## **Supplementary Material**

## 1. Comparison of Various Annealing Strategies

In our paper, we have devised a Gaussian-style function for rank annealing. This approach allows us to dynamically adjust ranks based on the amplitude of the loss. In addition to this final selection, we have also explored other rank annealing techniques involving a fixed step or factor, specifically,  $r_t = r_{t-1} - C$  or  $r_t = r_{t-1}/f$ . Here,  $r_t$  and  $r_{t-1}$  represent the ranks at time steps t and t-1, while t0 and t1 denote constants. As demonstrated in Table 1, our tailored rank annealing strategy has consistently resulted in optimal performance.

Method	Benchmark						
Method	LFW	CFP-FP	CPLFW	AgeDB			
Fixed step	99.39	96.21	89.76	94.83			
Fixed factor	99.43	96.29	89.81	84.89			
Gaussian	99.45	96.30	89.84	94.91			

Table 1. A comparison of models trained with various rank annealing strategies in terms of 1:1 verification accuracy (%) on CASIA-WebFace.

## 2. Visulisations of NPT and Ada<sup>2</sup>NPT

By incorporating the adaptive rank annealing strategy and distance measurement into NPT, the Ada<sup>2</sup>NPT yields a more discriminative and compact latent space. Despite the performance improvements achieved, we present the embeddings of ten identities extracted from the network trained with NPT or Ada<sup>2</sup>NPT in the first row of Fig 1. To facilitate a clearer comparison, we display the T-SNE distributions of two classes in the second row. It is evident that the embeddings learned with Ada<sup>2</sup>NPT exhibit reduced intra-class distances.

## 3. More results of Fairness

In addition to DebFace, we conducted comparisons with several other state-of-the-art methods to show fairness. All of these methods were re-implemented and tested using their originally published settings. As illustrated in Table 2, our approach consistently demonstrates superior fairness while maintaining recognition performance that is on par with these methods.

Method	Accuracy ↑				Bias↓	Accuracy ↑		Bias ↓
	Caucasian	African	Asian	Indian	Dias $\downarrow$	Female	Male	Dias $\downarrow$
CosFace	95.91	93.41	94.15	94.75	0.92	94.75	95.46	0.36
NPT	95.85	93.57	94.16	94.73	0.84	94.77	95.39	0.31
AdaFace	96.21	93.75	94.37	94.81	0.90	95.03	95.79	0.38
DebFace	95.95	93.67	94.33	94.78	0.83	94.86	95.43	0.29
Ours	95.96	93.82	94.42	94.80	0.78	95.01	95.45	0.22

Table 2. A comparison of the ethnicity and gender fairness between the proposed method and DebFace in terms of 1:1 verification accuracy (%). Our recognition performance is achieved by the embeddings from phase 1.

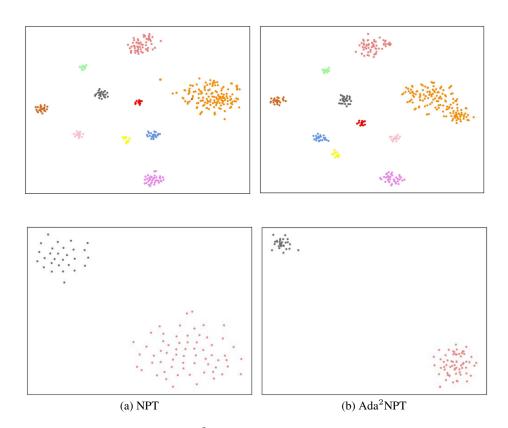


Figure 1. The T-SNE distributions of NPT and  $Ada^2NPT$ . We randomly selected ten classes and illustrated their 2D visualisations in the first row. For a clearer comparison, we show the T-SNE distributions of two classes in the second row.