Better Parallel Prefix Computation Circuits

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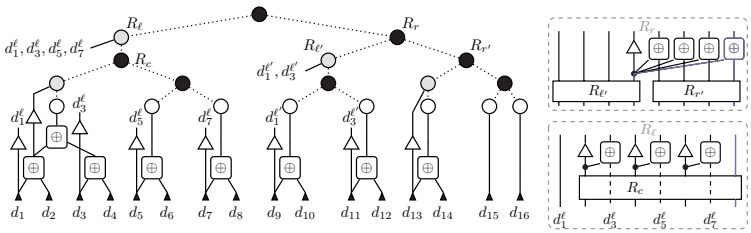

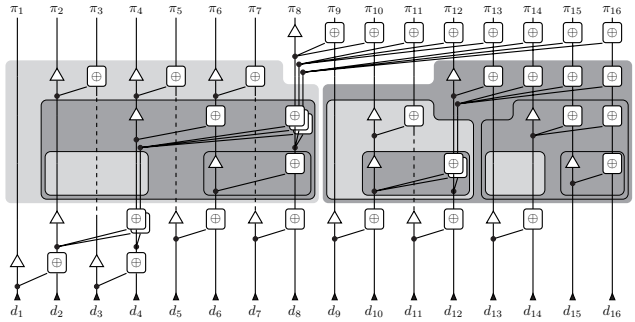

Parallel Prefix Computation (PPC)

Given:

- associative operator $\oplus\colon D \times D \to D$
- circuit C implementing $\oplus$
- inputs $d_i \in D$, $i \in \{1, \dots, n\}$

Compute:

- outputs $\pi_i = \bigoplus_{j=1}^{i} d_j$, $i \in \{1, \dots, n\}$

Goals:

- minimize size of circuit
- minimize delay of circuit

Simplifications:

- assign unit size to C, ignore buffers
- assign unit depth to C, assume buffers have smaller depth
- minimize depth and fan-out in lieu of delay

State of the Art and Our Results

Kogge and Stone:

- depth $\lceil \log n \rceil$ (optimal)
- size $n \log n$ (bad)
- fan-out 2 (optimal)

Ladner and Fischer (LF):

- depth $\lceil \log n \rceil$ (optimal)
- size $\approx 4n$ (asymptotically optimal)
- fan-out $\Theta(n)$ (bad)

Our Circuit:

- adds redundant gates to the LF construction to reduce fan-out
- achieves fan-out $F \geq 3$ by increasing size
- optimal depth and asymptotically optimal size
- can increase depth for smaller size (as for LF)

Applications: Adders, metastability-containing Gray code comparators

References

[KS] A Parallel Algorithm for the Efficient Solution of a General Class of Recurrence Equations. P. Kogge and S. Stone. Transactions on Computers, 1973.

[LF80] Parallel Prefix Computation. R. Ladner and M. Fischer. Journal of the ACM, 1980.

[BLM19] Optimal Metastability-Containing Sorting via Parallel Prefix Computation. J. Bund, C. Lenzen, and M. Medina. Transactions on Computers, 2019.

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