```
> with(linalg):
```

Warning, the protected names norm and trace have been redefined and unprotected

Maximum value of k in all calculations.

```
> maxk:=18:
```

Weakly semimagic homogeneous and affine 3x3 squares

We calculate the number of weakly semimagic squares: w[k] has upper bound x[i]<k, and wa[k] has magic sum k.

This is the raw data calculated by a simple method:

```
> for k from 1 to maxk do
    w[k]:=0:
    wa[k]:=0:
    for x[1] from 1 to (k-1) do
     for x[2] from 1 to (k-1) do
       for x[3] from 1 to (k-1) do
         rs:=x[1]+x[2]+x[3]:
         for x[4] from 1 to (k-1) do
           for x[5] from 1 to (k-1) do
>            x[6]:=rs-x[4]-x[5]:
             x[7]:=rs-x[1]-x[4]:
             x[8]:=rs-x[2]-x[5]:
             x[9]:=rs-x[3]-x[6]:
             if ( (x[6]>0) and (x[6]<k) and (x[7]>0) and (x[7]<k)
  and (x[8]>0) and (x[8]<k) and (x[9]>0) and (x[9]<k) ) then
>              w[k]:=w[k]+1:
               if (rs=k) then wa[k]:=wa[k]+1: fi:
>            fi:
           od:
>        od:
       od:
     od:
    od:
    print(k,w[k],wa[k]):
  od:
```

$$
\begin{array}{c}
1, 0, 0 \\
2, 1, 0 \\
3, 14, 1 \\
4, 87, 6 \\
5, 340, 21 \\
6, 1001, 55 \\
7, 2442, 120 \\
8, 5215, 231 \\
9, 10088, 406 \\
10, 18081, 666 \\
11, 30502, 1035 \\
12, 48983, 1540 \\
13, 75516, 2211 \\
14, 112489, 3081 \\
15, 162722, 4186 \\
16, 229503, 5565 \\
17, 316624, 7260 \\
18, 428417, 9316
\end{array} \tag{1}
$$

We expect a quasipolynomial for w (cubical) of degree 5 and a quasipolynomial for wa (affine) of degree 4. We don't know the period; the following calculations are set up to do any desired period. The variables:
p = assumed period of quasipolynomial,
r (1<=r<=p) = constituent residue,
deg = degree of polynomial, dp = deg+1.

The first step sets up the period and degree.

```
> p:=2;
```

$$p := 2 \quad (2)$$

```
> deg:=5;
  dp:=deg+1;
```

$$deg := 5 \quad (3)$$

$$dp := 6$$

Arrays to hold the coefficients of the cubical and affine polynomials. "coef" is a temporary working array.

```
> coef:=array(1..dp);
  wcoeff:=array(1..p,1..dp);
  wacoeff:=array(1..p,1..dp);
```

$$coef := array(1..6, [\,]) \quad (4)$$

$$wcoeff := array(1..2, 1..6, [\,])$$

$$wacoeff := array(1..2, 1..6, [\,])$$

The following procedure will generate all the p different weak polynomials and p different strong polynomials, factor them, and test by substituting the next value of the argument, comparing to the raw data of the surplus period that was calculated in the first procedure). The polynomials will be saved in "wpoly[r]" and "spoly[r]".

```
> for r from 1 to p do
```

The following procedure will generate the matrix of values for numbers mod r of the period for degree deg with any coefficients.

```
> V2:=array(1..dp,1..dp):
  V2a:=array(1..deg,1..deg):
> for n from 1 to dp do
    for k from 1 to dp do
>     V2[k,n]:=(p*(k-1)+r)^(n-1):
      if ( (k<dp) and (n<dp) ) then V2a[k,n]:=V2[k,n]: fi:
    od:
> od:
> print(V2);
```

This part assumes degree 3.

```
> print([w[r],w[r+p],w[r+2*p],w[r+3*p],w[r+4*p],w[r+5*p]]);
> coef:=linsolve(V2,[w[r],w[r+p],w[r+2*p],w[r+3*p],w[r+4*p],w
  [r+5*p]]);
> for j from 1 to dp do
>   wcoeff[r,j]:=coef[j]:
  od;
  print([wa[r],wa[r+p],wa[r+2*p],wa[r+3*p],wa[r+4*p],wa[r+5*p]]);
```

```
> coef:=linsolve(V2,[wa[r],wa[r+p],wa[r+2*p],wa[r+3*p],wa[r+4*p],
  wa[r+5*p]]);
> for j from 1 to dp do
>   wacoeff[r,j]:=coef[j]:
  od;

> wpoly[r]:=wcoeff[r,6]*x^5+wcoeff[r,5]*x^4+wcoeff[r,4]*x^3+
  wcoeff[r,3]*x^2+wcoeff[r,2]*x+wcoeff[r,1];
  subs(x=r+dp*p,wpoly[r]);
  factor(wpoly[r]);

> wapoly[r]:=wacoeff[r,6]*x^5+wacoeff[r,5]*x^4+wacoeff[r,4]*x^3+
  wacoeff[r,3]*x^2+wacoeff[r,2]*x+wacoeff[r,1];
  subs(x=r+dp*p,wapoly[r]);
  factor(wapoly[r]);

> od;
```

$$V2 := array(1..6, 1..6, [\,]) \quad \textbf{(5)}$$

$$V2a := array(1..5, 1..5, [\,])$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 & 81 & 243 \\ 1 & 5 & 25 & 125 & 625 & 3125 \\ 1 & 7 & 49 & 343 & 2401 & 16807 \\ 1 & 9 & 81 & 729 & 6561 & 59049 \\ 1 & 11 & 121 & 1331 & 14641 & 161051 \end{bmatrix}$$

$$[0, 14, 340, 2442, 10088, 30502]$$

$$coef := \begin{bmatrix} -1 & \frac{16}{5} & -\frac{9}{2} & \frac{7}{2} & -\frac{3}{2} & \frac{3}{10} \end{bmatrix}$$

$$[0, 1, 21, 120, 406, 1035]$$

$$coef := \begin{bmatrix} 1 & -\frac{9}{4} & \frac{15}{8} & -\frac{3}{4} & \frac{1}{8} & 0 \end{bmatrix}$$

$$wpoly_1 := \frac{3}{10} x^5 - \frac{3}{2} x^4 + \frac{7}{2} x^3 - \frac{9}{2} x^2 + \frac{16}{5} x - 1$$

$$75516$$

$$\frac{(x-1)\,(3\,x^2 - 6\,x + 5)\,(x^2 - 2\,x + 2)}{10}$$

$$wapoly_1 := 1 + \frac{1}{8} x^4 - \frac{3}{4} x^3 + \frac{15}{8} x^2 - \frac{9}{4} x$$

$$2211$$

$$\frac{(x-1)\,(x-2)\,(x^2-3\,x+4)}{8}$$

$$V2 := array(1..6, 1..6, [\ ])$$

$$V2a := array(1..5, 1..5, [\ ])$$

$$\begin{bmatrix} 1 & 2 & 4 & 8 & 16 & 32 \\ 1 & 4 & 16 & 64 & 256 & 1024 \\ 1 & 6 & 36 & 216 & 1296 & 7776 \\ 1 & 8 & 64 & 512 & 4096 & 32768 \\ 1 & 10 & 100 & 1000 & 10000 & 100000 \\ 1 & 12 & 144 & 1728 & 20736 & 248832 \end{bmatrix}$$

$$[1, 87, 1001, 5215, 18081, 48983]$$

$$coef := \begin{bmatrix} -1 & \frac{16}{5} & -\frac{9}{2} & \frac{7}{2} & -\frac{3}{2} & \frac{3}{10} \end{bmatrix}$$

$$[0, 6, 55, 231, 666, 1540]$$

$$coef := \begin{bmatrix} 1 & -\frac{9}{4} & \frac{15}{8} & -\frac{3}{4} & \frac{1}{8} & 0 \end{bmatrix}$$

$$wpoly_2 := \frac{3}{10}\,x^5 - \frac{3}{2}\,x^4 + \frac{7}{2}\,x^3 - \frac{9}{2}\,x^2 + \frac{16}{5}\,x - 1$$

$$112489$$

$$\frac{(x-1)\,(3\,x^2-6\,x+5)\,(x^2-2\,x+2)}{10}$$

$$wapoly_2 := 1 + \frac{1}{8}\,x^4 - \frac{3}{4}\,x^3 + \frac{15}{8}\,x^2 - \frac{9}{4}\,x$$

$$3081$$

$$\frac{(x-1)\,(x-2)\,(x^2-3\,x+4)}{8}$$

```
> for r from 1 to p do: r: wapoly[r]: 8*wapoly[r]: factor(wapoly
  [r]): od;
```

$$1 \qquad (6)$$

$$1 + \frac{1}{8}\,x^4 - \frac{3}{4}\,x^3 + \frac{15}{8}\,x^2 - \frac{9}{4}\,x$$

$$8 + x^4 - 6\,x^3 + 15\,x^2 - 18\,x$$

$$\frac{(x-1)\,(x-2)\,(x^2-3\,x+4)}{8}$$

$$2$$

$$1+\frac{1}{8}\,x^4-\frac{3}{4}\,x^3+\frac{15}{8}\,x^2-\frac{9}{4}\,x$$

$$8+x^4-6\,x^3+15\,x^2-18\,x$$

$$\frac{(x-1)\,(x-2)\,(x^2-3\,x+4)}{8}$$

```
> r:=0:
> for k from 1 to maxk do
>   r:=r+1:
>   if (r>p) then r:=1: fi:
>   print(k,r,eval(wapoly[r],x=k)-wa[k]):
> od:
```

$$1,\,1,\,0 \tag{7}$$

$$2,\,2,\,0$$

$$3,\,1,\,0$$

$$4,\,2,\,0$$

$$5,\,1,\,0$$

$$6,\,2,\,0$$

$$7,\,1,\,0$$

$$8,\,2,\,0$$

$$9,\,1,\,0$$

$$10,\,2,\,0$$

$$11,\,1,\,0$$

$$12,\,2,\,0$$

$$13,\,1,\,0$$

$$14,\,2,\,0$$

$$15,\,1,\,0$$

$$16,\,2,\,0$$

$$17,\,1,\,0$$

$$18,\,2,\,0$$

```
> for r from 1 to p do: r: wpoly[r]: 10*wpoly[r]: factor(wpoly[r]
): od;
```

$$1 \tag{8}$$

$$\frac{3}{10}x^5-\frac{3}{2}x^4+\frac{7}{2}x^3-\frac{9}{2}x^2+\frac{16}{5}x-1$$

$$3x^5-15x^4+35x^3-45x^2+32x-10$$

$$\frac{(x-1)(3x^2-6x+5)(x^2-2x+2)}{10}$$

$$2$$

$$\frac{3}{10}x^5-\frac{3}{2}x^4+\frac{7}{2}x^3-\frac{9}{2}x^2+\frac{16}{5}x-1$$

$$3x^5-15x^4+35x^3-45x^2+32x-10$$

$$\frac{(x-1)(3x^2-6x+5)(x^2-2x+2)}{10}$$

```
> r:=0:
  for k from 1 to maxk do
    r:=r+1:
    if (r>p) then r:=1: fi:
    print(k,r,eval(wpoly[r],x=k)-w[k]):
  od:
```

1, 1, 0 **(9)**

2, 2, 0

3, 1, 0

4, 2, 0

5, 1, 0

6, 2, 0

7, 1, 0

8, 2, 0

9, 1, 0

10, 2, 0

11, 1, 0

12, 2, 0

13, 1, 0

14, 2, 0

15, 1, 0

16, 2, 0

17, 1, 0

18, 2, 0