to the stage-discharge rating. USGS numbers streamgage ratings sequentially in time. Leading into the 2015 flooding, Rating 17 was in effect as shown on figure 5. Measurements that were made during either a rising portion or falling portion of the hydrograph are plotted on figure 5, with their number indicating the time sequence of measurement. The complexity inherent in this rating is readily evident, with the rising measurements plotting to the right of the falling measurements as would be expected for traditional hysteresis rating models for unsteady flows (Fig. 1).

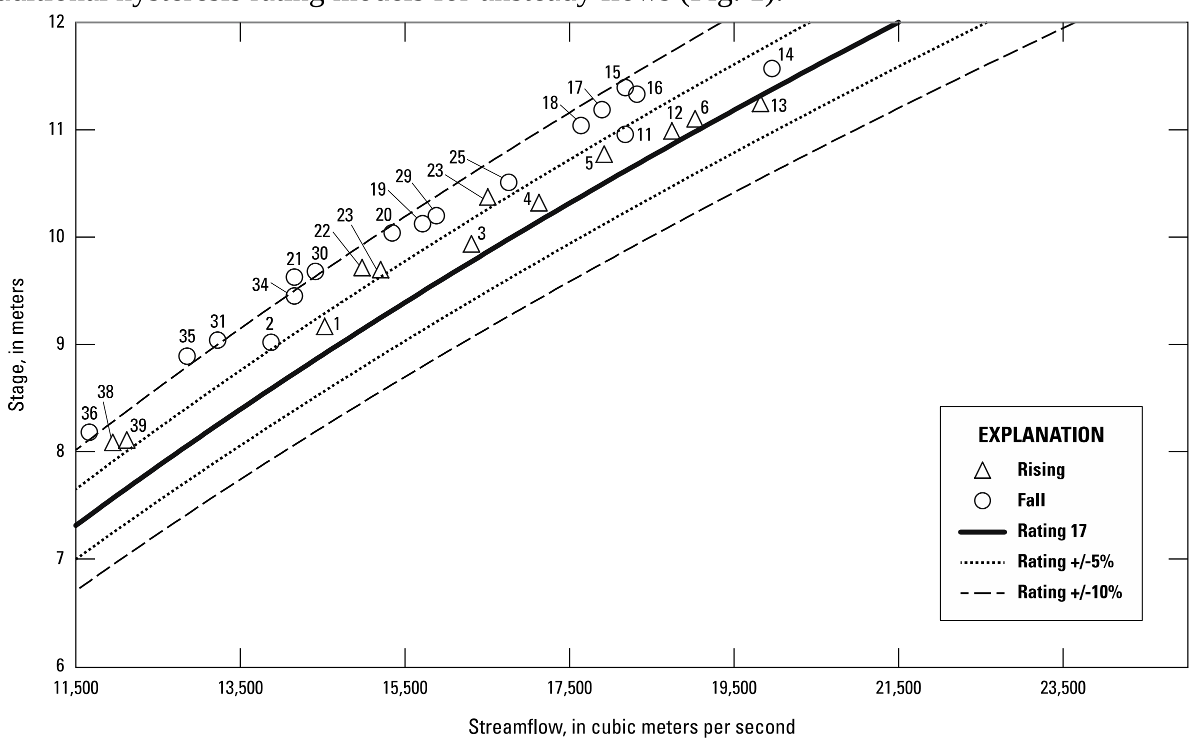


**Figure 5.** Base stage-discharge rating for the Mississippi River at St. Louis, Missouri (07010000) and discharge measurements during rising or falling hydrograph periods in June–July, 2015

USGS rates the quality of its records (continuous time series of discharge) as excellent (95% of the daily discharges are correct within 5%), good (within 10%), fair (within 15%), and poor (greater than 15%) (Kennedy, 1983). Individual discharge measurements have quality ratings of excellent (2%), good (5%), fair (8%) and poor (greater than 8%) (Turnipseed and Sauer, 2010). With this in mind, the ± 5% and ± 10 % bands for rating 17 are contained on figure 5 and nearly all measurements fit within the 10% error band of rating 17, although if rating 17 were used to compute continuous discharge a bias would be present as all measurements plot to the left of the rating.

For hydrographs with multiple peaks, multiple loops can be expected. Measurements made on either the rise or the fall from June 17 to July 8 are shown in figure 6. Measurements 3-6 are made on the rise and they form a nice outer loop. A dip in the hydrograph occurs between measurement 6 and measurement 12, but stage changes were so gradual during this period, that measurements 7 through 10 were characterized as trough measurements and thus not plotted as falling measurements. Measurement 11 is a falling measurement and plots very much to the left of measurement 6, which was made at nearly the same stage. When the rise begins again in earnest, measurements 12 and 13 plot once again back to the right, with 13 extending out more to the right than what would be envisioned from the original rising loop defined by measurements 3 through 6. Measurements 14 through 21 were all falling measurements, with the falling hydrograph continuously falling, but rates of fall varying throughout this 8-day period.

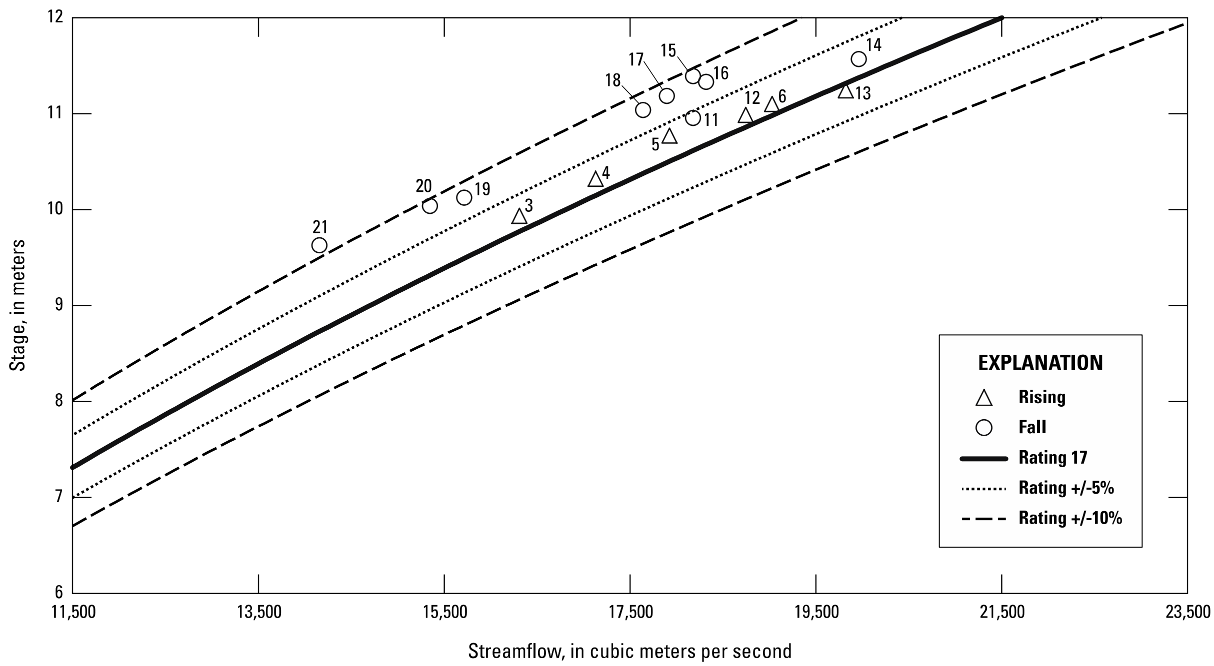


**Figure 6.** Base stage-discharge rating for the Mississippi River at St. Louis, Missouri (07010000) and discharge measurements on the rise and fall from June 17–July 8, 2015.

The USGS handles the complexity of the Mississippi River at St. Louis by the shifting control method. As part of this experiment, the USGS hydrographer tasked with computing the record for this site, used only measurements 2, 13, 14, 15, 21, 33, and 39 to do the computations. These measurements were deemed those that would have been made under normal operating procedures (absent this specific detailed data campaign). Figure 7 shows the shifted rating that was developed as part of working the record and it is noted that only 5 of the measurements plot outside the ± 5% error bands for the shifted rating, regardless of whether rise or falling measurements.  All measurements clearly plot within the ± 8% band.

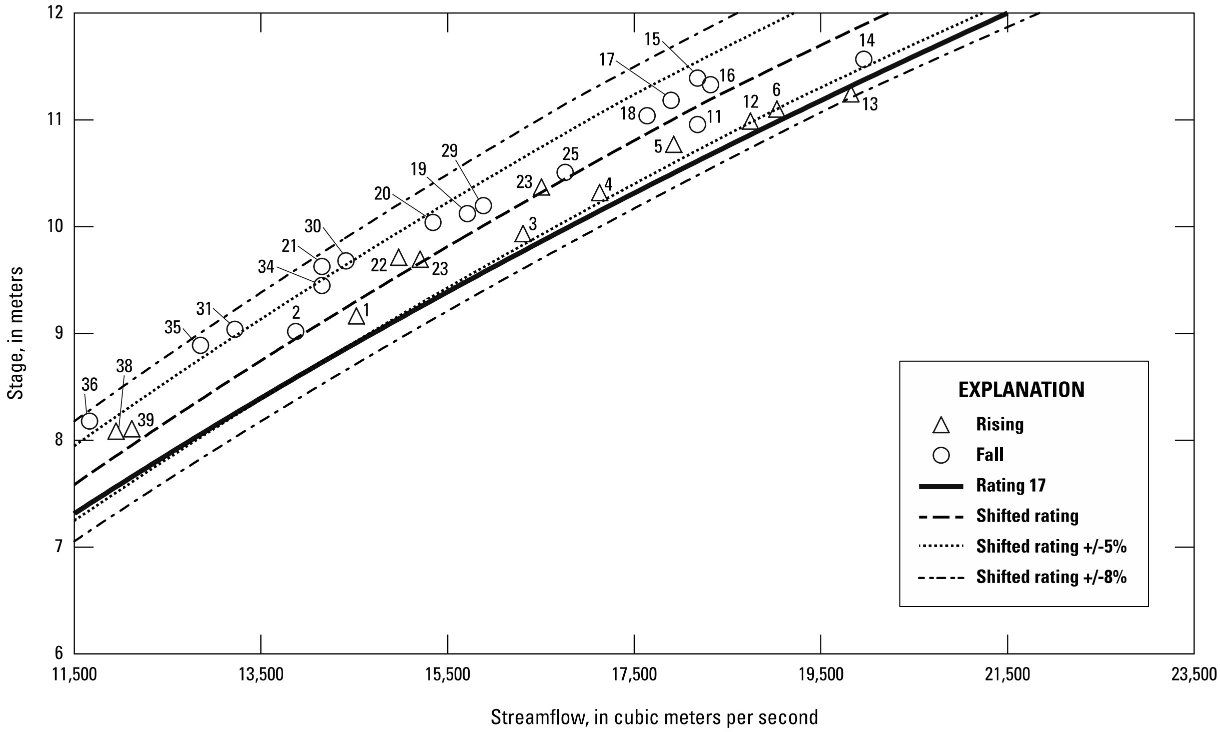


**Figure 7.** Base stage-discharge rating and shifted stage-discharge rating for the Mississippi River at St. Louis, Missouri (07010000) and discharge measurements during rising and falling hydrograph periods in June–July, 2015.

## 8. Conclusions

Rating complexity can induce large errors in sub-daily values of derived discharge if the complexity is not taken into account by one of the various complex rating methods.  Detailed campaigns to collect multiple measurements over a hydrograph range (both rising and falling) are good practice to examine for the presence of complexity.   In the absence of a detailed campaign, methods are under development to detect complexity when sufficient discharge measurements have been collected over time on both the rising and falling limb of the hydrograph.

The detailed campaign of 39 discharge measurements made over 56 days of flooding (including multiple flood peaks) documents the rating complexity for the Mississippi River at St. Louis.  The shifting control rating method used to compute discharge record during this period was within ± 8% for all measurements.

## References


Chow, V.T., 1959, Open-Channel Hydraulics, MaGraw-Hill, New York, 680 p

Corbett, D.M., 1943, Stream gaging procedure, U.S. Geological Survey Water-Supply Paper 888, 245 p

French, R.H., 1985, Open-channel hydraulics, McGraw-Hill, New York, 705 p.

Henderson, F.M., 1966, Open Channel Flow, McMillan Company, New York, 522 p
Jones, B.E., 1915, A method of correcting river discharge for changing stage, U.S. Geological Survey Water-Supply Paper 375, pp 117-130
Kennedy, E.J., 1983, Computation of continuous records of streamflow, U.S. Geological Survey Techniques of Water Resources Investigations Book 3, Chapter A13, 53 p.
Kennedy, E.J., 1984, Discharge ratings at gaging stations, U.S. Geological Survey Techniques of Water Resources Investigations Book 3, Chapter A10, 59 p.
Levesque, V.A. and Oberg, K.A., 2012, Computing discharge using the index velocity method, U.S. Geological Survey Techniques and Methods 3-A23, 148 p.
Rantz, S.E., 1982, Measurement and computation of streamflow, U.S. Geological Survey Water Supply Paper 2175, 631 p.
Shen, H.W., Mellema, W.J., and Harrison, A.S., 1978, Temperature and Missouri River stages near Omaha, Journal of the Hydraulics Division, Vol. 104, No. HY1, pp 1-20
Simons, D.B., and Richardson, E.V., 1966, Resistance to flow in alluvial channels, U.S. Geological Survey Professional Paper 422-J, 61 p.
Turnipseed, D.P. and Sauer, V.B., 2010, Discharge measurements at gaging stations, U.S. Geological Survey Techniques and Methods book 3, Chapter A8, 87 p., http:/pubs.usgs.gov/tm/tm3-A8/
Yen, B.C., 1991, Channel flow resistance: Centennial of Manning's formula, Water Resources Publications, Littleton, Colorado, 453 p.