Semimagic generating functions and sequences

(general form, with affine data)

Notation:

S: semimagic squares (all positive values).

s: normalized squares (symmetry types).

R: reduced squares (least element is 0).

r: reduced normalized squares (reduced symmetry types).

n: semimagic r.

gf: generating function in some form.

gfsum: generating function as a sum of simple terms.

c: Cubic (fixed strict upper bound; weak upper bound for reduced).

a: Affine (fixed magic sum).

p: Period of the quasipolynomial (known from geometry). (Period of the truncated quasipolynomial, in the affine count.)
d: Dimension of the geometry = degree of the quasipolynomials.
RtoSfactor: the rational function that multiplies Rgf to Sgf and rgf to sgf.

enddegree: The number of terms desired in the sequences, from degree 1 (but initial zeros will be omitted).

```
> enddegree:=500;
```

$$enddegree := 500$$

This is for **affine**: set up main constants.

```
> d:=4; p:=120;
  RtoSfactor:=x^3/(1-x^3);
```

$$d := 4$$

$$p := 120$$

$$RtoSfactor := \frac{x^3}{1 - x^3}$$

We start by computing r_s=rsgf from the semimagic count. From the Latte results we get the closed Ehrhart g.f. of each flat, which depends on whether we're doing cubic or affine.

Set up the simplex data for the faces and intersection polytopes in the semimagic part of the magilatin series.

This is for **affine.**

```
> simplexname[1]:="OABC": ehrgf[1]:= 1/((1-x)*(1-x^2)^3) : dimen[1]:=3:
  simplexname[2]:="OEE2": ehrgf[2]:= 1/((1-x)*(1-x^4)^2) : dimen[2]:=2:
  simplexname[3]:="OAE2": ehrgf[3]:= 1/((1-x)*(1-x^2)*(1-x^4)) :
  dimen[3]:=2:
  simplexname[4]:="ADE2": ehrgf[4]:= 1/((1-x^2)*(1-x^3)*(1-x^4)) :
  dimen[4]:=2:
  simplexname[5]:="DE1E2": ehrgf[5]:= 1/((1-x^3)*(1-x^4)^2) : dimen[5]:=2:
  simplexname[6]:="OCE": ehrgf[6]:= 1/((1-x)*(1-x^2)*(1-x^4)) :
```

```
dimen[6]:=2:
simplexname[7]:="BDE1": ehrgf[7]:= 1/((1-x^2)*(1-x^3)*(1-x^4)) :
dimen[7]:=2:
simplexname[8]:="ABD": ehrgf[8]:= 1/((1-x^2)^2*(1-x^3)) : dimen[8]:=2:
simplexname[9]:="FG1": ehrgf[9]:= 1/((1-x^5)*(1-x^8)) : dimen[9]:=1:
simplexname[10]:="EF": ehrgf[10]:= 1/((1-x^4)*(1-x^5)) : dimen[10]:=1:
simplexname[11]:="OG": ehrgf[11]:= 1/((1-x)*(1-x^6)) : dimen[11]:=1:
simplexname[12]:="FG": ehrgf[12]:= 1/((1-x^5)*(1-x^6)) : dimen[12]:=1:
simplexname[13]:="AF": ehrgf[13]:= 1/((1-x^2)*(1-x^5)) : dimen[13]:=1:
simplexname[14]:="DG": ehrgf[14]:= 1/((1-x^3)*(1-x^6)) : dimen[14]:=1:
simplexname[15]:="DG2": ehrgf[15]:= 1/((1-x^3)*(1-x^8)) : dimen[15]:=1:
simplexname[16]:="DE": ehrgf[16]:= 1/((1-x^3)*(1-x^4)) : dimen[16]:=1:
simplexname[17]:="H": ehrgf[17] := 1/(1-x^7) : dimen[17]:=0:
# for n from 1 to 17 do print(simplexname[n], dimen[n], ehrgf[n]); od;
```

The closed E.g.f. is converted to the open E.g.f. The first step is to compute the Mobius function of the intersection poset.

```
> for n from 1 to 17 do
    mu[n]:=(-1)^(dimen[1]-dimen[n]):
  od:
  mu[14]:=2*mu[14]:
  for n from 1 to 17 do
    openehrgf[n]:=simplify(-(-1)^dimen[n]*subs(x=1/x,ehrgf[n])):
  od:
```

Get the g.f. of reduced, normalized squares.

```
> for n from 1 to 17 do
    rgfterm[n]:=openehrgf[n]:
  od:
  rgfsum:=sum(mu[nn]*rgfterm[nn],nn=1..17):
  rgf:=simplify(rgfsum);
```

$$rgf := \frac{1}{(x^7-1)(x^6-1)(x^8-1)(x^5-1)(x^2+x+1)(x^3+x^2+x+1)}(x^{12}(18x^{19}+41x^{18}+79x^{17}+117x^{16}+166x^{15}+207x^{14}+249x^{13}+268x^{12}+274x^{11}+258x^{10}+233x^9+192x^8+152x^7+109x^6+73x^5+44x^4+24x^3+11x^2+4x+1))$$

Get the g.f. of reduced squares.

```
> Rgfsum:=72*rgfsum:
  Rgf:=simplify(Rgfsum);
```

$$Rgf := \frac{1}{(x^7-1)(x^6-1)(x^8-1)(x^5-1)(x^2+x+1)(x^3+x^2+x+1)}(72x^{12}(18x^{19}+41x^{18}+79x^{17}+117x^{16}+166x^{15}+207x^{14}+249x^{13}+268x^{12}+274x^{11}+258x^{10}+233x^9+192x^8+152x^7+109x^6+73x^5+44x^4+24x^3+11x^2+4x+1))$$

Hence S, the g.f. of the number of semimagic squares, equals

```
> Sgf:=simplify(RtoSfactor*Rgf);
```

$$Sgf := -\frac{1}{(x^3-1)(x^7-1)(x^6-1)(x^8-1)(x^5-1)(x^2+x+1)(x^3+x^2+x+1)}(72x^{15}(18x^{19}+41x^{18}+79x^{17}+117x^{16}+166x^{15}+207x^{14}+249x^{13}+268x^{12}+274x^{11}+258x^{10}+233x^9+192x^8+152x^7+109x^6+73x^5+44x^4+24x^3+11x^2+4x+1))$$

The g.f. of the total number of symmetry types, l_ml ("lgf"):

```
> sgf:=simplify(RtoSfactor*rgf);
```

$$sgf := -\frac{1}{(x^3-1)(x^7-1)(x^6-1)(x^8-1)(x^5-1)(x^2+x+1)(x^3+x^2+x+1)}(x^{15}(18x^{19}+41x^{18}+79x^{17}+117x^{16}+166x^{15}+207x^{14}+249x^{13}+268x^{12}+274x^{11}+258x^{10}+233x^9+192x^8+152x^7+109x^6+73x^5+44x^4+24x^3+11x^2+4x+1))$$

Generate the labelled sequence of magilatin square numbers of all four kinds. The first step is to compute the degree of the first non-zero term.

```
> Sgfdegree:=ldegree(numer(Sgf),x);
  Rgfdegree:=ldegree(numer(Rgf),x);
  sgfdegree:=ldegree(numer(sgf),x);
  rgfdegree:=ldegree(numer(rgf),x);
```

$$Sgfdegree := 15$$

$$Rgfdegree := 12$$

$$sgfdegree := 15$$

$$rgfdegree := 12$$

Generate the series expansions of the g.f.'s.

Expressing the rational function with standard denominator gives an orders-of-magnitude speedup in the series expansion.

Standard denominator (1-x^p)^{d+1}.

```
> pdenom:=(1-x^p):
  standenom:=pdenom^(d+1);
```

$$standenom := (1-x^{120})^5$$

G.f. as rational function with standard denominator.

```
> Sgfstandnum:=simplify(numer(Sgf)*standenom/denom(Sgf)):
  Sgf:=Sgfstandnum/standenom;
```

$$Sgf := \frac{1}{(x^6+x^5+x^4+x^3+x^2+x+1)(1-x^{120})^5}(72(1-x+x^3-x^4+x^8+x^6-x^7+x^{11}-x^{13}+x^{24}+x^{30}-x^{25}+x^{27}+x^{14}-x^{10}-x^{31}-x^{28}-x^{37}+x^{32}-x^{34}+x^{35}+x^{110}-x^{109}+x^{107}-x^{106}+x^{104}-x^{103}+x^{102}-x^{100}+x^{99}-x^{97}+x^{96}+x^{86}-x^{85}-x^{79}+x^{78}+x^{80}+x^{75}-x^{73}-x^{76}+x^{72}-x^{82}+x^{83}+x^{59}+x^{56}-x^{55}+x^{54}-x^{52}+x^{51}-x^{58}-x^{49}+x^{38}+x^{48}+x^{62}-x^{61})(1+x+x^3+x^2+x^4+x^5+x^8+x^6+x^7+x^9+x^{11}+x^{13}+x^{16}+x^{12}+x^{24}+x^{19}+x^{21}+x^{20}+x^{15}+x^{17}+x^{30}+x^{22}+x^{25}+x^{27}+x^{14}+x^{26}+x^{23}+x^{10}+x^{18}+x^{31}+x^{28}+x^{37}+x^{29}+x^{32}+x^{34}+x^{33}+x^{35}+x^{36}+x^{119}+x^{117}+x^{118}+x^{116}+x^{115}+x^{114}+x^{113}+x^{112}+x^{111}+x^{110}+x^{109}+x^{108}+x^{107}+x^{106}+x^{105}+x^{104}+x^{103}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{97}+x^{96}+x^{95}+x^{94}+x^{93}+x^{92}+x^{91}+x^{90}+x^{89}+x^{88}+x^{87}+x^{86}+x^{85}+x^{79}+x^{78}+x^{81}+x^{80}+x^{77}+x^{75}+x^{74}+x^{73}+x^{76}+x^{72}+x^{70}+x^{69}+x^{68}+x^{67}+x^{66}+x^{71}+x^{84}+x^{82}+x^{83}+x^{59}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{58}+x^{49}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{48}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61})$$

$$+x^{60})^2 (1 - x + x^2 + x^8 + x^6 - x^7 - x^{13} + x^{12} + x^{24} - x^{19} + x^{20} + x^{30} - x^{25} + x^{14} + x^{26} + x^{18} - x^{31} - x^{37} + x^{32} + x^{36} + x^{116} - x^{115} + x^{114} + x^{110} - x^{109} + x^{108} + x^{104} - x^{103} + x^{102} + x^{98} - x^{97} + x^{96} + x^{92} - x^{91} + x^{90} + x^{86} - x^{85} - x^{79} + x^{78} + x^{80} + x^{74} - x^{73} + x^{72} + x^{68} - x^{67} + x^{66} + x^{84} + x^{56} - x^{55} + x^{54} + x^{50} - x^{49} + x^{44} - x^{43} + x^{42} + x^{38} + x^{48} + x^{62} - x^{61} + x^{60}) (1 - x + x^3 - x^2 + x^4 + x^8 - x^6 - x^7 + x^9 - x^{11} + x^{24} + x^{20} + x^{15} - x^{17} - x^{22} - x^{26} - x^{31} + x^{28} - x^{37} + x^{33} + x^{108} - x^{107} - x^{106} + x^{105} + x^{104} - x^{102} - x^{101} + x^{100} + x^{99} - x^{97} + x^{93} - x^{91} + x^{88} - x^{86} + x^{80} - x^{77} + x^{75} + x^{69} + x^{68} - x^{67} - x^{66} - x^{71} + x^{84} - x^{82} - x^{47} - x^{46} + x^{45} + x^{44} - x^{42} - x^{41} + x^{40} + x^{39} + x^{48} + x^{64} + x^{63} - x^{62} - x^{61} + x^{60}) x^{15} (18 x^{19} + 41 x^{18} + 79 x^{17} + 117 x^{16} + 166 x^{15} + 207 x^{14} + 249 x^{13} + 268 x^{12} + 274 x^{11} + 258 x^{10} + 233 x^9 + 192 x^8 + 152 x^7 + 109 x^6 + 73 x^5 + 44 x^4 + 24 x^3 + 11 x^2 + 4 x + 1))$$

G.f. as rational function with standard denominator.

```
> Rgfstandnum:=simplify(numer(Rgf)*standenom/denom(Rgf)):
  Rgf:=Rgfstandnum/standenom;
```

$$Rgf := -\frac{1}{(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)(1 - x^{120})^5} (72 (18 x^{19} + 41 x^{18} + 79 x^{17} + 117 x^{16} + 166 x^{15} + 207 x^{14} + 249 x^{13} + 268 x^{12} + 274 x^{11} + 258 x^{10} + 233 x^9 + 192 x^8 + 152 x^7 + 109 x^6 + 73 x^5 + 44 x^4 + 24 x^3 + 11 x^2 + 4 x + 1) x^{12} (1 - x + x^3 - x^4 + x^8 + x^6 - x^7 + x^{11} - x^{13} + x^{24} + x^{30} - x^{25} + x^{27} + x^{14} - x^{10} - x^{31} - x^{28} - x^{37} + x^{32} - x^{34} + x^{35} + x^{110} - x^{109} + x^{107} - x^{106} + x^{104} - x^{103} + x^{102} - x^{100} + x^{99} - x^{97} + x^{96} + x^{86} - x^{85} - x^{79} + x^{78} + x^{80} + x^{75} - x^{73} - x^{76} + x^{72} - x^{82} + x^{83} + x^{59} + x^{56} - x^{55} + x^{54} - x^{52} + x^{51} - x^{58} - x^{49} + x^{38} + x^{48} + x^{62} - x^{61}) (1 + x + x^3 + x^2 + x^4 + x^5 + x^8 + x^6 + x^7 + x^9 + x^{11} + x^{13} + x^{16} + x^{12} + x^{24} + x^{19} + x^{21} + x^{20} + x^{15} + x^{17} + x^{30} + x^{22} + x^{25} + x^{27} + x^{14} + x^{26} + x^{23} + x^{10} + x^{18} + x^{31} + x^{28} + x^{37} + x^{29} + x^{32} + x^{34} + x^{33} + x^{35} + x^{36} + x^{119} + x^{117} + x^{118} + x^{116} + x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{79} + x^{78} + x^{81} + x^{80} + x^{77} + x^{75} + x^{74} + x^{73} + x^{76} + x^{72} + x^{70} + x^{69} + x^{68} + x^{67} + x^{66} + x^{71} + x^{84} + x^{82} + x^{83} + x^{59} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{58} + x^{49} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{38} + x^{48} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60})^2 (-1 + x + x^3 - x^2 - x^4 + x^5 - x^8 - x^6 + x^7 + x^9 + x^{11} + x^{13} - x^{16} - x^{12} - x^{24} + x^{19} + x^{21} - x^{20} + x^{15} + x^{17} - x^{30} - x^{22} + x^{25} + x^{27} - x^{14} - x^{26} + x^{23} - x^{10} - x^{18} + x^{31} - x^{28} + x^{37} + x^{29} - x^{32} - x^{34} + x^{33} + x^{35} - x^{36} + x^{119} + x^{117} - x^{118} - x^{116} + x^{115} - x^{114} + x^{113} - x^{112} + x^{111} - x^{110} + x^{109} - x^{108} + x^{107} - x^{106} + x^{105} - x^{104} + x^{103} - x^{102} + x^{101} - x^{100} + x^{99} - x^{98} + x^{97} - x^{96} + x^{95} - x^{94} + x^{93} - x^{92} + x^{91} - x^{90} + x^{89} - x^{88} + x^{87} - x^{86} + x^{85} + x^{79} - x^{78} + x^{81} - x^{80} + x^{77} + x^{75} - x^{74} + x^{73} - x^{76} - x^{72} - x^{70} + x^{69} - x^{68} + x^{67} - x^{66} + x^{71} - x^{84} - x^{82} + x^{83} + x^{59} + x^{57} - x^{56} + x^{55} - x^{54} + x^{53} - x^{52} + x^{51} - x^{50} - x^{58} + x^{49} + x^{47} - x^{46} + x^{45} - x^{44} + x^{43} - x^{42} + x^{41} - x^{40} + x^{39} - x^{38} - x^{48} + x^{65} - x^{64} + x^{63} - x^{62} + x^{61} - x^{60}) (1 - x + x^3 - x^2 + x^4 + x^8 - x^6 - x^7 + x^9 - x^{11} + x^{24} + x^{20} + x^{15} - x^{17} - x^{22} - x^{26} - x^{31} + x^{28} - x^{37} + x^{33} + x^{108} - x^{107} - x^{106} + x^{105} + x^{104} - x^{102} - x^{101} + x^{100} + x^{99} - x^{97} + x^{93} - x^{91} + x^{88} - x^{86} + x^{80} - x^{77} + x^{75} + x^{69} + x^{68} - x^{67} - x^{66} - x^{71} + x^{84} - x^{82} - x^{47} - x^{46} + x^{45} + x^{44} - x^{42} - x^{41} + x^{40} + x^{39} + x^{48} + x^{64} + x^{63} - x^{62} - x^{61} + x^{60}))$$

G.f. as rational function with standard denominator.

```
> sgfstandnum:=simplify(numer(sgf)*standenom/denom(sgf)):
  sgf:=sgfstandnum/standenom;
```

$$sgf := \frac{1}{(x^6+x^5+x^4+x^3+x^2+x+1)\,(1-x^{120})^5}((1-x+x^3-x^4+x^8+x^6-x^7+x^{11}-x^{13}+x^{24}+x^{30}$$
$$-x^{25}+x^{27}+x^{14}-x^{10}-x^{31}-x^{28}-x^{37}+x^{32}-x^{34}+x^{35}+x^{110}-x^{109}+x^{107}-x^{106}+x^{104}-x^{103}$$
$$+x^{102}-x^{100}+x^{99}-x^{97}+x^{96}+x^{86}-x^{85}-x^{79}+x^{78}+x^{80}+x^{75}-x^{73}-x^{76}+x^{72}-x^{82}+x^{83}+x^{59}$$
$$+x^{56}-x^{55}+x^{54}-x^{52}+x^{51}-x^{58}-x^{49}+x^{38}+x^{48}+x^{62}-x^{61})\,(1+x+x^3+x^2+x^4+x^5+x^8+x^6$$
$$+x^7+x^9+x^{11}+x^{13}+x^{16}+x^{12}+x^{24}+x^{19}+x^{21}+x^{20}+x^{15}+x^{17}+x^{30}+x^{22}+x^{25}+x^{27}+x^{14}$$
$$+x^{26}+x^{23}+x^{10}+x^{18}+x^{31}+x^{28}+x^{37}+x^{29}+x^{32}+x^{34}+x^{33}+x^{35}+x^{36}+x^{119}+x^{117}+x^{118}$$
$$+x^{116}+x^{115}+x^{114}+x^{113}+x^{112}+x^{111}+x^{110}+x^{109}+x^{108}+x^{107}+x^{106}+x^{105}+x^{104}+x^{103}$$
$$+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{97}+x^{96}+x^{95}+x^{94}+x^{93}+x^{92}+x^{91}+x^{90}+x^{89}+x^{88}+x^{87}$$
$$+x^{86}+x^{85}+x^{79}+x^{78}+x^{81}+x^{80}+x^{77}+x^{75}+x^{74}+x^{73}+x^{76}+x^{72}+x^{70}+x^{69}+x^{68}+x^{67}$$
$$+x^{66}+x^{71}+x^{84}+x^{82}+x^{83}+x^{59}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{58}+x^{49}$$
$$+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{48}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}$$
$$+x^{60})^2\,(1-x+x^2+x^8+x^6-x^7-x^{13}+x^{12}+x^{24}-x^{19}+x^{20}+x^{30}-x^{25}+x^{14}+x^{26}+x^{18}-x^{31}$$
$$-x^{37}+x^{32}+x^{36}+x^{116}-x^{115}+x^{114}+x^{110}-x^{109}+x^{108}+x^{104}-x^{103}+x^{102}+x^{98}-x^{97}+x^{96}$$
$$+x^{92}-x^{91}+x^{90}+x^{86}-x^{85}-x^{79}+x^{78}+x^{80}+x^{74}-x^{73}+x^{72}+x^{68}-x^{67}+x^{66}+x^{84}+x^{56}-x^{55}$$
$$+x^{54}+x^{50}-x^{49}+x^{44}-x^{43}+x^{42}+x^{38}+x^{48}+x^{62}-x^{61}+x^{60})\,(1-x+x^3-x^2+x^4+x^8-x^6-x^7$$
$$+x^9-x^{11}+x^{24}+x^{20}+x^{15}-x^{17}-x^{22}-x^{26}-x^{31}+x^{28}-x^{37}+x^{33}+x^{108}-x^{107}-x^{106}+x^{105}$$
$$+x^{104}-x^{102}-x^{101}+x^{100}+x^{99}-x^{97}+x^{93}-x^{91}+x^{88}-x^{86}+x^{80}-x^{77}+x^{75}+x^{69}+x^{68}-x^{67}$$
$$-x^{66}-x^{71}+x^{84}-x^{82}-x^{47}-x^{46}+x^{45}+x^{44}-x^{42}-x^{41}+x^{40}+x^{39}+x^{48}+x^{64}+x^{63}-x^{62}-x^{61}$$
$$+x^{60})\,x^{15}\,(18\,x^{19}+41\,x^{18}+79\,x^{17}+117\,x^{16}+166\,x^{15}+207\,x^{14}+249\,x^{13}+268\,x^{12}+274\,x^{11}$$
$$+258\,x^{10}+233\,x^9+192\,x^8+152\,x^7+109\,x^6+73\,x^5+44\,x^4+24\,x^3+11\,x^2+4\,x+1))$$

G.f. as rational function with standard denominator.

```
> rgfstandnum:=simplify(numer(rgf)*standenom/denom(rgf)):
  rgf:=rgfstandnum/standenom;
```

$$rgf := -\frac{1}{(x^6+x^5+x^4+x^3+x^2+x+1)\,(1-x^{120})^5}((18\,x^{19}+41\,x^{18}+79\,x^{17}+117\,x^{16}+166\,x^{15}$$
$$+207\,x^{14}+249\,x^{13}+268\,x^{12}+274\,x^{11}+258\,x^{10}+233\,x^9+192\,x^8+152\,x^7+109\,x^6+73\,x^5$$
$$+44\,x^4+24\,x^3+11\,x^2+4\,x+1)\,x^{12}\,(1-x+x^3-x^4+x^8+x^6-x^7+x^{11}-x^{13}+x^{24}+x^{30}-x^{25}$$
$$+x^{27}+x^{14}-x^{10}-x^{31}-x^{28}-x^{37}+x^{32}-x^{34}+x^{35}+x^{110}-x^{109}+x^{107}-x^{106}+x^{104}-x^{103}+x^{102}$$
$$-x^{100}+x^{99}-x^{97}+x^{96}+x^{86}-x^{85}-x^{79}+x^{78}+x^{80}+x^{75}-x^{73}-x^{76}+x^{72}-x^{82}+x^{83}+x^{59}+x^{56}$$
$$-x^{55}+x^{54}-x^{52}+x^{51}-x^{58}-x^{49}+x^{38}+x^{48}+x^{62}-x^{61})\,(1+x+x^3+x^2+x^4+x^5+x^8+x^6+x^7$$
$$+x^9+x^{11}+x^{13}+x^{16}+x^{12}+x^{24}+x^{19}+x^{21}+x^{20}+x^{15}+x^{17}+x^{30}+x^{22}+x^{25}+x^{27}+x^{14}$$
$$+x^{26}+x^{23}+x^{10}+x^{18}+x^{31}+x^{28}+x^{37}+x^{29}+x^{32}+x^{34}+x^{33}+x^{35}+x^{36}+x^{119}+x^{117}+x^{118}$$
$$+x^{116}+x^{115}+x^{114}+x^{113}+x^{112}+x^{111}+x^{110}+x^{109}+x^{108}+x^{107}+x^{106}+x^{105}+x^{104}+x^{103}$$
$$+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{97}+x^{96}+x^{95}+x^{94}+x^{93}+x^{92}+x^{91}+x^{90}+x^{89}+x^{88}+x^{87}$$
$$+x^{86}+x^{85}+x^{79}+x^{78}+x^{81}+x^{80}+x^{77}+x^{75}+x^{74}+x^{73}+x^{76}+x^{72}+x^{70}+x^{69}+x^{68}+x^{67}$$
$$+x^{66}+x^{71}+x^{84}+x^{82}+x^{83}+x^{59}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{58}+x^{49}$$
$$+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{48}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}$$
$$+x^{60})^2\,(-1+x+x^3-x^2-x^4+x^5-x^8-x^6+x^7+x^9+x^{11}+x^{13}-x^{16}-x^{12}-x^{24}+x^{19}+x^{21}-x^{20}$$
$$+x^{15}+x^{17}-x^{30}-x^{22}+x^{25}+x^{27}-x^{14}-x^{26}+x^{23}-x^{10}-x^{18}+x^{31}-x^{28}+x^{37}+x^{29}-x^{32}-x^{34}$$
$$+x^{33}+x^{35}-x^{36}+x^{119}+x^{117}-x^{118}-x^{116}+x^{115}-x^{114}+x^{113}-x^{112}+x^{111}-x^{110}+x^{109}-x^{108}$$

$$+x^{107}-x^{106}+x^{105}-x^{104}+x^{103}-x^{102}+x^{101}-x^{100}+x^{99}-x^{98}+x^{97}-x^{96}+x^{95}-x^{94}+x^{93}-x^{92}+x^{91}-x^{90}+x^{89}-x^{88}+x^{87}-x^{86}+x^{85}+x^{79}-x^{78}+x^{81}-x^{80}+x^{77}+x^{75}-x^{74}+x^{73}-x^{76}-x^{72}-x^{70}+x^{69}-x^{68}+x^{67}-x^{66}+x^{71}-x^{84}-x^{82}+x^{83}+x^{59}+x^{57}-x^{56}+x^{55}-x^{54}+x^{53}-x^{52}+x^{51}-x^{50}-x^{58}+x^{49}+x^{47}-x^{46}+x^{45}-x^{44}+x^{43}-x^{42}+x^{41}-x^{40}+x^{39}-x^{38}-x^{48}+x^{65}-x^{64}+x^{63}-x^{62}+x^{61}-x^{60})\,(1-x+x^{3}-x^{2}+x^{4}+x^{8}-x^{6}-x^{7}+x^{9}-x^{11}+x^{24}+x^{20}+x^{15}-x^{17}-x^{22}-x^{26}-x^{31}+x^{28}-x^{37}+x^{33}+x^{108}-x^{107}-x^{106}+x^{105}+x^{104}-x^{102}-x^{101}+x^{100}+x^{99}-x^{97}+x^{93}-x^{91}+x^{88}-x^{86}+x^{80}-x^{77}+x^{75}+x^{69}+x^{68}-x^{67}-x^{66}-x^{71}+x^{84}-x^{82}-x^{47}-x^{46}+x^{45}+x^{44}-x^{42}-x^{41}+x^{40}+x^{39}+x^{48}+x^{64}+x^{63}-x^{62}-x^{61}+x^{60}))$$

Expand as a series to find the first few values of the number of squares and symmetry types.

```
> Sseries:=series(Sgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

```
> sseries:=series(sgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

Expand as a series to find the first few values of the number of reduced squares and reduced symmetry types.

```
> Rseries:=series(Rgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

```
> rseries:=series(rgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

Find the counting sequences

List the coefficients of each series, i.e., the terms of the counting sequences.
The comment symbol # is for controlling the output. With large "enddegree" the output is huge so it's more convenient to run each sequence's output separately and copy it from the worksheet.

```
> for n from Sgfdegree to enddegree do
   co:=coeff(Sseries,x,n):
   printf("%d  %d \n",n,co);
  od:
  print("Coefficients complete.",n,co);
```

```
15  72
16  144
17  288
18  576
19  864
20  1440
21  2088
22  3024
23  3888
24  5904
25  6984
26  9432
```

27	12168
28	14904
29	17928
30	23832
31	26784
32	33048
33	39672
34	46584
35	53640
36	65592
37	72504
38	85248
39	98928
40	111816
41	125208
42	147528
43	160632
44	182808
45	206424
46	229176
47	252648
48	287928
49	310752
50	347040
51	384480
52	420120
53	457200
54	511488
55	547704
56	601344
57	657504
58	711504
59	766872
60	844128
61	897984
62	975672
63	1055736
64	1131984
65	1210752
66	1318104
67	1395072
68	1500912
69	1610784
70	1716624
71	1824624
72	1967184
73	2072592
74	2215224
75	2361384
76	2501136
77	2645064
78	2831688
79	2972448
80	3156624

81 3346200
82 3529368
83 3715920
84 3952728
85 4135608
86 4371264
87 4612392
88 4844088
89 5081400
90 5378976
91 5611608
92 5905080
93 6205896
94 6497136
95 6793704
96 7158456
97 7449336
98 7812144
99 8182080
100 8539128
101 8903592
102 9348480
103 9706536
104 10145592
105 10594368
106 11029680
107 11472120
108 12004200
109 12439296
110 12967992
111 13505904
112 14026896
113 14557392
114 15191352
115 15713496
116 16339536
117 16977888
118 17598456
119 18228528
120 18972432
121 19592496
122 20331216
123 21081528
124 21810096
125 22550976
126 23420808
127 24150816
128 25010496
129 25885296
130 26737848
131 27601776
132 28606896
133 29458728
134 30456576

135	31468968
136	32454000
137	33454296
138	34612272
139	35598960
140	36744408
141	37907784
142	39043440
143	40193424
144	41514552
145	42649560
146	43960968
147	45290016
148	46585800
149	47899944
150	49403448
151	50701032
152	52188984
153	53698248
154	55174032
155	56667096
156	58363848
157	59838840
158	61523496
159	63229104
160	64895112
161	66582576
162	68494104
163	70161768
164	72054504
165	73972512
166	75849912
167	77748264
168	79886016
169	81762768
170	83885616
171	86032656
172	88133328
173	90259056
174	92646144
175	94748832
176	97113600
177	99507600
178	101854080
179	104225040
180	106874064
181	109219680
182	111850560
183	114509592
184	117114480
185	119748528
186	122684112
187	125291088
188	128200608

189 131143752
190 134031600
191 136947600
192 140183280
193 143070120
194 146284200
195 149530680
196 152714664
197 155932200

198 159494184
199 162680544
200 166212792
201 169782696
202 173289384
203 176828328
204 180731304
205 184236984
206 188114400
207 192029040
208 195872184
209 199753416
210 204025464
211 207870984
212 212108616
213 216388872
214 220596984
215 224841960
216 229497912
217 233704944
218 238331520
219 242999856
220 247587624
221 252218160
222 257288256
223 261878472
224 266909760
225 271988784
226 276986016
227 282024936
228 287525088
229 292521312
230 297987768
231 303500952
232 308923632
233 314394048
234 320356008
235 325781424
236 331699392
237 337670784
238 343550304
239 349476624
240 355917168
241 361795392

242	368197704
243	374651784
244	381004992
245	387411624
246	394363944
247	400720032
248	407623392
249	414585864
250	421446024
251	428357952
252	435840264
253	442698984
254	450137952
255	457634664
256	465019128
257	472462632
258	480509568
259	487897128
260	495889704
261	503947080
262	511891056
263	519892632
264	528523128
265	536465736
266	545047776
267	553693392
268	562215096
269	570801960
270	580052736
271	588577608
272	597768120
273	607029696
274	616166208
275	625366152
276	635256504
277	644391648
278	654228072
279	664134480
280	673904880
281	683746272
282	694315080
283	704088504
284	714591072
285	725171328
286	735613848
287	746126352
288	757393632
289	767834784
290	779042592
291	790326504
292	801461664
293	812674512
294	824680872
295	835819488

296	847753056
297	859771296
298	871639128
299	883583136
300	896349600
301	908215848
302	920916576
303	933700248
304	946321776
305	959027976
306	972596448
307	985221504
308	998710776
309	1012291848
310	1025708976
311	1039208976
312	1053601848
313	1067017176
314	1081337976
315	1095748848
316	1109983320
317	1124309736
318	1139570280
319	1153808712
320	1168983288
321	1184256792
322	1199352672
323	1214538264
324	1230689880
325	1245783960
326	1261856808
327	1278027216
328	1294006680
329	1310085504
330	1327173480
331	1343156760
332	1360151352
333	1377252720
334	1394161560
335	1411167816
336	1429215840
337	1446122880
338	1464085296
339	1482152832
340	1500014592
341	1517983056
342	1537038432
343	1554904080
344	1573858656
345	1592928000
346	1611789552
347	1630756224
348	1650843504
349	1669703328

350	1689698016
351	1709805528
352	1729691568
353	1749692304
354	1770860736
355	1790750880
356	1811810448
357	1832993352
358	1853952480
359	1875024720
360	1897299504
361	1918256760
362	1940431320
363	1962726840
364	1984784760
365	2006965800
366	2030397480
367	2052459648
368	2075774760
369	2099221560
370	2122428600
371	2145756600
372	2170371960
373	2193576840
374	2218084128
375	2242721232
376	2267103384
377	2291617368
378	2317468536
379	2341855224
380	2367581400
381	2393447976
382	2419057944
383	2444797368
384	2471911704
385	2497519584
386	2524517712
387	2551654368
388	2578518792
389	2605524048
390	2633955552
391	2660824584
392	2689122528
393	2717569872
394	2745743184
395	2774054808
396	2803831632
397	2832002784
398	2861655048
399	2891455128
400	2920964976
401	2950624368
402	2981802456
403	3011316912

404	3042352368
405	3073546944
406	3104449056
407	3135498768
408	3168106848
409	3199006800
410	3231482040
411	3264113880
412	3296437200
413	3328919640
414	3363015528
415	3395343744
416	3429287712
417	3463400664
418	3497202504
419	3531161232
420	3566774808
421	3600574344
422	3636045792
423	3671683704
424	3706993944
425	3742472952
426	3779663184
427	3814978392
428	3852007272
429	3889215144
430	3926092752
431	3963136824
432	4001934744
433	4038809832
434	4077456624
435	4116279744
436	4154755176
437	4193409672
438	4233876048
439	4272356808
440	4312652184
441	4353136560
442	4393271088
443	4433581800
444	4475748456
445	4515880608
446	4557886344
447	4600078992
448	4641903648
449	4683917520
450	4727847096
451	4769676936
452	4813425360
453	4857373296
454	4900950936
455	4944715344
456	4990440096
457	5034015216

```
458  5079569472
459  5125321080
460  5170683888
461  5216246352
462  5263831080
463  5309199216
464  5356592352
465  5404196016
466  5451408360
467  5498817840
468  5548295520
469  5595505416
470  5644802592
471  5694307704
472  5743402704
473  5792708232
474  5844145392
475  5893245936
476  5944480776
477  5995937448
478  6046981056
479  6098232816
480  6151663512
481  6202704456
482  6255944136
483  6309402768
484  6362429256
485  6415677576
486  6471169416
487  6524201736
488  6579480456
489  6634992456

490  6690069648
491  6745365576
492  6802954056
493  6858028296
494  6915415176
495  6973032528
496  7030194840
497  7087590576
498  7147345032
499  7204513320
500  7264043496

                                  "Coefficients complete.", 501, 7264043496
```

```
> for n from Rgfdegree to enddegree do
   co:=coeff(Rseries,x,n):
   printf("%d  %d \n",n,co);
  od:
  print("Coefficients complete.",n,co);
12  72
```

13	144
14	288
15	504
16	720
17	1152
18	1512
19	2160
20	2448
21	3816
22	3960
23	5544
24	6264
25	7920
26	8496
27	11664
28	11880
29	15120
30	15840
31	19800
32	20592
33	25920
34	25920
35	31608
36	33336
37	39312
38	39960
39	48600
40	48816
41	57600
42	58896
43	68544
44	69840
45	81504
46	81576
47	94392
48	96552
49	109368
50	110160
51	127008
52	127584
53	144144
54	146016
55	163800
56	165528
57	186624
58	186480
59	208800
60	211608
61	234000
62	235080
63	262368
64	263088
65	290160
66	292680

67 321552
68 323712
69 356400
70 355968
71 390600
72 394200
73 428544
74 429840
75 470304
76 471312
77 511560
78 514512
79 556920
80 559296
81 606528
82 606240
83 655344
84 659664
85 708480
86 710136
87 766584
88 767520
89 823680
90 826920
91 885528
92 888624
93 952560
94 952200
95 1018440
96 1023624
97 1089792
98 1091448
99 1166400
100 1167408
101 1242000
102 1245888
103 1323144
104 1326528
105 1409832
106 1409616
107 1495872
108 1501704
109 1587600
110 1589400
111 1685448
112 1686600
113 1782144
114 1786536
115 1884960
116 1888992
117 1994544
118 1994040
119 2102688
120 2109096

121	2217600
122	2219760
123	2339280
124	2340720
125	2459520
126	2464488
127	2587032
128	2591280
129	2721600
130	2720880
131	2854800
132	2862072
133	2995272
134	2997720
135	3143304
136	3144960
137	3290112
138	3295512
139	3444480
140	3449016
141	3606768
142	3606120
143	3767544
144	3775464
145	3936240
146	3938976
147	4113432
148	4115232
149	4289040
150	4294800
151	4473000
152	4478112
153	4665600
154	4664808
155	4856400
156	4865256
157	5056272
158	5059080
159	5265000
160	5266656
161	5471928
162	5478408
163	5688144
164	5693760
165	5913504
166	5912856
167	6137352
168	6146640
169	6370560
170	6373440
171	6613488
172	6615504
173	6854544
174	6861456

175 7105248
176 7111440
177 7366464
178 7365600
179 7625520
180 7635528
181 7894800
182 7897968
183 8174520
184 8176608
185 8452080
186 8459640
187 8740512
188 8746992
189 9039528
190 9038520
191 9336600
192 9347400

193 9644544
194 9648000
195 9963504
196 9965880
197 10280592
198 10288512
199 10608840
200 10615536
201 10948608
202 10947600
203 11286072
204 11297736
205 11635200
206 11639016
207 11996424
208 11998800
209 12355200
210 12363408
211 12726000
212 12733344
213 13109040
214 13107960
215 13489560
216 13501944
217 13882680
218 13886640
219 14288400
220 14290848
221 14691600
222 14700528
223 15107544
224 15115176
225 15536304
226 15535296
227 15962832

228	15975864
229	16402320
230	16406280
231	16855056
232	16857792
233	17305344
234	17314776
235	17768880
236	17777232
237	18246384
238	18245088
239	18721080
240	18734616
241	19209600
242	19213920
243	19712160
244	19715040
245	20211768
246	20221920
247	20725992
248	20734560
249	21254400
250	21252960
251	21780000
252	21794400
253	22320144
254	22324680
255	22874904
256	22878000
257	23427072
258	23437512
259	23993928
260	24002928
261	24576048
262	24574680
263	25155144
264	25170264
265	25749360
266	25754184
267	26359344
268	26362512
269	26966160
270	26976960
271	27588600
272	27598032
273	28226808
274	28225440
275	28861920
276	28877976
277	29513232
278	29518200
279	30180600
280	30183624
281	30844800

282	30856248
283	31525344
284	31535280
285	32222304
286	32220936
287	32916240
288	32932872
289	33626880
290	33631920
291	34354368
292	34357824
293	35078544
294	35090424
295	35819640
296	35830080
297	36578304
298	36576720
299	37333440
300	37350648
301	38105928
302	38111400
303	38896200
304	38899728
305	39682800
306	39695400
307	40487472
308	40498200
309	41310000
310	41308200
311	42129000
312	42147000
313	42966144
314	42971760
315	43821432
316	43825392
317	44673552
318	44686512
319	45543960
320	45554976
321	46433088
322	46431288
323	47318544
324	47337336
325	48222720
326	48228696
327	49146264
328	49150080
329	50065848
330	50079240
331	51004800
332	51016464
333	51963120
334	51961320
335	52917480

336	52936992
337	53891712
338	53897760
339	54885600
340	54889488
341	55875600
342	55889568
343	56885472
344	56897568
345	57915504
346	57913776
347	58941792
348	58962024
349	59988240
350	59994288
351	61055208
352	61059312
353	62118144
354	62132616
355	63201600
356	63214272
357	64306152
358	64304280
359	65406600
360	65427336
361	66528000
362	66534480
363	67670640
364	67674888
365	68808960
366	68824080
367	69968952
368	69981840
369	71150400
370	71148240
371	72327528
372	72349272
373	73526544
374	73533240
375	74747304
376	74751840
377	75964032
378	75979440
379	77202720
380	77215968
381	78463728
382	78461640
383	79720344
384	79742664
385	80999208
386	81006336
387	82301184
388	82305792
389	83598480

390	83614320
391	84918600
392	84932280
393	86261760
394	86259600
395	87600240
396	87623496
397	88962192
398	88969320
399	90347328
400	90351936
401	91728000
402	91744488
403	93132144
404	93146400
405	94559904
406	94557744
407	95983272
408	96007032
409	97430400
410	97437600
411	98901648
412	98906544
413	100368072
414	100385136
415	101858760
416	101873520
417	103374144
418	103371840
419	104884560
420	104908896
421	106419600
422	106427160
423	107979480
424	107984448
425	109534320
426	109551960
427	111114360
428	111129552
429	112719600
430	112717080
431	114319800
432	114345000
433	115945344
434	115953048
435	117596304
436	117601632
437	119242512
438	119260512
439	120914280
440	120929616
441	122611896
442	122609520
443	124304544

444	124330536
445	126023040
446	126031176
447	127768104
448	127773288
449	129507840
450	129526200
451	131274000
452	131289984
453	133066800
454	133064280
455	134854128
456	134880984
457	136668672
458	136676880
459	138510000
460	138515328
461	140346000
462	140364936
463	142209144
464	142225488
465	144099504
466	144097056
467	145984752
468	146012184
469	147897288
470	147905640
471	149837688
472	149843232
473	151772544
474	151792056
475	153735120
476	153752040
477	155726064
478	155723400
479	157711320
480	157739256
481	159724800
482	159733440
483	161766648
484	161772480
485	163802880
486	163823040
487	165867912
488	165885120
489	167961600
490	167958648
491	170049600
492	170078472
493	172166544
494	172175400
495	174312504
496	174318480
497	176452920

```
498  176473512
499  178622640
500  178640208
```

"Coefficients complete.", 501, 178640208

```
> for n from sgfdegree to enddegree do
   co:=coeff(sseries,x,n):
   printf("%d  %d \n",n,co);
  od:
  print("Coefficients complete.",n,co);
```

```
15  1
16  2
17  4
18  8
19  12
20  20
21  29
22  42
23  54
24  82
25  97
26  131
27  169
28  207
29  249
30  331
31  372
32  459
33  551
34  647
35  745
36  911
37  1007
38  1184
39  1374
40  1553
41  1739
42  2049
43  2231
44  2539
45  2867
46  3183
47  3509
48  3999
49  4316
50  4820
51  5340
52  5835
53  6350
54  7104
55  7607
56  8352
57  9132
```

58 9882
59 10651
60 11724
61 12472
62 13551
63 14663
64 15722
65 16816
66 18307
67 19376
68 20846
69 22372
70 23842
71 25342
72 27322
73 28786
74 30767
75 32797
76 34738
77 36737
78 39329
79 41284

80 43842
81 46475
82 49019
83 51610
84 54899
85 57439
86 60712
87 64061
88 67279
89 70575
90 74708
91 77939
92 82015
93 86193
94 90238
95 94357
96 99423
97 103463
98 108502
99 113640
100 118599
101 123661
102 129840
103 134813
104 140911
105 147144
106 153190
107 159335
108 166725
109 172768
110 180111

111	187582
112	194818
113	202186
114	210991
115	218243
116	226938
117	235804
118	244423
119	253174
120	263506
121	272118
122	282378
123	292799
124	302918
125	313208
126	325289
127	335428
128	347368
129	359518
130	371359
131	383358
132	397318
133	409149
134	423008
135	437069
136	450750
137	464643
138	480726
139	494430
140	510339
141	526497
142	542270
143	558242
144	576591
145	592355
146	610569
147	629028
148	647025
149	665277
150	686159
151	704181
152	724847
153	745809
154	766306
155	787043
156	810609
157	831095
158	854493
159	878182
160	901321
161	924758
162	951307
163	974469
164	1000757

165	1027396
166	1053471
167	1079837
168	1109528
169	1135594
170	1165078
171	1194898
172	1224074
173	1253598
174	1286752
175	1315956
176	1348800
177	1382050
178	1414640
179	1447570
180	1484362
181	1516940
182	1553480
183	1590411
184	1626590
185	1663174
186	1703946
187	1740154
188	1780564
189	1821441
190	1861550
191	1902050
192	1946990
193	1987085
194	2031725
195	2076815
196	2121037
197	2165725
198	2215197
199	2259452
200	2308511
201	2358093
202	2406797
203	2455949
204	2510157
205	2558847
206	2612700
207	2667070
208	2720447
209	2774353
210	2833687
211	2887097
212	2945953
213	3005401
214	3063847
215	3122805
216	3187471
217	3245902
218	3310160

219	3374998
220	3438717
221	3503030
222	3573448
223	3637201
224	3707080
225	3777622
226	3847028
227	3917013
228	3993404
229	4062796
230	4138719
231	4215291
232	4290606
233	4366584
234	4449389
235	4524742
236	4606936
237	4689872
238	4771532
239	4853842
240	4943294
241	5024936
242	5113857
243	5203497
244	5291736
245	5380717
246	5477277
247	5565556
248	5661436
249	5758137
250	5853417
251	5949416
252	6053337
253	6148597
254	6251916
255	6356037
256	6458599
257	6561981
258	6673744
259	6776349
260	6887357
261	6999265
262	7109598
263	7220731
264	7340599
265	7450913
266	7570108
267	7690186
268	7808543
269	7927805
270	8056288
271	8174689
272	8302335

273	8430968
274	8557864
275	8685641
276	8823007
277	8949884
278	9086501
279	9224090
280	9359790
281	9496476
282	9643265
283	9779007
284	9924876
285	10071824
286	10216859
287	10362866
288	10519356
289	10664372
290	10820036
291	10976757
292	11131412
293	11287146
294	11453901
295	11608604
296	11774348
297	11941268
298	12106099
299	12271988
300	12449300
301	12614109
302	12790508
303	12968059
304	13143358
305	13319833
306	13508284
307	13683632
308	13870983
309	14059609
310	14245958
311	14433458
312	14633359
313	14819683
314	15018583
315	15218734
316	15416435
317	15615413
318	15827365
319	16025121
320	16235879
321	16448011
322	16657676
323	16868587
324	17092915
325	17302555

326	17525789
327	17750378
328	17972315
329	18195632
330	18432965
331	18654955
332	18890991
333	19128510
334	19363355
335	19599553
336	19850220
337	20085040
338	20334518
339	20585456
340	20833536
341	21083098
342	21347756
343	21595890
344	21859148
345	22124000
346	22385966
347	22649392
348	22928382
349	23190324
350	23468028
351	23747299
352	24023494
353	24301282
354	24595288
355	24871540
356	25164034
357	25458241
358	25749340
359	26042010
360	26351382
361	26642455
362	26950435
363	27260095
364	27566455
365	27874525
366	28199965
367	28506384
368	28830205
369	29155855
370	29478175
371	29802175
372	30144055
373	30466345
374	30806724
375	31148906
376	31487547
377	31828019
378	32187063
379	32525767

380	32883075
381	33242333
382	33598027
383	33955519
384	34332107
385	34687772
386	35062746
387	35439644
388	35812761
389	36187834
390	36582716
391	36955897
392	37348924
393	37744026
394	38135322
395	38528539
396	38942106
397	39333372
398	39745209
399	40159099
400	40568958
401	40980894
402	41413923
403	41823846
404	42254894
405	42688152
406	43117348
407	43548594
408	44001484
409	44430650
410	44881695
411	45334915
412	45783850
413	46234995
414	46708549
415	47157552
416	47628996
417	48102787
418	48572257
419	49043906
420	49538539
421	50007977
422	50500636
423	50995607
424	51486027
425	51978791
426	52495322
427	52985811
428	53500101
429	54016877
430	54529066
431	55043567
432	55582427
433	56094581

434	56631342
435	57170552
436	57704933
437	58241801
438	58803834
439	59338289
440	59897947
441	60460230
442	61017654
443	61577525
444	62163173
445	62720564
446	63303977
447	63889986
448	64470884
449	65054410
450	65664543
451	66245513
452	66853130
453	67463518
454	68068763
455	68676602
456	69311668
457	69916878
458	70549576
459	71185015
460	71815054
461	72447866
462	73108765
463	73738878
464	74397116
465	75058278
466	75714005
467	76372470
468	77059660
469	77715353
470	78400036
471	79087607
472	79769482
473	80454281
474	81168686
475	81850638
476	82562233
477	83276909
478	83985848
479	84697678
480	85439771
481	86148673
482	86888113
483	87630594
484	88367073
485	89106633
486	89877353
487	90613913

```
488  91381673
489  92152673
490  92917634
491  93685633
492  94485473
493  95250393
494  96047433
495  96847674
496  97641595
497  98438758
498  99268681
499  100062685
500  100889493
```

"Coefficients complete.", 501, 100889493

```
> for n from rgfdegree to enddegree do
   co:=coeff(rseries,x,n):
   printf("%d  %d \n",n,co);
  od:
  print("Coefficients complete.",n,co);
```

```
12  1
13  2
14  4
15  7
16  10
17  16
18  21
19  30
20  34
21  53
22  55
23  77
24  87
25  110
26  118
27  162
28  165
29  210
30  220
31  275
32  286
33  360
34  360
35  439
36  463
37  546
38  555
39  675
40  678
41  800
42  818
43  952
44  970
```

45 1132
46 1133
47 1311
48 1341
49 1519
50 1530
51 1764
52 1772
53 2002
54 2028
55 2275
56 2299
57 2592
58 2590
59 2900
60 2939
61 3250
62 3265
63 3644
64 3654
65 4030
66 4065
67 4466
68 4496
69 4950
70 4944
71 5425
72 5475
73 5952
74 5970
75 6532
76 6546
77 7105
78 7146
79 7735
80 7768
81 8424
82 8420
83 9102
84 9162
85 9840
86 9863
87 10647
88 10660
89 11440
90 11485
91 12299
92 12342
93 13230
94 13225
95 14145
96 14217
97 15136
98 15159

99 16200
100 16214
101 17250
102 17304
103 18377
104 18424
105 19581
106 19578
107 20776
108 20857
109 22050
110 22075
111 23409
112 23425
113 24752
114 24813
115 26180
116 26236
117 27702
118 27695
119 29204
120 29293
121 30800
122 30830
123 32490
124 32510
125 34160
126 34229
127 35931
128 35990
129 37800
130 37790
131 39650
132 39751
133 41601
134 41635
135 43657
136 43680
137 45696
138 45771
139 47840
140 47903
141 50094
142 50085
143 52327
144 52437
145 54670
146 54708
147 57131
148 57156
149 59570
150 59650
151 62125
152 62196

153 64800
154 64789
155 67450
156 67573
157 70226
158 70265
159 73125
160 73148
161 75999
162 76089
163 79002
164 79080
165 82132
166 82123
167 85241
168 85370
169 88480
170 88520
171 91854
172 91882
173 95202
174 95298
175 98684
176 98770
177 102312
178 102300
179 105910
180 106049
181 109650
182 109694
183 113535
184 113564
185 117390
186 117495
187 121396
188 121486
189 125549
190 125535
191 129675
192 129825
193 133952
194 134000
195 138382
196 138415
197 142786
198 142896
199 147345
200 147438
201 152064
202 152050
203 156751
204 156913
205 161600
206 161653

207 166617
208 166650
209 171600
210 171714
211 176750
212 176852
213 182070
214 182055
215 187355
216 187527
217 192815
218 192870
219 198450
220 198484
221 204050
222 204174
223 209827
224 209933
225 215782
226 215768
227 221706
228 221887
229 227810
230 227865
231 234098
232 234136
233 240352
234 240483
235 246790
236 246906
237 253422
238 253404
239 260015
240 260203
241 266800
242 266860
243 273780
244 273820
245 280719
246 280860
247 287861
248 287980

249 295200
250 295180
251 302500
252 302700
253 310002
254 310065
255 317707
256 317750
257 325376
258 325521
259 333249

260	333374
261	341334
262	341315
263	349377
264	349587
265	357630
266	357697
267	366102
268	366146
269	374530
270	374680
271	383175
272	383306
273	392039
274	392020
275	400860
276	401083
277	409906
278	409975
279	419175
280	419217
281	428400
282	428559
283	437852
284	437990
285	447532
286	447513
287	457170
288	457401
289	467040
290	467110
291	477144
292	477192
293	487202
294	487367
295	497495
296	497640
297	508032
298	508010
299	518520
300	518759
301	529249
302	529325
303	540225
304	540274
305	551150
306	551325
307	562326
308	562475
309	573750
310	573725
311	585125
312	585375
313	596752

314	596830
315	608631
316	608686
317	620466
318	620646
319	632555
320	632708
321	644904
322	644879
323	657202
324	657463
325	669760
326	669843
327	682587
328	682640
329	695359
330	695545
331	708400
332	708562
333	721710
334	721685
335	734965
336	735236
337	748496
338	748580
339	762300
340	762354
341	776050
342	776244
343	790076
344	790244
345	804382
346	804358
347	818636
348	818917
349	833170
350	833254
351	847989
352	848046
353	862752
354	862953
355	877800
356	877976
357	893141
358	893115
359	908425
360	908713
361	924000
362	924090
363	939870
364	939929
365	955680
366	955890
367	971791

368	971970
369	988200
370	988170
371	1004549
372	1004851
373	1021202
374	1021295
375	1038157
376	1038220
377	1055056
378	1055270
379	1072260
380	1072444
381	1089774
382	1089745
383	1107227
384	1107537
385	1124989
386	1125088
387	1143072
388	1143136
389	1161090
390	1161310
391	1179425
392	1179615
393	1198080
394	1198050
395	1216670
396	1216993
397	1235586
398	1235685
399	1254824
400	1254888
401	1274000
402	1274229
403	1293502
404	1293700
405	1313332
406	1313302
407	1333101
408	1333431
409	1353200
410	1353300
411	1373634
412	1373702
413	1394001
414	1394238
415	1414705
416	1414910
417	1435752
418	1435720
419	1456730
420	1457068
421	1478050

422 1478155
423 1499715
424 1499784
425 1521310
426 1521555
427 1543255
428 1543466
429 1565550
430 1565515
431 1587775
432 1588125
433 1610352
434 1610459
435 1633282
436 1633356
437 1656146
438 1656396
439 1679365
440 1679578
441 1702943
442 1702910
443 1726452
444 1726813
445 1750320
446 1750433
447 1774557
448 1774629
449 1798720
450 1798975
451 1823250
452 1823472
453 1848150
454 1848115
455 1872974
456 1873347
457 1898176
458 1898290
459 1923750
460 1923824
461 1949250
462 1949513
463 1975127
464 1975354
465 2001382
466 2001348
467 2027566
468 2027947
469 2054129
470 2054245
471 2081079
472 2081156
473 2107952
474 2108223
475 2135210

```
476  2135445
477  2162862
478  2162825
479  2190435
480  2190823
481  2218400
482  2218520
483  2246759
484  2246840
485  2275040
486  2275320
487  2303721
488  2303960
489  2332800
490  2332759
491  2361800
492  2362201
493  2391202
494  2391325
495  2421007
496  2421090
497  2450735
498  2451021
499  2480870
500  2481114

                                    "Coefficients complete.", 501, 2481114
>
```