Delegating Data Collection in Decentralized Machine Learning

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Abstract

Motivated by the emergence of decentralized machine learning (ML) ecosystems, we study the delegation of data collection. Taking the field of contract theory as our starting point, we design optimal and near-optimal contracts that deal with two fundamental information asymmetries that arise in decentralized ML: uncertainty in the assessment of model quality and uncertainty regarding the optimal performance of any model. We show that a principal can cope with such asymmetry via simple linear contracts that achieve $1 - 1/e$ fraction of the optimal utility. To address the lack of a priori knowledge regarding the optimal performance, we give a convex program that can adaptively and efficiently compute the optimal contract. We also analyze the optimal utility and linear contracts for the more complex setting of multiple interactions.

1 INTRODUCTION

We are seeing a flourishing new industry at the intersection of ML and operations which makes use of specialization and decentralization to achieve high performance and operational efficiency. Such an ML ecosystem creates a need for new tools for standard design choices such as how much and what kind of data to use for training, how much test data to use for verification, and how to tune hyper-parameters.

The consideration for these design choices is not how the designer should perform a task in this pipeline, but rather how she should delegate it to agents who are willing and capable of performing the task on her behalf. *How should the designer interact with this ecosystem? How should she evaluate and compensate other agents for their work? How does the outcome of the delegated pipeline compare with the outcome if the designer were to perform the task by herself?* In this work, we focus on the data collection aspect of the pipeline, initiate the study of its delegation through the lens of *contract theory*, and take a step towards answering these questions.

Contract theory provides a principal-agent perspective, where the principal—who is the designer interested in the outcome of the learning pipeline—can create a contractual arrangement—a menu of services and compensations—with an agent. At the heart of the issue is creating contracts that incentivize the agents, who may be more knowledgeable and skilled than the principal, to take the appropriate actions.

Consider a scenario where a firm delegates a predictive task to an ML service provider. In this context, the service provider may offer the firm either a dataset for learning or a pre-trained predictive model based on that dataset. To ensure aligned incentives, the firm needs to assess the dataset or predictive model and design the payment structure for the service provider accordingly. Since the accuracy of the model is crucial to the firm as it directly influences revenue, a natural evaluation approach involves directly measuring the accuracy of the model that the service provider produces for the firm. Several challenges arise during this evaluation process. Firstly, the firm generally only has limited data in the form of historical data or data acquired shortly after deploying the model for evaluation. So there is inherent noise in the evaluation. Secondly, the firm lacks knowledge about the baseline accuracy that is realistically achievable. This makes it harder for the firm to reward the service provider in a way that yields accuracy close to the optimal accuracy. These challenges are due to two sources of uncertainty and asymmetry that we study in this work:

- *Hidden actions* (aka Moral Hazard): Contracts must compensate the agent for his effort but this effort is not observable. But a proxy correlated with but not exactly determined by it is available to the principal.

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The principal thus is uncertain about the agent's effort while the agent knows it exactly resulting in a form of asymmetry we call hidden actions.

- *Hidden state* (aka Adverse Selection): There are parameters about the delegation problem instance the agent may know more about such as the cost to achieve an outcome. This could be because the cost depends on private information the agent possesses or because the agent deduces this through his computations.

As in many other delegation settings, the principal faces the hidden state and hidden action challenges when delegating learning. While the principal desires to construct a contract based on the true accuracy of the learned model, they can only obtain a noisy estimate of this value using test data. Our focus is on scenarios where the size of the test dataset is not excessively large. If the test dataset is too large, it becomes more beneficial for the principal to learn a model using their own test data rather than delegating the data collection process.

Even when the estimate of the learned model's accuracy has negligible noise, the principal still faces the hidden state challenge, i.e., the principal does not know how to value the accuracy since she does not know the optimal error achievable. Assigning a low payment for the model's accuracy when the optimal error is high would result in negative agent utility, discouraging agent participation. Conversely, assigning a high payment when the optimal error is low might incentivize the agent to collect a smaller dataset than is optimal for the principal.

1.1 Our Results

We consider performance-based contracts where the agent is compensated as a function of the estimated accuracy of the learned model. The principal's utility is the true accuracy of the learned classifier minus the monetary transfer she makes to the agent. We model the agent in two delegation settings. In each we contrast the principal's utility through contracting with and without information asymmetry. Borrowing terminology from the contract theory literature, we refer to the hypothetical scenario without information asymmetry the *first-best scenario* and the resulting optimal utility the *first-best utility*, which we use as a benchmark.

Single-round of interaction. We address both types of information asymmetry: hidden state and hidden action, creating contracts specific to each while also evaluating their efficacy when both asymmetries coexist. For hidden actions, our linear contract based on a single test point (Proposition 2) ensures at least $1 - 1/e$ fraction of the first-best utility. This guarantee continues to hold even with hidden state if agent's sampling cost is low (Theorem 1).

For the hidden state challenge with n possible states, we derive an optimal contract by solving a convex optimization (refer to Opt) with $O(n^2)$ constraints. Section B.3 describes how this contract's optimality improves as the principal's test set size increases.

Multiple rounds of interaction. In Section 4, we analyze a multi-round delegation setting where the agent is uncertain about the delegated task and uses feedback over rounds to learn the principal's requirements and collect relevant samples. For a notion of principal's regret, we provide a tight $\Theta(T^{3/4})$ regret bound through repeated linear contracts over T rounds. This shows that linear contracts are also powerful approximations of optimal utility in multi-round settings. In comparison, we obtain a strictly better regret of $O(T^{2/3})$ for multi-round first best contracts.

1.2 Related Work

There is a rich literature on contract theory in economics (see, e.g., Laffont and Martimort, 2009; Bolton and Dewatripont, 2004). More recently, there has been work on algorithmic and statistical aspects of contract theory (Carroll, 2015; Dütting et al., 2019; Dütting et al., 2020; Bates et al., 2022; Alon et al., 2022) which include results on approximation by simple contracts. These results hold for either finite actions or outcomes, and thus are not directly applicable to our setting, which involves infinite actions and a continuous space of outcomes. Working in such spaces requires utilizing the structure of our problem, and specifically exploiting fundamental results on statistical minimax rates.

The *pricing* of data has been considered for various purposes and considerations (Bergemann and Bonatti, 2019; Acemoglu et al., 2022; Cai et al., 2015; Ho et al., 2016) including in learning problems (Agarwal et al., 2019; Chen et al., 2022). The latter study the pricing of previously collected data to incentivize the seller and buyer to be forthright about the valuation and quality of their data, respectively. We are interested instead in pricing for the purpose of incentivizing the data collecting agent to exert effort to collect data. Some of these papers also consider incentivizing high-quality data labelling by relying on multiple labellers who can be compared. We study delegation of learning in the setting of a single agent.

An adversarial perspective on the delegation problem (Chiesa and Gur, 2018; Goldwasser et al., 2021) has been considered for machine learning from the lens

of interactive proofs. In this line of work, the principal wants to fully verify the effort of an agent who may be an adversary that is interested in getting his effort verified. While they deal with similar challenges, such as not knowing the optimal achievable error, they do not consider incentivizing the agent (via contracts and compensations) to improve the outcome.

Concurrent work by Saig et al. (2023) studies a similar setting of incentivizing data collection for classification. They characterize the optimal contract for a given test set size, under the hidden action challenge, as a threshold contract when the agent has two choices for actions. We propose a simple contract and show its near-optimality for an arbitrary number of actions. We also show that this simple contract is robust to the hidden state challenge to an extent.

2 MODEL

We have a task distribution D representing the joint distribution over the domain and label set. The principal aims to learn a classifier h that achieves high accuracy on D, denoted by $1 - L_D(h)$. To accomplish this, the principal delegates the task to an agent who selects the number of samples to collect and trains a classifier. We prioritize the collection of samples as the primary effort, considering it more significant than classifier training. The principal's primary objective is to incentivize high-quality data collection, leading to the development of an accurate classifier. To evaluate the performance of the model obtained through delegation, the principal possesses an independent test set consisting of independently and identically distributed (i.i.d.) samples from D. The principal utilizes this test set to evaluate the learned classifier's accuracy.

The delegation process begins with the principal publishing a contract which is a mapping from test accuracy to payment for the agent. Seeing the contract, the agent collects data and provides a classifier to the principal. The principal then executes the contract by evaluating the classifier on her own test set. The principal pays the agent the amount specified by the contract for the measured test accuracy. We assume that the principal can commit to a test set in advance and that this test set is not accessible to the agent until the contract is executed after the agent's data collection.

Principal and Agent Utilities. Upon receiving a classifier with true accuracy a on the task distribution D and paying the agent t, the principal gets utility $a - \beta t$ for some *accuracy-payment* scaling constant $\beta > 0$.[1] The agent exerts effort α per sample it collects. So the utility for the agent receiving payment t by collecting n samples is $t - \alpha n$.

First-best Contracts. To study the effects of information asymmetry, we create a benchmark without any information asymmetry between the principal and the agent i.e., no hidden state or hidden action. We call this idealized scenario, the *first-best scenario*. We call the utility of the optimal contract in this scenario, the *first-best utility*. We will say the effects of information asymmetry are limited if the utility achievable through incentive-compatible and individually rational contracts are close to the first-best utility.

2.1 Single Round of Delegation

Most of our analysis focuses on a single round of delegation. The agent collects a batch of data once and one model is learned. The payment is provided commensurate to the test accuracy of this single model. We consider (risk neutral) utility-maximizing agents, who collect a number of samples that maximizes their expected utility. We further make assumptions on how the agent's effort translates to the observed outcome as described below.

Assumption 1 (Outcome as a function of the agent's action.)**.** *We assume that when the agent collects n samples,*[2] *the classifier's observed accuracy on the principal's test set drawn from D is drawn from a distribution with mean $1 - \theta - \frac{d}{n^p}$, where θ denotes the optimal error achievable on the task distribution D.*

The constant d depends on the complexity of the training algorithm and the constant p describes the rate of decay of the excess error. These rates are motivated in part by minimax statistical rates and scaling laws of classification and regression tasks.

Remark 1 (VC dimension bound)**.** *An algorithm that PAC-learns a function class $\mathcal{H}$ with VC dimension d using n i.i.d. samples drawn from $\mathcal{D}$ and returns a classifier h satisfying $L_{\mathcal{D}}(h) \leq \theta(\mathcal{D}, \mathcal{H}) + C\sqrt{d/n}$, where $\theta(\mathcal{D}, \mathcal{H}) = \min_{h \in \mathcal{H}} L_{\mathcal{D}}(h)$. This is minimax-optimal as there is a distribution $\mathcal{D}$ such that $L_{\mathcal{D}}(h) \geq \theta(\mathcal{D}, \mathcal{H}) + C\sqrt{d/n}$.*

Remark 2 (Linear regression model)**.** *In a d-dimensional linear model with covariates $x_i \sim \mathcal{N}(0, \Sigma)$ and outcomes $y_i = \beta^t x_i + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$ for $i \in [n]$, the Ordinary Least Squares (OLS) estimator $\hat{\beta}$ satisfies the property $\mathbb{E}[(x^t\hat{\beta} - y_i)^2] = \sigma^2\,(1 + O(d/n))$.*

Even though minimax rates are typically upper bounds, we treat them as exact rates in the main body

[1] We note that $1/\beta$ can be thought of as the benefit the principal gained per unit of accuracy.

[2] We will consider agent's action as continuous and the true sample size is a rounding of the action.

and defer the discussion on the implications of treating them as upper bounds to Appendix B.4.

Using this assumption on outcome as a function of the agent's action, we can compute the first-best contract as below.

Proposition 1 (First-best contract)**.** *For any set of problem parameters with optimal error $\theta \in [0,1)$, learning parameters d and p, agent's cost-per-sample $\alpha > 0$, and principal's accuracy-payment scaling parameter $\beta > 0$, the first-best contract offers payment αn^* when the test accuracy is at least $1 - \theta - d/n^{*p}$, where $n^* = (pd/\alpha\beta)^{1/(p+1)}$.*

One way to interpret the first-best contract is that it asks the agent to collect n^* samples and compensates the agent exactly for n^* samples. Without hidden state or hidden action, the first-best contract yields zero utility to the agent and the first-best utility to the principal.

While first-best utility is used as a benchmark, the first-best contract itself may not be optimal due to existing randomness in test accuracy (hidden action). Additionally, each optimal error value θ leads to a different first-best contract, which is not implementable when the principal doesn't know the θ parameter exactly (hidden state).

Linear Contracts. As opposed to first-best contracts that can be quite complex, *linear contracts* are simple contracts that compensate an agent by a linear function of the test accuracy. That is, a c-linear contract for parameter $c \in \mathbb{R}^+$ assigns payment $T_c(a_{\text{test}}) = c \times a$ when the test accuracy is a. Linear contracts must have non-negative parameter c, since the principal cannot make negative payments to the agent.

3 OPTIMALITY OF LINEAR CONTRACTS

In this section, we aim to find near-optimal contracts in the realistic scenario with hidden state and hidden actions, recognizing that the first-best contract may not be optimal. Our first result is that a linear contract compensating the agent based on the test (and not true) accuracy is approximately optimal across all possible contracts for the principal. Moreover, the slope of the linear contract has an explicit value that is the same across a wide range of θ making it possible to deal with both hidden state and hidden action challenges. Our results in fact show a stronger comparison, that linear contracts approximate not just the optimal utility but also the first-best utility. This is quite a strong guarantee as there is often no contract that can achieve the first-best utility in presence of the hidden action challenge.

A crucial advantage of our linear contract is that it works with any unbiased estimator of the accuracy of the learned model. Therefore, even a test set of size one suffices to enact this contract. We state our main results in this section and defer their formal proofs to Appendix A.

Before we state this result in its full generality, we start with the following proposition which deals only with the hidden action challenge while assuming that optimal error θ is known to the principal (as well as the learning parameters p and d).

Our main theorem in this section, stated in Theorem 1, then follows from this proposition and shows that the linear contract in this proposition not only handles the hidden action challenge but also can be implemented without knowing θ, for a large range of parameters.

Proposition 2 (Linear contracts are approximately optimal when optimal error is known)**.** *For any set of problem parameters $\theta \in [0,1), d, p, \alpha, \beta > 0$, a principal who knows these parameters can construct a* linear contract *whose expected principal utility is at least $1 - 1/e$ fraction of the first-best utility. Furthermore this contract only requires a single test sample.*

In more detail, the c^*-linear contract that achieves this claim uses

$$c^* = \max\left(\frac{1}{\beta(p+1)^{\frac{p+1}{p}}}, \frac{\alpha d^{\frac{1}{p}}}{p} \cdot \left(\frac{p+1}{1-\theta}\right)^{\frac{p+1}{p}}\right)$$

and approximates the first-best utility to a factor of

$$1 - \frac{1}{(p+1)^{\frac{p+1}{p}}} \geq 1 - \frac{1}{e}.$$

Linear contracts are known to approximate optimal contracts in very limited settings that do not apply to our problem setting. For example, Alon et al. (2022); Dütting et al. (2019) gave constant approximation guarantees for linear contracts when the agent's action set is finite or the ratio of the maximum and minimum reward for the principal is bounded by H. In the former case, an approximation ratio of $1/2$ is obtained and in the latter the ratio is $1/2\log(H)$. Neither of these conditions hold in our setting, as the number of agent's action corresponds to the number of samples collected and is unbounded and the reward can take any value in $(0,1)$. Instead, we use the structure of first-best contract (Proposition 1), the linearity of contracts, and the structure of the utility functions to obtain this $1 - 1/e$ approximation guarantees.

Proof sketch of Proposition 2. The full details are deferred to the appendices; here we provide some intuition and a proof sketch. Underlying the proof is the

linearity of expectation and the fact that the agent is expectation-maximizing. Under a linear contract c, the expectation-maximizing agent aims to maximize $\mathbb{E}[c \cdot a(n) - \alpha n] = c \cdot \mathbb{E}[a(n)] - \alpha n$, where $a(n)$ is the test-set accuracy of a model trained on n samples drawn from an unknown distribution with mean $1-\theta-d/n^p$. The only distribution-dependent quantity in this maximizing objective is the expected accuracy $\mathbb{E}[a(n)] = 1-\theta-d/n^p$. So the agent's action and hence the principal's contract design only depends on the expectation of the test accuracy and not on the exact distribution of the test accuracy. Next we sketch a proof for the approximation result and use the structured way the expected accuracy depends on the number of samples drawn.

Note that c^* is the maximum of two terms. Let us denote these terms by c_1, c_2. Given a c-linear contract, the agent's best response is to choose n so as to maximize $u(n; c) = c\,(1 - \theta - d/n^p) - \alpha n$. The maximizing value is $n(c) = (cdp/\alpha)^{\frac{1}{p+1}}$. By setting c large enough, we have $u(n(c), c) \geq 0$ where c_2 is the threshold above which this holds. So the value of c_2 is set to ensure the agent gets non-negative utility from participating.

When $c_1 \geq c_2$, c_1 satisfies the participation constraint. By computing the principal's utility from the c_1-linear contract using the expression for the agent's best response, we see that it is $1-\beta c_1$ times the first-best utility. Moreover, we have $1 - \beta c_1 = 1 - 1/(p+1)^{\frac{p+1}{p}}$. It turns out the same upper bound holds for the approximation ratio of the c_2-linear contract to the first-best utility when $c_2 \geq c_1$. This upper bound is decreasing in p and the limit as $p \to 0$ is $1 - 1/e$. □

Importantly, by inspecting the contract in Proposition 2, we see that in many cases it does not depend on problem-specific parameters like the optimum error. This makes c^* deployable in practice.

The optimal-error-parameter-agnostic linear contract is appropriate when the cost per sample collection is small enough and when the optimal error is low enough. As a result, when α is small, we can relax the assumption that the principal knows the exact optimum error θ to that the principal knows that θ lies in a certain range. Moreover, even under this relaxation, linear contracts are still approximately optimal. This is stated as the following theorem.

Theorem 1. *For any $d, p, \beta > 0$, consider the $\bar{c}$-linear contract for $\bar{c} = \frac{1}{\beta(p+1)^{\frac{p+1}{p}}}$. Let $\overline{\theta} \in [0, 1)$ be any parameter for which $0 < \alpha \leq \frac{p}{\beta d^{1/p}} \left(\frac{1-\overline{\theta}}{(p+1)^2}\right)^{\frac{p+1}{p}}$. Then, $\bar{c}$-linear contract obtains a principal utility that is at least $(1 - 1/e)$ times the first-best utility, for any unknown optimal error parameter $\theta \in [0, \overline{\theta})$.*

Note that $\bar{c}$ is constructed based on p (error decay rate) and β (how the principal values accuracy relative to payment). The principal knows these quantities. In contrast, the optimal contract requires additional knowledge, such as θ (optimum error) and α (agent's cost per sample). The theorem demonstrates a simple contract that requires less knowledge but remains approximately optimal in utility.

4 MULTI-ROUND DELEGATION

So far, we analyzed delegated learning that occurs through a single round of interaction between the principal and the agent. However, delegation often occurs over multiple rounds to allow the agent to learn more about the principal's requirements. Here we model such a scenario and analyze what happens when the principal uses a linear contract c in each round. We introduce a notion of regret and show that repeated linear contracting over T rounds results in $\Theta(T^{3/4})$ regret for the principal which is worse than the $O(T^{2/3})$ regret achievable without information asymmetry i.e., the *first best regret*. We provide proof sketches of our results in the main body and provide the full proofs in the appendices.

The Model. To model uncertainty about the principal's requirements, we assume that the target distribution D^* belongs to a class $\mathcal{D} = \{D_1, \ldots, D_k\}$. The agent knows the class $\mathcal{D}$ but does not know D^* apriori. The principal deploys classifiers for T rounds and contracts learning for $N \leq T$ of these T rounds.

Contracting Protocol. The principal decides on $N \leq T$ rounds to contract. For each round $i = 1, \ldots, N$:

1. The principal announces payment ρ_i which is a randomized mapping from a classifier to a positive, real-valued payment to the agent.
2. The agent chooses number of i.i.d samples to collect from each distribution. Denote this by $\mathbf{n_i} = (n_i^1, \ldots, n_i^k)$ where n_i^j is the number of i.i.d samples the agent draws from distribution D_j in round i. This is not observed by the principal
3. The agent provides classifier h_i to the principal.
4. The principal pays $\rho_i(h_i)$ according to the announced payment rule ρ_i to the agent.

Utilities. Over these N rounds, the agent's utility is $\sum_{i=1}^{N} \left(\rho_i(h_i) - \alpha \sum_{j=1}^{k} n_i^k\right)$ and the principal's utility is $\sum_{i=1}^{N} (1 - L_{D^*}(h_i) - \beta\rho_i(h_i))$.

Test-Accuracy Based Payments. Our results deal with contracts based on test accuracy. In round $i \in [N]$ where the agent provides classifier h_i, the payment

is a function of the test accuracy $1-L_{D^*}(h_i)+\eta_i$, where η_i is a mean-zero, random variable resulting in the principal's randomness of testing. The c-linear contract offers payment c times this test accuracy in each round.

Non-Contracting Rounds. In the remaining, non-contracting rounds ($j = N+1, \ldots, T$), the principal chooses a classifier h_j, possibly based on $(h_i)_{i=1}^N$ to deploy. The agent gets zero utility in these rounds. The principal gets utility $1 - L_{D^*}(h_j)$.

In these repeated interactions, the payments serve as feedback to the agent to learn the principal's requirements. The principal hopes to get an increasing quality of classifiers to deploy

Definition 1 ($\mathcal{H}$-regret)**.** *Let $\mathcal{H}$ be any class of classifiers. Let $((\mathbf{n}_t, h_t, \rho_t(h_t))_{t=1}^T$ be the sequence of actions by the principal and agent. The principal's $\mathcal{H}$-regret ($R_T^P(\mathcal{H})$) is the difference in the utility of the sequence and the utility of deploying the most accurate classifier in $\mathcal{H}$ without payments. $R_T^P(\mathcal{H}) =$*

$$T \max_{h\in\mathcal{H}}(1 - L_{D^*}(h)) - \sum_{t=1}^{T}(1 - L_{D^*}(h_t) - \mathbb{E}\left[\rho_t(h_t)\right])$$

The agent's $\mathcal{H}$-regret ($R_T^A(\mathcal{H})$) is the difference in the utility of the sequence and the utility of deploying the highest expected payment yielding classifier in $\mathcal{H}$ and collecting no samples.

$$R_T^A(\mathcal{H}) = \sum_{t=1}^{T}\max_{h\in\mathcal{H}}\mathbb{E}\left[\rho_t(h)\right] - \sum_{t=1}^{T}\left(\rho_t(h_t) - \alpha\sum_{j=1}^{k} n_t^j\right).$$

We start by establishing the benchmark of what can be achieved without the information asymmetry between the principal and the agent. This is the first-best benchmark.

First-best. As in the previous setting, the principal's lack of information is two-fold. The first is that the principal cannot observe the size and quality of the dataset and the principal potentially knows less about D^* than the agent. The first-best scenario we set here will be one where the principal knows exactly how the agent draws samples. That is, the principal knows the number of i.i.d samples the agent collects from each distribution in $\mathcal{D}$. But we do not assume the principal knows more about D^* than its membership in $\mathcal{D}$. Note that this is a weaker benchmark and so showing that we cannot compete with this benchmark through linear contracts is therefore a stronger result.

In the first-best scenario, the principal observes the agent's actions which is the set of samples S_i the agent collects at each round i, $\mathbf{n_i}$ which is the number of samples drawn from each distribution to construct S_i, and h_i. Thus the principal can dictate the agent's actions $(\mathbf{n_i}, h_i)$ by assigning non-zero payment only if the agent executes $(\mathbf{n_i}, h_i)$ in each round. This is true even if h_i is a function of samples that the agent has collected so far $(S_1, \ldots, S_{i-1})$. The following proposition shows an upper bound on the regret the principal can achieve through dictating the agent's actions.

Proposition 3. *[Multi-round first-best $\mathcal{H}$-regret] Consider any class $\mathcal{D}$ of k distributions and consider any $D^* \in \mathcal{D}$ as the task distribution. Let $\mathcal{H}$ be a function class with VC dimension d. The the principal's $\mathcal{H}$-regret in the first-best scenario (*first-best $\mathcal{H}$-regret*) is $R_T^P(\mathcal{H}) \in O\left(\left(\sqrt{T^{2/3}kd}\right) + T^{\frac{1}{3}}k\log T + T^{2/3}\right)$.*

Proof sketch. The principal contracts for $N \in O(T^{2/3})$ rounds. As discussed above, the first-best contract allows the principal to dictate the agent's actions $(\mathbf{n_i}, h_i)_{i=1}^N$. Below, we describe such an algorithm.

The principal further divides the first N rounds into p phases. The j-th phase has $N_j = 2^j$ rounds. We will describe the sampling strategy in this phase and the classifier choice separately. First the sampling strategy is to divide the rounds in the phase equally among the k distributions. So in phase j, the principal seeks to collect N_j/k samples from each distribution D_i for $i \in [k]$. As for the choice of classifiers, let h_{j-1}^i be the ERM classifier based on data collected in phase $j-1$ from distribution D_i. The principal's classifier selection strategy in phase j is to dictate the agent to play a bandit algorithm with arms $\{h_{j-1}^1, \ldots, h_{j-1}^k\}$ and mean rewards $1-L_{D^*}(h_{j-1}^i)$. So over the p phases, the principal ends up playing p bandit algorithms to determine classifier selection. Say a few more words about the proof if there is room. like where the regret bound is coming from

The principal's regret in the N rounds is $O(N+\sqrt{kdN})$ where the first term comes from payments for N samples and the second term is due to the bandit algorithm for classifier selection. To analyze the classifier selection algorithm, we use standard bandit regret bounds to bound the regret of chosen actions relative to the best arm. Then we show that the there exists an arm has mean close to θ^*. The arm that is the ERM over samples from D^* satisfies this.

The principal is able to attain a classifier with $O(\sqrt{kd/N})$ error more than θ^* from the N rounds and so in the remaining $T-N$ rounds incurs regret $O(T\sqrt{kd/n})$. Choosing $N \in O(T^{2/3})$, the above approach results in a total regret of order $O(T^{\frac{2}{3}})$ □

Now we bring back the information asymmetry where

the principal cannot observe the number of samples the agent collects. The principal can only assign payments based on the classifier the agent provides. We analyze the utility the principal can achieve with this information asymmetry by modeling the agent as achieving min-max optimal regret as formalized below:

Assumption 2 ($\mathcal{H}$-Min-max optimal agent)**.** *Consider any algorithm $\mathcal{A}$ that selects an action for the agent to select at a round, possibly depending on all previous actions and payments. Let R_T, R'_T be the agent's expected $\mathcal{H}$-regret over T rounds of the sequence of agent's actions and the sequence of actions according to algorithm $\mathcal{A}$ respectively. Then the agent is $\mathcal{H}$-min-max regret optimal if $\lim_{T\to\infty} R_T/R'_T \leq C$ for some constant C for every algorithm $\mathcal{A}$.*

The following theorem compares the principal's regret achievable when delegating with a min-max optimal agent to the first-best regret. The theorem shows that regret through delegation is $\Theta(T^{3/4})$ in contrast to the $O(T^{2/3})$ first-best regret.

Theorem 2. *Let $\mathcal{H}$ be a function class. Suppose the principal delegates with a linear contract c over multiple rounds to a $\mathcal{H}$-min-max optimal agent. Then there is a problem instance for which the principal's $\mathcal{H}$-regret over T rounds $R_T^P(\mathcal{H}) \in \Theta(T^{3/4})$.*

Proof sketch of Theorem 2. Suppose the principal contracts for $N \leq T$ rounds.

The proof of first-best regret bound (Proposition 3) provides an approach for the agent to achieve $O(N^{2/3})$ regret so a min-max agent achieves this bound.

Upper bound. The principal's regret during the N contracting rounds is $O(N)$. Suppose the agent provides classifiers $h_1, \ldots, h_N$ in these rounds, the principal could deploy $\bar{h}$ which is picked uniformly at random from $\{h_1, \ldots, h_N\}$ in the remaining $T - n$ rounds.

$\mathbb{E}[L_{D^*}(\bar{h})] \in O(N^{2/3}/N)$. Deploying $\bar{h}$ in the $T - N$ rounds yields regret $O(TN^{-1/3})$. Choosing $N \in \Theta(T^{3/4})$ results in the regret $O(N)+O(TN^{-1/2})$ being $O(T^{3/4})$.

Lower bound. Clearly, a min-max optimal agent would not collect $\omega(N^{2/3})$ samples.

The upper bound on the samples collected provides a lower bound on the error of the resulting classifier. And this lower bound will provide a lower bound on the principal's regret through delegation.

Usual sample complexity lower bounds provide lower bounds on the error of learning using a number of i.i.d samples. However, in our setting, the agent has more than just the samples he collects to learn classifiers. Through the linear payments he receives, he also has access to estimates of the accuracy of classifiers he provides in each round. This is a form of (noisy) query access to the distribution.

We provide a min-max lower bound on learning using both i.i.d samples and queries of the form answering the expected error on D^* of a classifier in the following proposition. We show that in a min-max sense, the queries do not allow for more accurate classifiers compared to using just the i.i.d samples.

Proposition 4 (Lower bound on error from using samples and queries)**.** *Consider a learning algorithm that uses m i.i.d samples and q queries of accuracies of classifiers. Then there exists a distribution D for which the expected error of the learned classifier is $\Omega(1/\sqrt{m})$ more than the optimal error of a one-dimensional halfspace on D.*

The main intuition of this lowerbound is that without any structure on the distribution, it is hard to know what classifiers are useful to query.

So when the principal contracts for N rounds, the agent collects $O(N^{2/3})$ samples and due to the above proposition, the classifiers the principal deploys have an average excess error of $\Omega(1/N^{1/3})$ resulting in regret $\Omega(T/N^{1/3})$. The principal also incurs a cost of order $O(cN)$ due to contracting for N rounds. The optimal choice of N to balance these two terms is $N \in \Theta(T^{3/4})$ resulting in regret of order $\Omega(T^{3/4})$.

□

5 OPTIMAL CONTRACTS FOR HIDDEN STATE

In this section, we focus on the hidden state challenge and derive optimal contracts when there is only hidden state but not hidden action.

Previously in Theorem 1, we showed how to deal with hidden state to a certain extent i.e., when the optimal error θ is low enough. Without hidden action, we compute contracts dealing with hidden state with θ outside this range. In Appendix B.3, we show how this contract continues to be good even with hidden action when the principal's test set is large enough.

When we ignore the hidden action challenge, we can assume that the observed accuracy is deterministic in the agent's action. That is, when the agent collects n samples, the observed accuracy is $a(n, \theta) = 1 - \theta - d/n^p$. We assume that the principal holds a prior belief on the optimal error that is supported on a finite set, but does not know the exact value. The agent knows more about the optimal error since he

collects data that informs him more about the optimal error. We start by assuming that the agent knows the exact optimal error. Later we describe how to design contracts in the more realistic setting of the agent learning the optimal error instead of knowing this value exactly. We discuss this in more detail in Appendix B.1. There we also numerically show that the utility guarantees by making the perfectly aware agent assumption still hold approximately in the more realistic case with a learning agent.

Let us analyze the optimization problem for computing the optimal contract. Let the finite support of the prior (ν) over optimal error be $\{\theta_1, \ldots, \theta_N\}$. The principal puts forth a contract of accuracy-payment pairs $\{(a_i, t_i) : i \in [N]\}$ with the pair i intended for when the optimal error is θ_i.[3] Let us denote the expected accuracy from collecting n_i when optimal error is θ_i by $a_i = a(n_i, \theta_i)$. Here n_i is the number of samples the agent would collect to achieve accuracy a_i when optimal error is θ_i. The principal optimizes over $(n_i, t_i)_{i\in[N]}$. The constraints of the optimization problem for the principal's contract design for hidden state are one of two types. The first type of constraint is the participation constraint, which ensures that the agent is adequately compensated for his effort when he chooses the contract intended for the optimal error. For each $i \in [N]$, the participation constraint (PC_i) can be expressed as $\alpha n_i \leq t_i$, where α represents the compensation rate.

The second type of constraint is the incentive compatibility constraint to ensure that the agent chooses the option intended in the contract for the optimal error. For any $i, j \in [N]$, the corresponding incentive compatibility constraint is that when the optimal error is θ_i, the utility of choosing (a_j, t_j) is worse for the agent than choosing (a_i, t_i). The number of samples the agent would choose to achieve a_j accuracy under optimal error θ_i is n_{ij} such that $a_j = a(n_{ij}, \theta_i)$.[4] The constraint ($\text{IC}_{ij}$) is $t_j - \alpha n_{ij} \leq t_i - \alpha n_i$. Due to the structure of $a(n, \theta)$, the IC constraints are convex (shown in Appendix B.2). The principal's expected utility which it maximizes is $\sum_{i=1}^{N} \nu(\theta_i)(a_i - \beta t_i)$. So the contract design problem is the following optimization problem:

$$\begin{aligned} \min_{(n_i,t_i)_{i=1}^N} \quad & \sum_{i=1}^{N} \nu(\theta_i)(a_i - \beta t_i) \\ \text{s.t.} \quad & \alpha n_i \leq t_i, \quad i \in [N] \\ & t_j - \alpha n_{ij} \leq t_i - \alpha n_i, \quad i, j \in [N] \\ & n_i, t_i \geq 0, \quad i \in [N]. \end{aligned} \tag{Opt}$$

Qualitative Insights on the Optimal Contract. We derive the following insights when there are two values for the optimal error, $\theta_1 < \theta_2$, in the Appendix B.2. These properties also hold more generally for finitely supported beliefs and have been studied for classical contract design for many other delegation problems Laffont and Martimort (2009).

- *Decreased utility.* The principal gets lower utility than the first-best utility and this utility decreases as $\Delta\theta = \theta_2 - \theta_1$ increases.
- *Information rent.* In the first-best contract, the agent gets no more payment than to compensate his effort. That is, $t = \alpha n$. Under hidden state, for problems with lower optimal error, the agent gets positive utility. This information rent is to incentivize the agent to not pretend the problem is harder and exert lower effort to achieve an accuracy that requires more effort if the problem was harder.
- *Downward distortion.* The first-best contract calls for the agent to collect a particular number of samples regardless of the optimal error. Under hidden state, when the problem is harder, agents are asked to collect fewer samples compared to the first-best contract. When the problem is the easiest in the support, the agent is asked to collect the same number of samples as in the first contract.

State-Learning Agents. In the analysis above, we assumed perfect knowledge of the hidden state (θ) by the agent. However, in reality, the agent does not know the optimal error beforehand. Instead, as the agent executes the contract, he learns more about the optimal error and adapts his actions accordingly.

To design a contract for such a state-learning agent, the principal must infer the agent's response to the contract. However, this is challenging for arbitrary contracts since the principal would require knowledge of the agent's exact learning strategy, which is often unreasonable. Therefore, we focus simple contracts for which we can easily derive the agent's response.

We demonstrate numerically in Section B.1 that the utility achieved with these simple contracts is close to the utility we previously derived for state-aware agents, which we refer to as "state-aware utility."

[3] This is implied by the revelation principle that states that, with hidden state, any delegation mechanism is equivalent to an *incentive compatible* mechanism where all agents inform their private information to a planner who then recommends actions.

[4] Note that all accuracies cannot be achieved for all optimal errors. If no such n_{ij} exists, an incentive compatibility constraint is not needed.

The simple contract we consider, which we call the *state-learning contract*, is the best of two simple contracts: optimal *pooling* and *separating* contracts. A pooling contract elicits the same action for every state and therefore is not affected by the agent's learning of the state. A separating contract is designed so that through its execution, the agent accurately learns the state. Therefore, the learning agent's response to a separating contract is similar to a state-aware agent's response, which we demonstrated how to compute in Opt. We compute the optimal pooling and separating contracts in Appendix B.1.

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Checklist

1. For all models and algorithms presented, check if you include:
   (a) A clear description of the mathematical setting, assumptions, algorithm, and/or model. [**Yes**/No/Not Applicable] Section 2 describes the general model and the single-round model. The multi-round interaction model is described in Section 4. The assumptions on the agent in these models is stated as Assumption 1 and Assumption 2 respectively.
   (b) An analysis of the properties and complexity (time, space, sample size) of any algorithm. [**Yes**/No/Not Applicable] We propose an algorithm in Proposition 3 and describe it's sample complexity. The algorithm contains as a subroutine ERM over a function class. The space and time complexity depend on this subroutine.

(c) (Optional) Anonymized source code, with specification of all dependencies, including external libraries. [Yes/No/**Not Applicable**]

2. For any theoretical claim, check if you include:
   (a) Statements of the full set of assumptions of all theoretical results. [**Yes**/No/Not Applicable]
   (b) Complete proofs of all theoretical results. [**Yes**/No/Not Applicable] We provide proof sketches in the main body and provide the full proofs in the appendices.
   (c) Clear explanations of any assumptions. [**Yes**/No/Not Applicable]

3. For all figures and tables that present empirical results, check if you include:
   (a) The code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL). [**Yes**/No/Not Applicable]
   (b) All the training details (e.g., data splits, hyperparameters, how they were chosen). [**Yes**/No/Not Applicable]
   (c) A clear definition of the specific measure or statistics and error bars (e.g., with respect to the random seed after running experiments multiple times). [**Yes**/No/Not Applicable]
   (d) A description of the computing infrastructure used. (e.g., type of GPUs, internal cluster, or cloud provider). [Yes/No/**Not Applicable**]

4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets, check if you include:
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5. If you used crowdsourcing or conducted research with human subjects, check if you include:
   (a) The full text of instructions given to participants and screenshots. [Yes/No/**Not Applicable**]
   (b) Descriptions of potential participant risks, with links to Institutional Review Board (IRB) approvals if applicable. [Yes/No/**Not Applicable**]
   (c) The estimated hourly wage paid to participants and the total amount spent on participant compensation. [Yes/No/**Not Applicable**]

CONTENTS OF SUPPLEMENTARY MATERIAL

A Omitted proofs

A.1 Proof of Proposition 2

Proof. The linear contract c^* that achieves this approximately optimal utility is the following:

$$c^* = \max\left(\frac{1}{\beta(p+1)^{\frac{p+1}{p}}}, \frac{\alpha d^{\frac{1}{p}}}{p} \cdot \left(\frac{p+1}{1-\theta}\right)^{\frac{p+1}{p}}\right)$$

We first show that this contract satisfies the participation constraint for the agent. When the linear contract is c times the accuracy, the agent the number of samples n to maximize the agent's utility $c\left(1-\theta-\frac{d}{n^p}\right)-\alpha n$. The number of samples the agent chooses as a function of c is $\left(\frac{cdp}{\alpha}\right)^{\frac{1}{p+1}}$. The contract c satisfies the participation constraint if the utility from choosing this number of samples is non-negative. This utility is:

$$\begin{aligned}
&c\left(1-\theta-d\left(\frac{\alpha}{cdp}\right)^{\frac{p}{p+1}}\right)-\alpha\left(\frac{cdp}{\alpha}\right)^{\frac{1}{p+1}} \\
&= c(1-\theta) - c^{\frac{1}{p+1}}\left(\frac{\alpha d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}} - c^{\frac{1}{p+1}}\left(\alpha d^{\frac{1}{p}\frac{p}{p+1}}\right)p^{\frac{1}{p+1}} \\
&= c(1-\theta) - c^{\frac{1}{p+1}} \cdot \frac{p+1}{1-\theta} \cdot \left(\frac{\alpha d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}}.
\end{aligned}$$

This utility is non-negative when

$$c \geq \frac{\alpha d^{\frac{1}{p}}}{p} \left(\frac{p+1}{1-\theta}\right)^{\frac{p+1}{p}}.$$

By the definition of c^*, it is greater than the above quantity and so satisfies the participation constraint.

When the principal chooses a linear contract c, it achieves a utility

$$(1-\beta c)\left(1-\theta-\left(\frac{\alpha d^{\frac{1}{p}}}{pc}\right)^{\frac{p}{p+1}}\right).$$

We can provide an upper bound on the optimum utility using the optimum utility of the principal when there is no noise in the observed accuracy. In this case, the principal gets the agent to collect $\left(\frac{dp}{\alpha\beta}\right)^{\frac{1}{p+1}}$ and pays the agent α times this amount. So the optimum utility is at most

$$1-\theta-(p+1)\left(\frac{\alpha\beta d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}}.$$

To show that c^* achieves the approximation guaranteed in the theorem, we consider two cases. The first case is when $c^* = \frac{1}{\beta(p+1)^{\frac{p+1}{p}}}$. In this case the utility of c^* is

$$\left(1-\frac{1}{(p+1)^{\frac{p+1}{p}}}\right)\left(1-\theta-(p+1)\left(\frac{\alpha\beta d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}}\right)$$

The other case is when $c^* = \frac{\alpha d^{\frac{1}{p}}}{p}\left(\frac{p+1}{1-\theta}\right)^{\frac{p+1}{p}}$. In this case,

$$\frac{1}{\beta(p+1)^{\frac{p+1}{p}}} \leq \frac{\alpha d^{\frac{1}{p}}}{p}\left(\frac{p+1}{1-\theta}\right)^{\frac{p+1}{p}}$$

$$\implies \frac{1}{p+1} \leq \frac{p+1}{1-\theta}\cdot\left(\frac{\alpha\beta d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}}.$$

The ratio of the utility of c^* to the optimum utility in this case is at least

$$\frac{\left(1-\frac{\alpha\beta d^{\frac{1}{p}}}{p}\left(\frac{p+1}{1-\theta}\right)^{\frac{p+1}{p}}\right)\frac{p(1-\theta)}{p+1}}{1-\theta-(p+1)\left(\frac{\alpha\beta d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}}}$$

For $t = \frac{p+1}{1-\theta}\cdot\left(\frac{\alpha\beta d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}}$,

$$= \frac{\frac{p}{p+1}\left(1-t^{\frac{p+1}{p}}\right)}{1-t}.$$

This quantity is increasing in t. From the condition on c^*, we have $\frac{1}{p+1} \leq t$. So we can obtain a lower bound on the above quantity by setting $t = \frac{1}{p+1}$.

$$\geq 1-\frac{1}{(p+1)^{\frac{p+1}{p}}}$$

□

A.2 Proof of Proposition 3

Proof. The principal contracts for $N \in O(T^{2/3})$ rounds. The first-best contract is described by a sequence of classifiers $(h_t)_{t=1}^N$ and sequence of indices of sampling distribution $(a_t)_{t=1}^N$. The first-best contract offers payment α for each round if the agent draws a sample from distribution D_{a_t} in round t of the N rounds and provides classifier h_t. Otherwise the payment is zero. Below we described how to construct the sequences $(h_t)_{t=1}^N$ and $(a_t)_{t=1}^N$ to guarantee the principal's regret bound in the proposition.

Algorithm to determine first-best contract. Divide the first N rounds into p phases. The j-th phase has $N_j = 2^j$ rounds. We will describe the sampling strategy in this phase and the classifier choice separately. First the sampling strategy is to divide the rounds in the phase equally among the k distributions. So in phase j, the principal seeks to collect N_j/k samples from each distribution D_i for $i \in [k]$. As for the choice of classifiers, let h^i_{j-1} be the ERM classifier based on data collected in phase $j-1$ from distribution D_i. The principal's classifier selection strategy in phase j is to dictate the agent to play a bandit algorithm with arms $\{h^1_{j-1}, \ldots, h^k_{j-1}\}$ and mean rewards $1 - L_{D^*}(h^i_{j-1})$. So over the p phases, the principal ends up playing p bandit algorithms to determine classifier selection.

Regret in first N rounds. We start by analyzing the regret in the first N rounds. The principal's regret in the N rounds is $O(N + \sqrt{kdN})$ where the first term comes from payments for N samples and the second term is due to the bandit algorithm for classifier selection. To analyze the classifier selection algorithm, we use standard bandit regret bounds to bound the regret of chosen actions relative to the best arm. Then we show that the there exists an arm has mean close to θ^*. The arm that is the ERM over samples from D^* satisfies this.

The principal is able to attain a classifier with $O(\sqrt{kd/N})$ error more than θ^* from the N ronunds and so in the remaining $T - N$ rounds incurs regret $O(T\sqrt{kd/n})$. Choosing $N \in O(T^{2/3})$, the above approach results in a total regret of order $O(T^{\frac{2}{3}})$

Let us denote the classifiers provided in the N rounds of contracting as $(h_i)_{i=1}^N$, $\min_{h\in\mathcal{H}} L_{D^*}(h)$ by θ^*, the principal's regret over the N rounds R^P_N as R_N. We can write the regret R_N as the sum of regrets in each phase $R_1, \ldots, R_m$ where $m = \lceil \log N \rceil$. Recall that the length of phase j is $n_j = 2^j$. So,

$$R_j = \sum_{t=n_{j-1}+1}^{n_{j-1}+n_j} \left(L_{D^*}(h_t) - \theta^* + \alpha\right).$$

Recall that to choose classifiers in phase j, we play a bandit algorithm treating $\{h^1_{j-1}, \ldots, h^k_{j-1}\}$ as arms, where h^a_{j-1} is the ERM classifier trained on the samples drawn from D_a in phase $j-1$. h^a_{j-1} is the ERM classifier trained from n_{j-1}/k samples from D_a. Note that one of these arms h^*_j is the ERM classifier trained using $n_{j-1} = n_j/2$ samples from D^*. Let $L^*_j = L_{D^*}(h^*_j)$ be the random variable denoting the expected loss of this classifier under D^*. Note that this is a random variable since h^*_j is a random variable. We however know that $\mathbb{E}[L^*_j] \le \theta^* + C_1\sqrt{kd/n_{j-1}}$, where d is the VC dimension of $\mathcal{H}$.

By the tower property we can write

$$\begin{aligned} R_j &= \mathbb{E}\left[\mathbb{E}\left[\sum_{t=n_{j-1}+1}^{n_{j-1}+n_j} \left(L_{D^*}(h_t) - \theta^* - \alpha\right) | L^*_j\right]\right] \\ &= \mathbb{E}\left[\mathbb{E}\left[\left(R^1_j + R^2_j\right) | L^*_j\right]\right] \end{aligned}$$

where,

$$\begin{aligned} R^1_j &= \sum_{t=n_{j-1}+1}^{n_{j-1}+n_j} \left(L_{D^*}(h_t) - \theta^*\right) \\ R^2_j &= n_j(L^*_j - L^*) + \alpha n_j \end{aligned}$$

$\mathbb{E}[R_j^1|L_j^*]$ is simply expected regret of a bandit algorithm with k arms and bounded suboptimalities of arms. So for a constant C_2,

$$\begin{aligned}\mathbb{E}[R_j^1|L_j^*] &\leq C_2\sqrt{kn_j} + k\\ \mathbb{E}[R_j^2|L_j^*] &\leq n_j\left(\mathbb{E}[L_j^*] - L^*\right)\\ &\leq C_1\sqrt{dn_j \log n_j}.\\ \implies R_j &\leq C_3\sqrt{kdn_j \log n_j} + k.\end{aligned}$$

Finally, summing over regrets from each phase,

$$\begin{aligned}R_N &\leq C_3(\sqrt{kd})\sum_{j=1}^{\log N} 2^{j/2}\log N + k\log N\\ &\leq C\left(\sqrt{kdN\log N} + k\log N + \alpha N\right).\end{aligned}$$

Principal's regret in the latter $T-N$ rounds. After the N rounds of contracting as described above, the principal can deploy a classifier chosen uniformly at random from classifiers $h_1,\ldots,h_N$ for the remaining $T-N$ rounds without any further sampling.

As shown in the regret analysis for the first N rounds, $\frac{1}{N}\sum_{i=1}^N(\mathbb{E}[L_{D^*}(h_i)]) \leq \theta^* + O(kd\log N/\sqrt{N})$. The quantity on the left-hand side is also the expected error of $\bar{h}$. $\mathbb{E}[L_{D^*}(\bar{h})] = \mathbb{E}\left[\frac{1}{N}\sum_{i=1}^N L_{D^*}(h_i)\right] = \frac{1}{N}\sum_{i=1}^N \mathbb{E}[L_{D^*}(h_i)]*$. So deploying $\bar{h}$ for $T-n$ rounds results in expected regret $O(T\sqrt{kdN\log N})$.

Total regret. The total regret is of order $O(\sqrt{kdN\log N} + k + N + T\sqrt{kdN\log N})$. Choosing $N \in O(T^{2/3})$ results in the regret bound of the proposition. □

A.3 Proof of Theorem 2

Proof. **Upper bound.** We first show that the principal can achieve the upper bound regret of $T^{3/4}$ by choosing an appropriate N number of rounds to contract through linear contracts c. The principal's regret during the N contracting rounds is $O(N)$. Suppose the agent provides classifiers $h_1,\ldots,h_N$ in these rounds, the principal could deploy $\bar{h}$ which is picked uniformly at random from $\{h_1,\ldots,h_N\}$ in the remaining $T-n$ rounds.

The proof of regret in the first-best scenario given by Proposition 3 provides a way for the agent to achieve regret $O(N^{2/3})$. Since the agent is min-max optimal, the agent achieves regret $O(N^{2/3})$. Again, let $\theta^* = \min_{h\in\mathcal{H}} L_{D^*}(h)$. As a result of agent's min-max optimality,

$$\begin{aligned}R_N^A &\leq c\left(N\theta^* - \sum_{i=1}^N \mathbb{E}[L_{D^*}(h_i)]\right)\\ &\in O(N^{2/3})\\ \implies \sum_{i=1}^N \mathbb{E}[L_{D^*}(h_i)] &\leq N\theta^* + O(N^{2/3})\end{aligned}$$

The expected error of $\bar{h}$ is $\mathbb{E}[L_{D^*}(\bar{h})] = \frac{1}{N}\sum_{i=1}^N \mathbb{E}[L_{D^*}(h_i)] \in \theta^* + O(N^{-1/3})$. Deploying $\bar{h}$ in the $T-N$ rounds yields regret $O(TN^{-1/3})$. Choosing $N \in \Theta(T^{3/4})$ results in the regret $O(N) + O(TN^{-1/2})$ being $O(T^{3/4})$.

Lower bound. We can provide a lower bound for regret due to the excess error of classifiers deployed and a lower bound for regret due to payments provided.

First we analyze the regret due to excess error of classifiers deployed. Clearly, a min-max optimal agent would not collect $\omega(N^{2/3})$ samples. The upper bound on the samples collected provides a lower bound on the error of the resulting classifier. And this lower bound will provide a lower bound on the principal's regret through delegation.

Usual sample complexity lower bounds provide lower bounds on the error of learning using a number of i.i.d samples. However, in our setting, the agent has more than just the samples he collects to learn classifiers. Through the linear payments he receives, he also has access to estimates of the accuracy of classifiers he provides in each round. This is a form of (noisy) query access to the distribution.

We provide a min-max lower bound on learning using both i.i.d samples and queries of the form answering the expected error on D^* of a classifier in the following proposition. We show that in a min-max sense, the queries do not allow for more accurate classifiers compared to using just the i.i.d samples. This is shown in Proposition 4 which is restated below and is proved after this proof.

Restating Proposition 4 Consider a learning algorithm that uses n i.i.d samples and q queries of accuracies of classifiers. Then there exists a distribution D for which the expected error of the learned classifier is $\Omega(1/\sqrt{n})$ more than the optimal error of a one-dimensional halfspace on D.

Using this proposition, we can show that $\mathcal{H}$-regret with N rounds of contracting when $\mathcal{H}$ is the class of one-dimensional halfspaces is $\Omega(TN^{-1/3})$. With N rounds of contracting we know a min-max optimal agent collects $O(N^{2/3})$. With $O(N^{2/3})$ samples and any number of accuracy queries, the proposition says for any learning algorithm, there is a distribution on which resulting classifier has excess error over $\mathcal{H}$ $\Omega(N^{-1/3})$. As a result the $\mathcal{H}$-regret is $\Omega(TN^{-1/3})$.

Next we analyze regret due to the payments in N rounds of contracting. Suppose the principal's payment to the agent in these N rounds is ρ. Since the agent achieves $O^(N^{2/3})$ regret through contracting, $N\alpha\theta^* - \rho \in O(N^{2/3})$. So, $\rho \in \Omega(N\alpha\theta^* - N^{2/3}) \in \Omega(N)$. Since the construction for the lower bound has $\theta^* = \frac{1}{2} - N^{-1/3}$, the payment $\rho \in \Omega(N)$. As a result, the principal's regret from N rounds of contracting is $\Omega(N)$.

Combining both regret terms, the principal's regret is $\Omega(N + TN^{-1/3})$. Minimizing this regret is setting $N \in \Theta(T^{3/4})$ resulting in the $\Omega(T^{3/4})$ regret lower bound.

□

Next we prove the proposition used in the above theorem's proof.

A.3.1 Proof of Proposition 4

Proof. For each n, q, we will construct a class of distributions such that for any learning algorithm with access to n i.i.d. samples and q queries, there is a distribution D^* in the class for which the learning algorithm will have excess error at least $\theta^* + \Omega(1/\sqrt{n})$ where θ^* is the optimal error achieved by the class of one-dimensional half-spaces on D^*. This is the class $\mathcal{H}_{1d-HS}$ which we will refer by $\mathcal{H}$

$$\mathcal{H}_{1d-HS} = \{\mathbb{1}\{x \geq \theta\} : \theta \in \mathbb{R}\} \cup \{\mathbb{1}\{x \leq \theta\} : \theta \in \mathbb{R}\}.$$

As a construction for the lower bound, consider the class of distributions over a domain of M points, each having a uniform marginal distribution supported on $m < M$ of those points. The labelling distribution $\Pr(y = 1|x)$ is $1/2 \pm 1/2\sqrt{n}$. We later describe how to choose m, M so that the lower bound holds for n, q.

We will show that for any set of q queries, there are two distributions D_1, D_2 such that

1. All query values are the same for D_1, D_2.
2. No algorithm can distinguish between D_1, D_2 with probability more than $1/2$ using n samples drawn.
3. Any classifier with error $\min_{h \in \mathcal{H}_{1d-DS}} L_{D_1}(h) + O(1/\sqrt{n})$ on D_1 necessarily has error $\min_{h \in \mathcal{H}_{1d-DS}} L_{D_2}(h) + \Omega(1/\sqrt{n})$ on D_2.

The above properties suffice to show the required lower bound. This is because with the above properties, a learning algorithm that achieves $o(1\sqrt{n})$ expected excess error necessarily distinguishes between D_1 and D_2 with probability at least $1/2$. Distinguishing between D_1, D_2 should not be possible with n samples and q queries if the above properties hold.

Now let us show how to choose m, M and construct D_1, D_2 to make the above properties hold.

Each query assigns a value of 0 or 1 to each point. So there is a sequence of length q indicating the labels the queries assign for each point. We can partition the domain into points having the same sequence of query labels. By constructing D_1, D_2, so that the number of points in each partition with labelling function $1/2 + 1/2\sqrt{n}$ is the same for D_1 and D_2, we can guarantee that all query values are the same for D_1 and D_2

Let us set M so that $M > 2^q m$. Since there are 2^q query sequences, at least one of the partition sets has size m. Consider the m points distributions D_1, D_2 are supported on to be the n points of samples drawn and the remaining points are points from the partition of size m. There are at least $m - n$ points of the support from the partition of size m.

Let the points in the support that are in the query partition of size m be $x_1 \leq \ldots \leq x_s$ where $s \geq m - n$. D_1 has probability of $+1$ label $1/2 + 1/\sqrt{n}$ for points $x_1, \ldots, x_{s/2}$ and probability of $+1$ label $1/2 - 1/\sqrt{n}$ for points $x_{s/2+1}, \ldots, x_m$. D_2 has probability of $+1$ label $1/2 - o1/\sqrt{n}$ for points $x_1, \ldots, x_{s/2}$ and probability of $+1$ label $1/2 + 1/\sqrt{n}$ for points $x_{s/2+1}, \ldots, x_m$.

Outside of this partition, let the labelling distribution of any other point in the support be the same for D_1, D_2. By this construction, property (1) is satisfied.

Restricted to the query partition, the errors on distributions D_1, D_2 sum up to $1/2 + 1/\sqrt{n}$. By choosing $m > 2n$, the query partition makes up at least half the fraction of the support. Therefore any classifier h has $L_{D_1}(h) + L_{D_2}(h) = 1/4 + 1/2\sqrt{n}$. and due to this property (3) holds.

The KL divergence between the sampling distributions from D_1, D_2 is at most $\sqrt{n}\frac{1}{2\sqrt{n}}$, and therefore with probability $\geq 1/2$, we cannot distinguish between D_1 and D_2 using n samples. This shows property (2) holds.

□

A.4 Proof of Proposition 1

Proof. Since in the first-best scenario, the principal knows θ and the test accuracy is deterministic in the number of samples collected, the principal can deduce the number of samples agent draws exactly based on the test accuracy. Therefore, we can focus on contracts directly based on the number of samples.

The optimal number of samples to maximize principal's utility subject to compensating the agent is given by the following optimization problem: $\max_n 1 - \theta - \frac{d}{n^p} - \alpha\beta n$

The first-best contract asks agent to collect n^* samples which is the optimal value to the above optimization problem and provides payment $t^* = \alpha n^*$.

The objective maximized is convex and so the optimal value n^* is obtained by setting gradient to zero. Thus $n^* = (pd/\alpha\beta)^{1/(p+1)}$. □

B Miscellaneous results and discussions

B.1 Designing contracts against state-learning agents

In Section 5 and particularly Opt, we assumed perfect knowledge of the hidden state (θ) by the agent. However, in reality, the agent does not know the optimal error beforehand. Instead, as the agent executes the contract, he learns more about the optimal error and adapts his actions accordingly. To design a contract for such a state-learning agent, the principal would need to predict the agent's response to the contract. However, this is challenging for arbitrary contracts since the principal would require knowledge of the agent's exact learning strategy, which is often unreasonable. Therefore, we focus on analyzing simple contracts for which we can easily derive the agent's response. We demonstrate numerically that the utility achieved with these simple contracts is close to the utility we previously derived for state-aware agents, which we refer to as "state-aware utility." This provides evidence that qualitative insights we derived about the state-aware utility in Section 2 and Section 3 are applicable in the more realistic case of a state-learning agent. We focus on the case where the optimal error can take one of two possible values, $\theta_1 < \theta_2$, but these design principles also extend to more possible values of the optimal error. The simple contract we consider, which we call the *state-learning contract*, is the best of two simple contracts: optimal *pooling* and *separating* contracts.

Separating contract. Separating contracts allow the agent to perfectly infer the hidden state while executing the contract. These are incentive-compatible contracts that ask agents to collect n_1, n_2 samples under optimal errors $\theta_1 < \theta_2$ respectively. Additionally, n_1, n_2 are such that the agent can successfully infer the optimal error after collecting $\min(n_1, n_2)$ samples. The agent's response to this contract would be to first collect $\min(n_1, n_2)$ samples and decide whether to collect more depending on the inferred optimal error. The agent's successful inference of the optimal error makes computing optimal separating contracts similar to the contract design problem against a state-aware agent, which was solving the optimization problem Opt. The new optimization problem that yields optimal separating contracts has the same objective and constraints as Opt with the added constraint that $n_1, n_2 > n_0$ for some n_0 that we will describe soon. The additional constraint ensures that the agent knows the optimal error (with high probability) after collecting $\min(n_1, n_2)$ samples.

To determine the value of n_0, we rely on assumptions about the agent's learning strategy. We assume that the agent can distinguish between θ_1, θ_2 with high probability using $k/(\Delta\theta)^2$ samples. Here $\Delta\theta = \theta_2 - \theta_1$ and k is a constant reflecting the degree of assumption made about the agent's efficiency. A lower value of k is a stronger assumption, assuming a more efficient agent. This assumption on the agent's learning strategy is more reasonable compared to assuming precise knowledge of the agent's learning strategy.

When n_0 is small enough that the added constraint $n_1, n_2 > n_0$ is not active, the state-learning agent is behaving exactly as the state-aware agent, so our results from Section 5 apply. On the other hand, if n_0 is large (which happens when $\Delta\theta$ is small) the additional constraint becomes too restrictive and the utility becomes low. In this case, another approach works well.

Pooling contract. In pooling contracts, the agent has no incentive to learn the optimal error. The pooling contract asks the agent to achieve one accuracy level $\bar{a}$ regardless of the optimal error. The payment for this accuracy is set to ensure the agent can get nonnegative utility regardless of the optimal error. It is again straightforward to understand the agent's response to this contract. Suppose $\bar{a}$ can be achieved by collecting $\bar{n}_1 < \bar{n}_2$ samples under optimal errors $\theta_1 < \theta_2$ respectively. To execute this contract, the agent starts collecting $\bar{n}_1$ and sees if it achieves $\bar{a}$ accuracy. If it does not, he collects $\bar{n}_2 - \bar{n}_1$ more samples since this action is guaranteed to yield nonnegative utility. Furthermore, collecting fewer or no additional samples results in less than $\bar{a}$ expected accuracy and hence zero payment even though the agent exerted effort.

A pooling contract does not let agents differentiate actions for different optimal errors and would be sub-optimal for this reason. However, when the difference in both problems is not significant i.e., $\Delta\theta$ is low, the benefit to the principal for distinguishing the agents is low. In summary, the separating contract has good utility when $\Delta\theta$ is large and the pooling contract has good utility when $\Delta\theta$ is small. By deploying the contract of the two with the higher utility, we can hope to have good utility for all values of $\Delta\theta \in [0, 0.5]$.

The optimal pooling and separating contracts for the two hidden states case are computed in Sections B.2. In this case, we numerically show in Appendix C that the best of pooling and separating contracts achieve close to the state-aware contract's utility.

B.2 Closed-form solution for the two optimal error, hidden state problem

Here we solve the state-aware optimization problem Opt when there are two optimal errors, $\underline{\theta} \le \overline{\theta}$, with a prior probability of ν for $\underline{\theta}$. Let $L(n) = d/n^p$. We solve the following optimization problem and show that the solution is the optimal solution we are looking for. Note that this problem omits the incentive-compatibility constraint for the problem $\overline{\theta}$ and the participation constraint for the easy problem.

$$\begin{aligned} \min_{\underline{n},\overline{n},\underline{t},\overline{t}} \quad & \nu(L(\underline{n}) + \beta\underline{t}) + (1-\nu)(L(\overline{n}) + \beta\overline{t}) \\ \text{s.t.} \quad & \overline{t} - \frac{\alpha\overline{n}}{(1+\Delta\overline{n}^p)^{1/p}} - \underline{t} + \alpha\underline{n} \le 0 \\ & \alpha\overline{n} - \overline{t} \le 0. \end{aligned}$$

First note that this is a convex optimization problem, where the objective is convex by the convexity of the loss and the PC constraint is a linear constraint. All that is left is to check that the IC constraint is convex. This is the sum of linear terms and the term $\frac{-\alpha d^{1/p}\overline{n}}{(d+\Delta\overline{n}^p)^2}$. The second derivative of this term is $\frac{3\alpha d^{1/p}\Delta}{2\overline{n}^p(d+\Delta\overline{n}^p)^4}$. Since the second derivative is positive, the IC constraint is convex.

Consider the Lagrangian

$$L(\underline{n},\overline{n},\underline{t},\overline{t};\lambda_1,\lambda_2) = \nu(L(\underline{n})+\beta\underline{t}) + (1-\nu)(L(\overline{n})+\beta\overline{t}) + \lambda_1\left(\overline{t} - \frac{\alpha d^{1/p}\overline{n}}{(d+\Delta\overline{n}^p)^{1/p}} - \underline{t} + \alpha\underline{n}\right) + \lambda_2\left(\alpha\overline{n}-\overline{t}\right),$$

which has the following gradients:

$$\nabla_{\underline{n}} L = \nu L'(\underline{n}) + \alpha\lambda_1 \tag{G1}$$
$$\nabla_{\underline{t}} L = \nu\beta - \lambda_1 \tag{G2}$$
$$\nabla_{\overline{t}} L = (1-\nu)\beta - \lambda_2 + \lambda_1 \tag{G3}$$
$$\nabla_{\overline{n}} L = (1-\nu)L'(\overline{n}) + \alpha\lambda_2 - \lambda_1\left(\frac{\alpha d^{1/p}}{(d+\Delta\overline{n}^p)^{(p+1)/p}}\right) \tag{G4}$$

To choose values $\underline{n}^*, \overline{n}^*, \underline{t}^*, \overline{t}^*, \lambda_1^*, \lambda_2^*$ that satisfy the KKT conditions, first we set the gradients to zero:

$$L'(\underline{n}^*) = -\alpha\beta \qquad \text{(From (G1), (G2))}$$
$$\lambda_1^* = \nu\beta \qquad \text{(From (G2)}$$
$$\lambda_2^* = \beta \qquad \text{(From (G3) and value of } \lambda_1^*)$$
$$(1-\nu)L'(\overline{n}^*) + \alpha\beta - \frac{\nu\alpha\beta}{(1+\Delta\overline{n}^p)^{(p+1)/p}} = 0 \qquad \text{(From (G4) and values of } \lambda_1^*, \lambda_2^*)$$
$$\implies L'(\overline{n}^*) = -\frac{\alpha\beta}{1-\nu}\left(1 - \frac{\nu d^{1/p}}{(d+\Delta\overline{n}^{*p})^{(p+1)/p}}\right)$$

By complementary slackness,

$$\alpha\overline{n}^* = \overline{t}^*$$
$$\underline{t}^* = \alpha\left(\underline{n}^* - \frac{d^{1/p}\overline{n}^*}{(d+\overline{n}^{*p})^{1/p}} + \overline{n}^*\right).$$

The contract described by $(\underline{n}^*, \overline{n}^*, \underline{t}^*, \overline{t}^*)$ satisfies the properties of the second-best contract in the classical contract theory setting. We list these properties here:

P1 No output distortion for the easy problem: $\underline{n}^*$ is the solution of $L'(\underline{n}^*) = -\alpha\beta$ which is also the value of n^{fb}. So for the easy problem, the agent gathers the same number of samples as in the full information case.

P2 Downward distortion for the hard problem:

$$\begin{aligned} L'(\overline{n}^*) &= -\frac{\alpha\beta}{1-\nu}\left(1 - \frac{\nu d^{1/p}}{(d+\Delta\overline{n}^{*p})^{(p+1)/p}}\right) \\ &< -\alpha\beta \\ &= L'(n^{fb}). \end{aligned}$$

So $\overline{n}^* < n^{fb}$. For the hard problem, the agent gathers fewer samples than in the full information case.

P3 When the problem is easy, the agent gets positive information rent:

$$\begin{aligned} \underline{t}^* - \alpha\underline{n}^* &= \overline{n}^* - \frac{d^{1/p}\overline{n}^*}{(d+\Delta\overline{n}^{*p})^{(p+1)/p}} \\ &> 0. \end{aligned}$$

We now check that the contract that is the solution to the above optimization problem also satisfies the omitted constraints. First we start with the participation constraint for the easy problem. By the positive information rent property (P3) we know that $\alpha\underline{n}^* < \underline{t}^*$. Next consider the incentive-compatibility constraint for the hard

problem. We only need to check when $\Delta \underline{n}^{*p} < d$. Otherwise, the IC constraint automatically holds. The difference in agent's utility between choosing the $\overline{\ell}^* = \overline{\theta} + d/\overline{n}^{*p}$ option and the $\underline{\ell}^* = \underline{\theta} + d/\underline{n}^{*p}$ option is:

$$
\begin{aligned}
&- \alpha \underline{n}^* + \overline{t} + \frac{\alpha d^{1/p} \underline{n}^*}{(d - \Delta \underline{n}^{*p})^p} - \underline{t}^* \\
&= -\frac{\alpha d^{1/p} \overline{n}^*}{(d + \Delta \overline{n}^{*p})^{1/p}} + \alpha \overline{n}^* + \frac{\alpha d^{1/p} \underline{n}^*}{(d - \Delta \underline{n}^{*p})^{1/p}} - \alpha \underline{n}^* && \text{(Using values of } \underline{t}^*, \overline{t}^*) \\
&= \alpha \left(\frac{d^{1/p} \underline{n}^*}{(d - \Delta \underline{n}^{*p})^{1/p}} - \underline{n}^* + \frac{d^{1/p} \overline{n}^*}{(d + \Delta \overline{n}^{*p})^{1/p}} - \overline{n}^* \right) \\
&\geq \alpha \overline{n}^* \left(\frac{1}{(1 - \Delta \underline{n}^{*p})^{1/p}} + \frac{1}{(1 + \Delta \underline{n}^{*p})^{1/p}} - 2 \right) && \text{(Since } \underline{n}^* < \overline{n}^*)
\end{aligned}
$$

Note that the function $\frac{d^{1/p}}{(d-x)^q} + \frac{d^{1/p}}{(d+x)^q}$ is increasing in the interval $[0, 1)$ for every q. The derivative of that function is $q \left(\frac{d^{1/p}}{(d-x)^{q+1}} - \frac{d^{1/p}}{(d+x)^{q+1}} \right)$. This is nonnegative and lies in $[0, 1)$.

$$
\geq 0 \qquad \text{(Since we assume } 0 < \Delta \underline{n}^{p*} < 1).
$$

This solution finds the optimal contract under hidden state.

B.2.1 Separating contracts

To be able to compute any separating contract, it suffices to solve the above optimization problem with the additional constraint $\underline{n}, \overline{n} \geq n_0$ for some $n_0 \geq 0$. The new optimizers $\underline{n}(n_0), \overline{n}(n_0), \underline{t}(n_0).\overline{t}(n_0)$ are as follows:

$$
\begin{aligned}
\underline{n}(n_0) &= \max(\underline{n}^*, n_0) \\
\overline{n}(n_0) &= \max(\overline{n}^*, n_0) \\
\underline{t}(n_0) &= \alpha \underline{n}(n_0) \\
\overline{t}(n_0) &= \alpha \left(\underline{n}(n_0) - \frac{d^{1/p} \overline{n}(n_0)}{(d + \overline{n}(n_0))^{1/p}} + \overline{n}(n_0) \right).
\end{aligned}
$$

B.2.2 Optimal pooling contract

The optimal pooling contract optimizes over $\overline{n}$. $t = \alpha \overline{n}$. $\underline{n}$ is chosen such that

$$
\begin{aligned}
\underline{\theta} + \frac{d}{\underline{n}^p} &= \overline{\theta} + \frac{d}{\overline{n}^p} \\
\implies \underline{n} &= \frac{d^{1/p} \overline{n}}{(d + \Delta \overline{n}^p)^{1/p}}.
\end{aligned}
$$

Thus the optimization problem is choosing $\overline{n}$ to be the minima of

$$
\nu \frac{d}{\left(\frac{d^{1/p} n}{(d + \Delta n^p)^{1/p}} \right)^p} + (1 - \nu) \frac{d}{n^p} + \alpha \beta n.
$$

B.3 Medium test-set regime

In our analysis, we have examined the impact of the hidden action challenge when dealing with a small test set size. The significance of the hidden action challenge diminishes as the test set size increases, as the principal can obtain highly accurate estimates of the model's accuracy. However, when the test set becomes too large, delegation loses its value since the principal can independently learn an accurate model without delegation. Is there a regime in which the test set size is large enough for hidden action to not be significant while also being small enough for the principal to benefit from delegating data collection? In this section, we demonstrate the existence of such a regime, referred to as the "medium test set regime." Later, we outline how we can capitalize on the larger size of the test set to achieve stronger results.

The sample complexity for learning an ϵ-optimal model is $\Theta(d/\epsilon^2)$. In particular, this bound is linear in the training algorithm's complexity which can be problematic when using highly complex training algorithms. We say that the medium test set regime exists, if the sample complexity for hidden action is significantly smaller than $\Theta(d/\epsilon^2)$, where ϵ captures the significance level of hidden action which we will make precise in the following definition.

Definition 2 (Insignificance of hidden action at level ϵ)**.** *In a finite test set setting with hidden action, for any optimal error parameter θ, let* OPT *denote the optimal expected utility of contracting. We say that hidden action is insignificant at level ϵ, for any $\epsilon > 0$, if the expected utility of the first-best contract based on θ in this setting is at least* $\text{OPT} - \epsilon$.

We next state a theorem giving the sample complexity of the principal's test set to achieve insignificance of the hidden action. The sample complexity stated in the theorem is logarithmic in d while learning would have required a number of samples linear in d. This demonstrates the existence of a medium test set regime where it is possible to employ delegation without considering hidden action.

Proposition 5 (Sample complexity for insignificant hidden action)**.** *For any $\epsilon > 0$, if the principal has a test set of size $O\left(\frac{1}{\epsilon^2}\log\frac{d}{\epsilon}\right)$, then hidden action is insignificant at the level ϵ.*

Proof. Recall that the first-best contract has a threshold form. The contract offers payment t^* when the test error is less than or equal to ℓ^* and offers payment zero otherwise. Let us denote the sample complexity to get expected loss at most ℓ by $n(\ell)$. That is,

$$n(\ell) = \left(\frac{d}{\ell - \theta}\right)^{1/p}.$$

The optimal contract offers $t^* = \alpha n(\ell^*)$ where α is the cost per sample for the agent. Let us denote $n(\ell^*)$ by n^*. And $n^* = (pd/\alpha\beta)^{1/(p+1)}$.

We will show that the best response for the agent against this contract is never to collect samples less than $n(\ell + \epsilon)$ when the test set has size $O\left(\frac{1}{\epsilon^2}\log\frac{d}{\epsilon}\right)$. We show this by showing that the agent's utility in choosing $n(\ell + \Delta)$ is less than the agent's utility in collecting $n(\ell - \Delta)$ for all $\Delta > \epsilon$.

The number of samples the agent would collect to get expected error $l^* + \Delta$ is such that:

$$\theta + \frac{d}{n_1^p} = \theta + \frac{d}{n^{*p}} + \Delta$$
$$n_1 = \frac{n^* d^p}{(d + \Delta)^p}.$$

Similarly, the number of samples needed to get expected error $l^* - \Delta$ is

$$n_2 = \frac{n^* d^p}{(d - \Delta)^p}.$$

For any action of the agent, the probability that the observed loss is ϵ or more away from the expected loss is less than $2\exp(-2m\epsilon^2)$. This is by applying Hoeffding's inequality on the observed loss random variable which is bounded between 0 and 1. As a result, for $\Delta > \epsilon$, the expected payment when collecting n_1 and n_2 samples is $\leq 2t^*\exp(-2m\epsilon^2)$ and $\geq t^*(1 - 2\exp(-2m\epsilon^2))$ respectively. The agent's utility due to n_1 is less than the utility due to n_2 when

$$\alpha n^* \left(1 - 4\exp(-2m\epsilon^2)\right) \geq \alpha n^* d^p \left(\frac{1}{(d - 2\Delta n^{*1/p})^p} - \frac{1}{(d + 2\Delta n^{*1/p})^p}\right)$$

Let us denote $\kappa = d^p\left(\frac{1}{(d-2\Delta n^{*1/p})^p} - \frac{1}{(d+2\Delta n^{*1/p})^p}\right)$. So this occues when

$$m \geq \frac{1}{2\epsilon^2}\log\frac{4}{1-\kappa}.$$

Note that $\frac{1}{1-\kappa}$ is polynomial in both d and $\frac{1}{\epsilon}$.

□

B.4 Treating error curves as upper bounds

For most of our results we have made use of the structured form of error curves reflecting how expected error of a learned model is assumed to vary with the number of samples used for training. This structure is inspired by statistical minimax bounds and are upper bounds rather the true error curves. We designed contracts assuming the bounds to be actual error curves. Here we discuss what we can say about these contracts without assuming the bounds to be exact error curves.

From the principal's perspective, these contracts result in accuracy that is just as good as that of learned models. However, the principal would end up paying the agent more than it could have if the principal knew the exact error curve. We can view the shape of the true error curve as another piece of information the principal is unaware of in addition to the optimal error. This hidden information results in more information rent but does not impact the accuracy of the model obtained from delegation.

The agent's perspective of what changes is more complicated. Our contracts assumed that the agent responded assuming that the upper bound was the true error curve. It may be reasonable that before starting the delegation process, the agent believes the upper bounds to be the true curves having no other frame of reference. However after starting to collect data, it is possible that the agent will learn more about the form of the true error curve and respond differently. This is similar to how the agent can learn the optimal error while executing the contract. Analyzing how this error curve learning occurs will allow us to design truly incentive-compatible contracts. However, learning the error curve shape is learning from a much broader class and is likely to be more challenging.

B.5 Variable label quality model

The setting above models the scenario where the agent does not have the option to choose the quality of the data is collects. However, the agent might be able to control the quality of the data as a function of the cost per sample. We study a model of quality of data where the quality corresponds to the quality of the labels of the data. The quality parameter $q = 1 - 2\eta$ captures the likelihood of the labels being correct. Here, $\eta \in (0, 1/2)$ is the probability of the label being incorrect. In this setting, the expected accuracy on the principal's test set from the agent collecting n samples at quality level q when the optimal error is θ is $1 - \theta - \frac{1}{qn^p}$ for some $p > 0$. We assume that the cost of collecting a single sample at quality level q for the agent is given by $\alpha(q)$ a function increasing in q and convex. So the cost for the agent of collecting n samples at quality level q is $C(n, q) = \alpha(q)n$.

In this section, we provide results for $\alpha(q) = q^b + \alpha_0$ for $b > 0$. We can think of α as the cost of collecting an unlabelled sample and q^b as the cost of labelling an unlabelled point. The main message of this section is that even though the quality of labels is an action that the agent chooses, effectively, this choice is not information that is private from the principal. It turns out that whatever the contract is, the utility-maximizing agent executes the contract by choosing a single quality value q^*. The principal can also compute q^* so the quality is not a reflection of information asymmetry. Therefore, this regime is essentially the same as the one studied in the previous section.

Theorem 3 (Constant quality level)**.** *For any problem parameters $d, p, \alpha_0 > 0, b > 1$, when $\alpha(q) = q^b + \alpha_0$, for any expected accuracy a the agent wishes to achieve, the agent chooses a constant q^* that only depends on α_0, b as the quality level.*

Proof. If the agent aims to achieve an expected accuracy of at least s, then the agent chooses the number of samples and quality level by solving the following optimization problem:

$$\begin{aligned} \min_{q,n} \quad & \left(q^b + \alpha_0\right) n \\ \text{s.t.} \quad & \theta + \frac{1}{qn^p} \leq 1 - a \\ & 0 \leq q \leq 1. \end{aligned}$$

The solution of this optimization problem can be calculated as follows:

$$\begin{aligned}\mathcal{L} &= \left(q^b + \alpha\right) n + \lambda_1 \left(\theta - l + \frac{1}{qn^p}\right) + (\lambda_2 - \lambda_3)q \\ \nabla_n \mathcal{L} &= q^b + \alpha - \frac{p\lambda_1}{bn^{p+1}} \\ \nabla_q \mathcal{L} &= bq^{b-1}n - \frac{\lambda_1}{q^2 n^p} + \lambda_2 - \lambda_3.\end{aligned}$$

If $\lambda_2^* = \lambda_3^* = 0$, we obtain

$$\begin{aligned}&\implies \lambda_1^* = bq^{*b+1}n^{p+1} \\ &\implies q^* = \left(\frac{\alpha}{b-1}\right)^{\frac{1}{b}},\end{aligned}$$

and n^* is obtained by solving

$$\theta + \frac{1}{q^* n^{*p}} = l.$$

If $(\alpha/(b-1))^{1/b}$ is not in $[0,1]$, then λ_2^* or λ_3^* is non-zero and q^* is either 0 or 1. □

B.6 Tightness of linear contracts approximation

Our main result (Theorem 1) gave a linear contract that provably approximates the optimal contract up to a constant factor. This approximation factor stated in Proposition 2 is also tight as stated in the following theorem, which shows that there is a problem instance for which no linear contract can do better than a given approximation factor. The problem instance for which the approximation ratio is tight is one that has deterministic test error distribution, which arises when the size of the test set tends to infinity.

Theorem 4 (Tightness of approximation bound)**.** *For every $\theta \in [0,1), p, d > 0$, there are problem parameters $\alpha, \beta > 0$ such that for the problem instance with these parameters, all linear contracts have at most $1 - \frac{1}{(p+1)^{\frac{p+1}{p}}}$ times the optimal utility.*

Proof. We will show that there exist α, β such that the contract $\frac{1}{\beta(p+1)^{\frac{p+1}{p}}}$ is the optimal contract chosen by the principal. This contract will also satisfy the participation constraint for our chosen values of α, β. Recall that the participation constraint is

$$\begin{aligned}\frac{1}{\beta(p+1)^{\frac{p+1}{p}}} &\geq \frac{\alpha d^{\frac{1}{p}}}{p} \cdot \left(\frac{p+1}{1-\theta}\right)^{\frac{p+1}{p}} \\ &\equiv 1 - \theta \geq \left(\frac{\alpha\beta d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}} (p+1)^2.\end{aligned}$$

The principal chooses the contract that sets the derivative of the above quantity to zero as long as that contract satisfies the participation constraint. If setting $\frac{1}{\beta(p+1)^{\frac{p+1}{p}}}$ yields a zero derivative and it satisfies the participation constraint, then it is the optimal linear contract. The derivative relative to c is

$$(1 - \beta c)\left(\frac{\alpha d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}} \cdot \frac{p}{p+1} \cdot \frac{1}{c^{\frac{2p+1}{p+1}}} - \beta\left(1 - \theta - \left(\frac{\alpha d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}}\right).$$

Setting $c = \frac{1}{\beta(p+1)^{\frac{p+1}{p}}}$, the derivative is

$$\left(1 - \frac{1}{(p+1)^{\frac{p+1}{p}}}\right) \frac{p}{p+1} \left(\frac{\alpha d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}} \beta^{\frac{2p+1}{p+1}} (p+1)^{\frac{2p+1}{p}}$$
$$- \beta \left(1 - \theta - (p+1)\left(\frac{\alpha\beta d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}}\right).$$

We can choose α, β to set this derivative to zero by choosing α, β satisfying:

$$1 - \theta = \left(\frac{\alpha\beta d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}} (p+1) \left(1 + \left(1 - \frac{1}{(p+1)^{\frac{p+1}{p}}}\right) p(p+1)^{\frac{1}{p}}\right).$$

Note that for every $p > 0$, $\left(1 + \left(1 - \frac{1}{(p+1)^{\frac{p+1}{p}}}\right) p(p+1)^{\frac{1}{p}}\right) > p + 1$. So,

$$\geq \left(\frac{\alpha\beta d^{\frac{1}{p}}}{p}\right)^{\frac{p}{p+1}} (p+1)^2.$$

This shows that there are problem parameters that make $c^* = \frac{1}{\beta(p+1)^{\frac{p+1}{p}}}$ the optimal linear contract. In the proof of Proposition 2, we showed that this linear contract achieves at least $1 - \frac{1}{(p+1)^{\frac{p+1}{p}}}$ times the optimum utility. When the problem involves a deterministic mapping between the number of samples and the observed accuracy, this ratio is exact. □

C Numerical simulations

C.1 Contracts for state-learning agents

In the two states case where states can be $\theta_1 < \theta_2$, we compute the utility difference between the state-aware contract and the state-learning contract, varying problem parameters $\Delta\theta = \theta_2 - \theta_1$ and k. We highlight a few observations (see Fig 1), that reflect the intuition we used to design the approach for state-learning contracts.

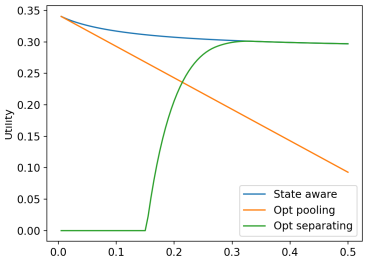
These simulations are for error rate problem parameter $p = 0.5$, inspired by binary classification. We vary k and $\Delta\Theta$ from 0 to 0.5 covering all states in the binary classification problem.

- Figure 1a shows that the state-learning contract is pooling when $\Delta\theta$ is less than some threshold and is separating otherwise. For small and large values of $\Delta\theta$, the state-learning contract has utility close to the state-aware utility.
- Figure 1b shows that when it is more difficult to distinguish between θ_1, θ_2, the pooling contract is better than the separating contract for more values of $\Delta\theta$.
- Figure 1c shows that the worst-case sub-optimality over all $\Delta\theta$ values of the state-learning contract compared to the state-aware utility increases as k increases. When the agent can test more efficiently, the state-learning contract has greater utility for the principal.

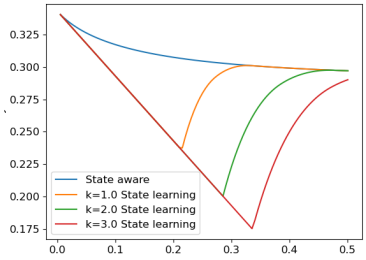
C.2 State-agnostic linear contracts

In Theorem 1, we proposed a linear contract $\bar{c}$ that does not depend on the hidden state θ and provide a range of values of θ for which this linear contract achieves multiplicatively approximate utility relative to the first-best utility.

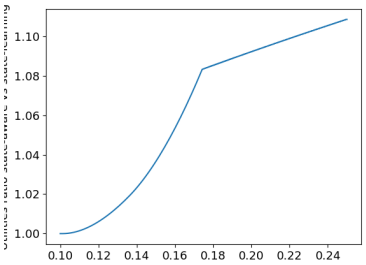
Now we numerically show the ratio of the utility of this linear contract to the first-best utility outside this range for θ. We show in Figure 2that the approximation ratio decreases with how far the state θ lies outside the range given in Theorem 1.



(a) Utilities of different contracts vs $\Delta\theta$



(b) Utilities of state-aware and state-learning contracts for various k's vs $\Delta\theta$



(c) Worst-case sub-optimality of state-learning contracts relative to state-aware contracts

Figure 1: Figure 1a plots the utilities of the state-aware, separating, and the pooling contract against ($\Delta\theta$). Figure 1b again plots the utilities of contracts on the y-axis and $\Delta\theta$ on the x-axis. It plots the state-aware utility and the utilities of state-learning contracts of different levels k of agent's testing efficiency. Figure 1c plots the worst-case sub-optimality of state-learning contracts against k. The sub-optimality is the ratio of the state-learning contract's utility and the state-aware utility. The worst-case sub-optimality is the largest sub-optimality over all $\Delta\theta \in [0, 0.5]$.

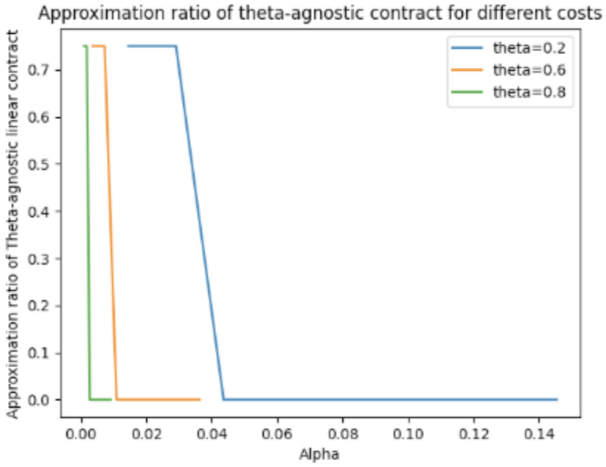


Figure 2: Approximation ratio of linear contract $\bar{c}$ versus θ