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Conformal Decision Rules

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Abstract

This paper proposes conformal decision rules. They are defined as decision rules with their own conformal predictors. Given a test instance, conformal decision rules provide a point prediction, an explanation, a p-value for that prediction plus a prediction set.
Keywords: Classification, Inductive Conformal Prediction, Rule Learning

1. Background

Integrating interpretable prediction and reliable prediction is a must in practical machine learning (Johansson et al., 2022). There are several methods to train conformal interpretable models in classification (van Prehn and Smirnov, 2008; Johansson et al., 2013) and regression (Johansson et al., 2018) all based on decision/regression trees. This paper makes one step further: it proposes to "conformalize" decision rules (Furnkranz et al., 2012) since they are more interpretable and algorithmically transparent than decision trees.

2. Conformal Decision Rules

The learning algorithm of conformal decision rules is given in Algorithm 1. It first trains ordered decision rules using a standard algorithm such as IREP (Furnkranz et al., 2012) (step 1). The rules represent a Mondrian taxonomy (Boström and Johansson, 2020) that partitions the data (steps 2-5). Therefore, a Mondrian inductive conformal predictor is trained on taxonomized data using a local/global strategy (steps 6-11). The algorithm outputs final predictor h as set of conformal decision rules s.t. each rule is a decision rule r with its own inductive conformal predictor ICP_r (Papadopoulos et al., 2002).

The global strategy for Mondrian ICP trains global nonconformity function A shared by all ICPs on global proper training subset T^t of the data generated by the original distribution. It assumes that larger data (e.g. T^t) results in more accurate non-conformity functions than smaller (e.g. T_r^t). However, the global function A can be less accurate on calibration sets T_r^c since they are biased to classes assigned by decision rules r. This implies less accurate nonconformity scores which might decrease the informational efficiency.

The local strategy is proposed in this paper for the class-imbalanced problem above: the local nonconformity functions A_r are trained on local proper training sets T_r^t. Thus, if the sets T_r are split in a stratified manner, the functions A_r can be accurate on calibration sets

Algorithm 1: Conformal Decision Rule Learning

Input: Training set T, calibration set ratio $c \in (0, 1.0)$, and Boolean variable *local*;

Train point predictor h of ordered decision rules r on training set T;

for *each rule* $r \in h$ **do**

  Determine training subset $T_r \subseteq T$ covered by rule r and remove T_r from T;

  Randomly split T_r into proper training set T_r^t and calibration set T_r^c according to c;

end

if *local* **then**

  Train inductive conformal predictor ICP_r using T_r^t and T_r^c for each rule $r \in h$;

else

  $T^t := \bigcup_{r \in h} T_r^t$;

  Train inductive conformal predictor ICP_r using T^t and T_r^c for each rule $r \in h$;

end

Output: Point predictor h of decision rules r and set $\{ICP_r\}_{r \in h}$.

T_r^c. This implies more accurate nonconformity scores which can boost the informational efficiency. Practically this happens if sets T_r^t and T_r^c are not small; i.e. the local strategy has to be used for relatively large data T.

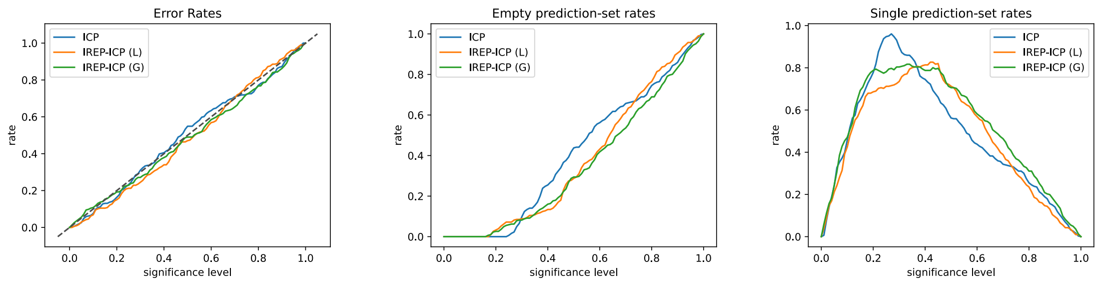
The classification procedure of conformal decision rules is simple: given test instance x, decision rules $r \in h$ are visited in the order imposed on h. If x matches the antecedent of rule r, it receives a class prediction associated with r, an explanation (of how x matches the antecedent), and a p-value for that prediction plus a prediction set $\Gamma^{\epsilon}(x)$ provided by ICP_r on a given significance level ϵ. We note that conformal decision rules are valid class set predictors. This is due to the fact that they are essentially Mondrian conformal predictors.

3. Preliminary Experiments

We experiment with pure ICP and conformal decision rules based on IREP with ICP denoted by IREP-ICP. IREP-ICP with the local (global) strategy is denoted by IREP-ICP(L) (IREP-ICP(G)). Standard settings are used for ICP and IREP-ICPs. The predictors are tested by 5-fold cross validation procedure using error rate e plus information efficiency metrics, rate r^e of empty prediction sets and rate r^s of single prediction sets.

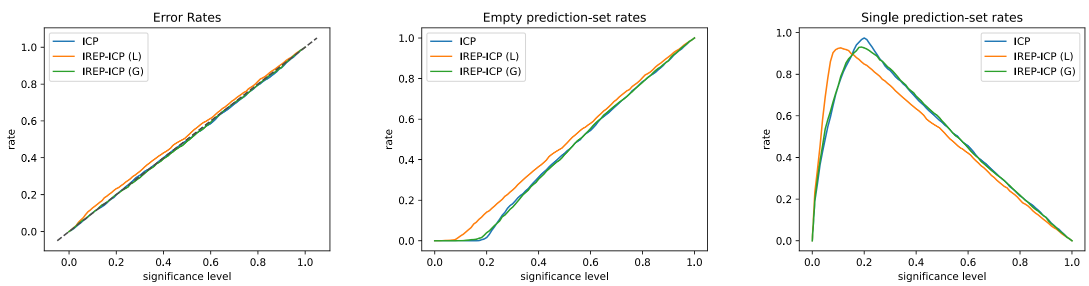
The results are given for the Haberman data and Spam base data. Figures 1(*a*) and 2(*a*) show that ICP, IREP-ICP(L) and IREP-ICP(G) are valid set predictors. Figures 1(*b*), 1(*c*), 2(*b*), and 2(*c*) show that the informational efficiency of ICP is usually better than that of IREP-ICP(L) and (G) since ICP employs all the data for the nonconformity function A and calibration. However, for some data (e.g. Spam base data) IREP-ICPs are better when decision-rule taxonomies make easier learning local nonconformity functions A_r.

Information efficiency of IREP-ICP(G) and (L) depends on the distance between the probability distributions that generate the global proper training set T^t and calibration sets T_r^c. When this distance is small (e.g. for the Haberman data) IREP-ICP(G) outperforms IREP-ICP(L). When this distance is big (e.g. for the Spam base data) IREP-ICP(L) outperforms IREP-ICP(G).



(a) Error rates (b) Empty prediction-set rates (c) Single prediction-set rates

Figure 1: Error Rate and Prediction-Set Size Plots for the Haberman Dataset



(a) Error rates (b) Empty prediction-set rates (c) Single prediction-set rates

Figure 2: Error Rate and Prediction-Set Size Plots for the Spambase Dataset

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