Magilatin generating functions and constituents

(general form, with affine data)

Notation:

L, S: magilatin, semimagic squares (all positive values).

ml: magilatin, except in g.f.'s.

l, s: normalized squares (symmetry types).

R: reduced squares (least element is 0).

r: reduced normalized squares (reduced symmetry types).

n: semimagic r.

gf: generating function in some form.

gfsum: generating function as a sum of simple terms.

c: Cubic (fixed strict upper bound; weak upper bound for reduced).

a: Affine (fixed magic sum).

p: Period of the quasipolynomial (known from geometry). (Period of the truncated quasipolynomial, in the affine count.)
d: Dimension of the geometry = degree of the quasipolynomials.
RtoLfactor: the rational function that multiplies Rgf to Lgf and rgf to lgf.

enddegree: The number of terms desired in the sequences, from degree 1 (but initial zeros will be omitted).

```
> enddegree:=500;
```

$$enddegree := 500$$

This is for **affine**: set up main constants.

```
> d:=4; p:=120;
  RtoLfactor:=x^3/(1-x^3);
```

$$d := 4$$

$$p := 120$$

$$RtoLfactor := \frac{x^3}{1 - x^3}$$

We start by recomputing R_s=rsgf from the semimagic count. From the Latte results we get the closed Ehrhart g.f. of each flat, which depends on whether we're doing cubic or affine.

Set up the simplex data for the faces and intersection polytopes in the semimagic part of the magilatin series.

This is for **affine.**

```
> simplexname[1]:="OABC": ehrgf[1]:= 1/((1-x)*(1-x^2)^3) : dimen[1]:=3:
  simplexname[2]:="OEE2": ehrgf[2]:= 1/((1-x)*(1-x^4)^2) : dimen[2]:=2:
  simplexname[3]:="OAE2": ehrgf[3]:= 1/((1-x)*(1-x^2)*(1-x^4)) :
  dimen[3]:=2:
  simplexname[4]:="ADE2": ehrgf[4]:= 1/((1-x^2)*(1-x^3)*(1-x^4)) :
  dimen[4]:=2:
  simplexname[5]:="DE1E2": ehrgf[5]:= 1/((1-x^3)*(1-x^4)^2) : dimen[5]:=2:
```

```
simplexname[6]:="OCE": ehrgf[6]:= 1/((1-x)*(1-x^2)*(1-x^4)) :
dimen[6]:=2:
simplexname[7]:="BDE1": ehrgf[7]:= 1/((1-x^2)*(1-x^3)*(1-x^4)) :
dimen[7]:=2:
simplexname[8]:="ABD": ehrgf[8]:= 1/((1-x^2)^2*(1-x^3)) : dimen[8]:=2:
simplexname[9]:="FG1": ehrgf[9]:= 1/((1-x^5)*(1-x^8)) : dimen[9]:=1:
simplexname[10]:="EF": ehrgf[10]:= 1/((1-x^4)*(1-x^5)) : dimen[10]:=1:
simplexname[11]:="OG": ehrgf[11]:= 1/((1-x)*(1-x^6)) : dimen[11]:=1:
simplexname[12]:="FG": ehrgf[12]:= 1/((1-x^5)*(1-x^6)) : dimen[12]:=1:
simplexname[13]:="AF": ehrgf[13]:= 1/((1-x^2)*(1-x^5)) : dimen[13]:=1:
simplexname[14]:="DG": ehrgf[14]:= 1/((1-x^3)*(1-x^6)) : dimen[14]:=1:
simplexname[15]:="DG2": ehrgf[15]:= 1/((1-x^3)*(1-x^8)) : dimen[15]:=1:
simplexname[16]:="DE": ehrgf[16]:= 1/((1-x^3)*(1-x^4)) : dimen[16]:=1:
simplexname[17]:="H": ehrgf[17] := 1/(1-x^7) : dimen[17]:=0:
# for n from 1 to 17 do print(simplexname[n], dimen[n], ehrgf[n]); od;
```

The closed E.g.f. is converted to the open E.g.f. The first step is to compute the Mobius function of the intersection poset.

```
> for n from 1 to 17 do
    mu[n]:=(-1)^(dimen[1]-dimen[n]):
  od:
  mu[14]:=2*mu[14]:
  for n from 1 to 17 do
    openehrgf[n]:=simplify(-(-1)^dimen[n]*subs(x=1/x,ehrgf[n])):
  od:
```

Set up basic g.f.'s.

```
> for n from 1 to 17 do
    rsgfterm[n]:=openehrgf[n]:
  od:
  rsgfsum:=sum(mu[nn]*rsgfterm[nn],nn=1..17):
  rsgf:=simplify(rsgfsum):
  sgf:=simplify(RtoLfactor*rsgf):
```

The additional faces and intersection polytopes involved in the magilatin computation. They depend on whether we're cubic or affine.

These are for **affine**.

```
> mlsimplexname[1]:="OAB": mlehrgf[1]:= 1 / ((1-x)*(1-x^2)^2) :
  mldimen[1]:=2:
  mlsimplexname[2]:="OE": mlehrgf[2]:= 1 / ((1-x)*(1-x^4)) :
  mldimen[2]:=1:
  mlsimplexname[3]:="OAC": mlehrgf[3]:= 1 / ((1-x)*(1-x^2)^2) :
  mldimen[3]:=2:
  mlsimplexname[4]:="AD": mlehrgf[4]:= 1 / ((1-x^3)*(1-x^2)) :
  mldimen[4]:=1:
  mlsimplexname[5]:="DE1": mlehrgf[5]:= 1 / ((1-x^3)*(1-x^4)) :
  mldimen[5]:=1:
  mlsimplexname[6]:="OBC": mlehrgf[6]:= 1 / ((1-x)*(1-x^2)^2) :
  mldimen[6]:=2:
  mlsimplexname[7]:="OE2": mlehrgf[7]:= 1 / ((1-x)*(1-x^4)) :
  mldimen[7]:=1:
  mlsimplexname[8]:="BD": mlehrgf[8]:= 1 / ((1-x^2)*(1-x^3)) :
  mldimen[8]:=1:
  mlsimplexname[9]:="DE2": mlehrgf[9]:= 1 / ((1-x^3)*(1-x^4)) :
```

```
mldimen[9]:=1:
mlsimplexname[10]:="F": mlehrgf[10]:= 1/(1-x^5) : mldimen[10]:=0:
mlsimplexname[11]:="OB": mlehrgf[11]:= 1/((1-x)*(1-x^2)) :
mldimen[11]:=1:
# for n from 1 to 11 do print(mlsimplexname[n], mldimen[n], mlehrgf[n]);
od;
```

Now a general computation. First, open Ehrhart g.f.'s.

```
> for n from 1 to 11 do
    openmlehrgf[n]:=simplify(-(-1)^mldimen[n]*subs(x=1/x,mlehrgf[n])):
  od:
```

(-1)^3 n_OAB(1/x) equals mlehrgf[1]+mlehrgf[2], and hence n_OAB(x) is, by another method that gives a nicer appearance, summing mu(,)E^o(x):

```
> mlnnew[1] := openmlehrgf[1]-openmlehrgf[2]:
```

(-1)^3 n_OAC(1/x) equals mlehrgf[3]+mlehrgf[4]+mlehrgf[5]. Hence n_OAC(x) equals

```
> mlnnew[2] := openmlehrgf[3]-openmlehrgf[4]-openmlehrgf[5]:
```

(-1)^3 n_OBC(1/x) equals mlehrgf[6]+mlehrgf[7]+mlehrgf[8]+mlehrgf[9]+mlehrgf[10]. So n_OBC(x) equals

```
> mlnnew[3] :=
  openmlehrgf[6]-openmlehrgf[7]-openmlehrgf[8]-openmlehrgf[9]+openmlehrgf[
  10]:
```

Finally, OB gives mlehrgf[11], so that n_OB(x) is

```
> mlnnew[4] := openmlehrgf[11]:
```

To compute r, we need rs=n from semimagic, which equals rgf:

```
> Rgfsum:=72*rsgfsum+36*(mlnnew[1]+mlnnew[2]+mlnnew[3])+12*mlnnew[4]:
  Rgf:=simplify(Rgfsum);
```

$$Rgf := \frac{1}{(x^7-1)(x^6-1)(x^8-1)(x^5-1)(x^2+x+1)(x^3+x^2+x+1)}(12x^3(1+3x+15x^3+7x^2+33x^4 + 65x^5+316x^8+128x^6+208x^7+434x^9+676x^{11}+852x^{13}+967x^{16}+784x^{12}+624x^{24} + 995x^{19}+955x^{21}+1000x^{20}+936x^{15}+967x^{17}+893x^{22}+456x^{25}+174x^{27}+911x^{14} + 322x^{26}+752x^{23}+566x^{10}+1001x^{18}+79x^{28}))$$

Hence L, the g.f. of the number of magilatin squares, equals

```
> Lgf:=simplify(RtoLfactor*Rgf);
```

$$Lgf := -\frac{1}{(x^3-1)(x^7-1)(x^6-1)(x^8-1)(x^5-1)(x^2+x+1)(x^3+x^2+x+1)}(12x^6(1+3x+15x^3+7x^2 + 33x^4+65x^5+316x^8+128x^6+208x^7+434x^9+676x^{11}+852x^{13}+967x^{16}+784x^{12} + 624x^{24}+995x^{19}+955x^{21}+1000x^{20}+936x^{15}+967x^{17}+893x^{22}+456x^{25}+174x^{27} + 911x^{14}+322x^{26}+752x^{23}+566x^{10}+1001x^{18}+79x^{28}))$$

Now compute the number of reduced symmetry types:

```
> rgfsum:=rsgfsum+mlnnew[1]+mlnnew[2]+mlnnew[3]+mlnnew[4]:
  rgf:=simplify(rgfsum);
```

$$rgf := \frac{1}{(x^7-1)(x^6-1)(x^8-1)(x^5-1)(x^2+x+1)(x^3+x^2+x+1)}(x^3(1+3x+13x^3+7x^2+23x^4$$

$$+ 37 x^{5} + 118 x^{8} + 60 x^{6} + 86 x^{7} + 149 x^{9} + 199 x^{11} + 208 x^{13} + 145 x^{16} + 212 x^{12} + 54 x^{24}$$
$$+ 79 x^{19} + 67 x^{21} + 72 x^{20} + 171 x^{15} + 115 x^{17} + 66 x^{22} + 43 x^{25} + 19 x^{27} + 196 x^{14} + 33 x^{26}$$
$$+ 59 x^{23} + 180 x^{10} + 96 x^{18} + 9 x^{28}))$$

The g.f. of the total number of symmetry types, l_ml ("lgf"):

```
> lgf:=simplify(RtoLfactor*rgf);
```

$$lgf := - \frac{1}{(x^{3} - 1) (x^{7} - 1) (x^{6} - 1) (x^{8} - 1) (x^{5} - 1) (x^{2} + x + 1) (x^{3} + x^{2} + x + 1)} (x^{6} (1 + 3 x + 13 x^{3} + 7 x^{2}$$
$$+ 23 x^{4} + 37 x^{5} + 118 x^{8} + 60 x^{6} + 86 x^{7} + 149 x^{9} + 199 x^{11} + 208 x^{13} + 145 x^{16} + 212 x^{12} + 54 x^{24}$$
$$+ 79 x^{19} + 67 x^{21} + 72 x^{20} + 171 x^{15} + 115 x^{17} + 66 x^{22} + 43 x^{25} + 19 x^{27} + 196 x^{14} + 33 x^{26}$$
$$+ 59 x^{23} + 180 x^{10} + 96 x^{18} + 9 x^{28}))$$

Generate the series expansions of the g.f.'s.

Expressing the rational function with standard denominator gives an orders-of-magnitude speedup in the series expansion.

Standard denominator (1-x^p)^{d+1}.

```
> pdenom:=(1-x^p):
  standenom:=pdenom^(d+1);
```

$$standenom := (1 - x^{120})^{5}$$

G.f. as rational function with standard denominator.

```
> Lgfstandnum:=simplify(numer(Lgf)*simplify(standenom/denom(Lgf))):
  Lgf:=Lgfstandnum/standenom;
```

$$Lgf := \frac{1}{(x^{6} + x^{5} + x^{4} + x^{3} + x^{2} + x + 1) (1 - x^{120})^{5}} (12 x^{6} (1 + 3 x + 15 x^{3} + 7 x^{2} + 33 x^{4} + 65 x^{5} + 316 x^{8}$$
$$+ 128 x^{6} + 208 x^{7} + 434 x^{9} + 676 x^{11} + 852 x^{13} + 967 x^{16} + 784 x^{12} + 624 x^{24} + 995 x^{19} + 955 x^{21}$$
$$+ 1000 x^{20} + 936 x^{15} + 967 x^{17} + 893 x^{22} + 456 x^{25} + 174 x^{27} + 911 x^{14} + 322 x^{26} + 752 x^{23}$$
$$+ 566 x^{10} + 1001 x^{18} + 79 x^{28}) (1 - x + x^{3} - x^{4} + x^{8} + x^{6} - x^{7} + x^{11} - x^{13} + x^{24} + x^{30} - x^{25} + x^{27}$$
$$+ x^{14} - x^{10} - x^{31} - x^{28} - x^{37} + x^{32} - x^{34} + x^{35} + x^{38} + x^{102} + x^{99} + x^{96} + x^{78} + x^{75} + x^{72} + x^{54} + x^{51}$$
$$+ x^{48} + x^{110} - x^{109} + x^{107} - x^{106} + x^{104} - x^{103} - x^{100} - x^{97} + x^{86} - x^{85} + x^{83} + x^{80} - x^{79} - x^{76} - x^{82}$$
$$- x^{73} + x^{62} - x^{61} + x^{59} + x^{56} - x^{55} - x^{52} - x^{49} - x^{58}) (1 + x + x^{3} + x^{2} + x^{4} + x^{5} + x^{8} + x^{6} + x^{7} + x^{9}$$
$$+ x^{11} + x^{13} + x^{16} + x^{12} + x^{24} + x^{19} + x^{21} + x^{20} + x^{15} + x^{17} + x^{30} + x^{22} + x^{25} + x^{27} + x^{14} + x^{26}$$
$$+ x^{23} + x^{10} + x^{18} + x^{31} + x^{28} + x^{37} + x^{29} + x^{32} + x^{34} + x^{33} + x^{35} + x^{36} + x^{41} + x^{38} + x^{39} + x^{40}$$
$$+ x^{117} + x^{111} + x^{114} + x^{108} + x^{105} + x^{102} + x^{99} + x^{96} + x^{93} + x^{90} + x^{87} + x^{81} + x^{78} + x^{75} + x^{84}$$
$$+ x^{72} + x^{66} + x^{63} + x^{60} + x^{57} + x^{54} + x^{69} + x^{51} + x^{45} + x^{42} + x^{48} + x^{119} + x^{118} + x^{116} + x^{115}$$
$$+ x^{113} + x^{112} + x^{110} + x^{109} + x^{107} + x^{106} + x^{104} + x^{103} + x^{101} + x^{100} + x^{98} + x^{97} + x^{95} + x^{92} + x^{91}$$
$$+ x^{94} + x^{88} + x^{86} + x^{85} + x^{89} + x^{83} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{82} + x^{73} + x^{71} + x^{70} + x^{68}$$
$$+ x^{67} + x^{65} + x^{64} + x^{62} + x^{61} + x^{59} + x^{56} + x^{55} + x^{53} + x^{52} + x^{50} + x^{49} + x^{47} + x^{46} + x^{44} + x^{43}$$
$$+ x^{58})\hat{\ }2 (1 - x + x^{2} + x^{8} + x^{6} - x^{7} - x^{13} + x^{12} + x^{24} - x^{19} + x^{20} + x^{30} - x^{25} + x^{14} + x^{26} + x^{18} - x^{31}$$
$$- x^{37} + x^{32} + x^{36} + x^{38} + x^{114} + x^{108} + x^{102} + x^{96} + x^{90} + x^{78} + x^{84} + x^{72} + x^{66} + x^{60} + x^{54} + x^{42}$$
$$+ x^{48} + x^{116} - x^{115} + x^{110} - x^{109} + x^{104} - x^{103} + x^{98} - x^{97} + x^{92} - x^{91} + x^{86} - x^{85} + x^{80} - x^{79} + x^{74}$$

$$-x^{73}+x^{68}-x^{67}+x^{62}-x^{61}+x^{56}-x^{55}+x^{50}-x^{49}+x^{44}-x^{43})\,(1-x+x^{3}-x^{2}+x^{4}+x^{8}-x^{6}-x^{7}$$
$$+x^{9}-x^{11}+x^{24}+x^{20}+x^{15}-x^{17}-x^{22}-x^{26}-x^{31}+x^{28}-x^{37}+x^{33}-x^{41}+x^{39}+x^{40}+x^{108}+x^{105}$$
$$-x^{102}+x^{99}+x^{93}+x^{75}+x^{84}-x^{66}+x^{63}+x^{60}+x^{69}+x^{45}-x^{42}+x^{48}-x^{107}-x^{106}+x^{104}-x^{101}$$
$$+x^{100}-x^{97}-x^{91}+x^{88}-x^{86}+x^{80}-x^{77}-x^{82}-x^{71}+x^{68}-x^{67}+x^{64}-x^{62}-x^{61}-x^{47}-x^{46}+x^{44}))$$

G.f. as rational function with standard denominator.

```
> Rgfstandnum:=simplify(numer(Rgf)*standenom/denom(Rgf)):
  Rgf:=Rgfstandnum/standenom;
```

$$Rgf := -\frac{1}{(x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x+1)\,(1-x^{120})^{5}}\,(12\,(1+3\,x+15\,x^{3}+7\,x^{2}+33\,x^{4}+65\,x^{5}+316\,x^{8}$$
$$+128\,x^{6}+208\,x^{7}+434\,x^{9}+676\,x^{11}+852\,x^{13}+967\,x^{16}+784\,x^{12}+624\,x^{24}+995\,x^{19}+955\,x^{21}$$
$$+1000\,x^{20}+936\,x^{15}+967\,x^{17}+893\,x^{22}+456\,x^{25}+174\,x^{27}+911\,x^{14}+322\,x^{26}+752\,x^{23}$$
$$+566\,x^{10}+1001\,x^{18}+79\,x^{28})\,x^{3}\,(1-x+x^{3}-x^{4}+x^{8}+x^{6}-x^{7}+x^{11}-x^{13}+x^{24}+x^{30}-x^{25}+x^{27}$$
$$+x^{14}-x^{10}-x^{31}-x^{28}-x^{37}+x^{32}-x^{34}+x^{35}+x^{38}+x^{102}+x^{99}+x^{96}+x^{78}+x^{75}+x^{72}+x^{54}+x^{51}$$
$$+x^{48}+x^{110}-x^{109}+x^{107}-x^{106}+x^{104}-x^{103}-x^{100}-x^{97}+x^{86}-x^{85}+x^{83}+x^{80}-x^{79}-x^{76}-x^{82}$$
$$-x^{73}+x^{62}-x^{61}+x^{59}+x^{56}-x^{55}-x^{52}-x^{49}-x^{58})\,(1+x+x^{3}+x^{2}+x^{4}+x^{5}+x^{8}+x^{6}+x^{7}+x^{9}$$
$$+x^{11}+x^{13}+x^{16}+x^{12}+x^{24}+x^{19}+x^{21}+x^{20}+x^{15}+x^{17}+x^{30}+x^{22}+x^{25}+x^{27}+x^{14}+x^{26}$$
$$+x^{23}+x^{10}+x^{18}+x^{31}+x^{28}+x^{37}+x^{29}+x^{32}+x^{34}+x^{33}+x^{35}+x^{36}+x^{41}+x^{38}+x^{39}+x^{40}$$
$$+x^{117}+x^{111}+x^{114}+x^{108}+x^{105}+x^{102}+x^{99}+x^{96}+x^{93}+x^{90}+x^{87}+x^{81}+x^{78}+x^{75}+x^{84}$$
$$+x^{72}+x^{66}+x^{63}+x^{60}+x^{57}+x^{54}+x^{69}+x^{51}+x^{45}+x^{42}+x^{48}+x^{119}+x^{118}+x^{116}+x^{115}$$
$$+x^{113}+x^{112}+x^{110}+x^{109}+x^{107}+x^{106}+x^{104}+x^{103}+x^{101}+x^{100}+x^{98}+x^{97}+x^{95}+x^{92}+x^{91}$$
$$+x^{94}+x^{88}+x^{86}+x^{85}+x^{89}+x^{83}+x^{80}+x^{79}+x^{77}+x^{76}+x^{74}+x^{82}+x^{73}+x^{71}+x^{70}+x^{68}$$
$$+x^{67}+x^{65}+x^{64}+x^{62}+x^{61}+x^{59}+x^{56}+x^{55}+x^{53}+x^{52}+x^{50}+x^{49}+x^{47}+x^{46}+x^{44}+x^{43}$$
$$+x^{58})\hat{}2\,(-1+x+x^{3}-x^{2}-x^{4}+x^{5}-x^{8}-x^{6}+x^{7}+x^{9}+x^{11}+x^{13}-x^{16}-x^{12}-x^{24}+x^{19}+x^{21}-x^{20}$$
$$+x^{15}+x^{17}-x^{30}-x^{22}+x^{25}+x^{27}-x^{14}-x^{26}+x^{23}-x^{10}-x^{18}+x^{31}-x^{28}+x^{37}+x^{29}-x^{32}-x^{34}$$
$$+x^{33}+x^{35}-x^{36}+x^{41}-x^{38}+x^{39}-x^{40}+x^{117}+x^{111}-x^{114}-x^{108}+x^{105}-x^{102}+x^{99}-x^{96}+x^{93}$$
$$-x^{90}+x^{87}+x^{81}-x^{78}+x^{75}-x^{84}-x^{72}-x^{66}+x^{63}-x^{60}+x^{57}-x^{54}+x^{69}+x^{51}+x^{45}-x^{42}-x^{48}$$
$$+x^{119}-x^{118}-x^{116}+x^{115}+x^{113}-x^{112}-x^{110}+x^{109}+x^{107}-x^{106}-x^{104}+x^{103}+x^{101}-x^{100}-x^{98}$$
$$+x^{97}+x^{95}-x^{92}+x^{91}-x^{94}-x^{88}-x^{86}+x^{85}+x^{89}+x^{83}-x^{80}+x^{79}+x^{77}-x^{76}-x^{74}-x^{82}+x^{73}$$
$$+x^{71}-x^{70}-x^{68}+x^{67}+x^{65}-x^{64}-x^{62}+x^{61}+x^{59}-x^{56}+x^{55}+x^{53}-x^{52}-x^{50}+x^{49}+x^{47}-x^{46}$$
$$-x^{44}+x^{43}-x^{58})\,(1-x+x^{3}-x^{2}+x^{4}+x^{8}-x^{6}-x^{7}+x^{9}-x^{11}+x^{24}+x^{20}+x^{15}-x^{17}-x^{22}-x^{26}$$
$$-x^{31}+x^{28}-x^{37}+x^{33}-x^{41}+x^{39}+x^{40}+x^{108}+x^{105}-x^{102}+x^{99}+x^{93}+x^{75}+x^{84}-x^{66}+x^{63}$$
$$+x^{60}+x^{69}+x^{45}-x^{42}+x^{48}-x^{107}-x^{106}+x^{104}-x^{101}+x^{100}-x^{97}-x^{91}+x^{88}-x^{86}+x^{80}-x^{77}$$
$$-x^{82}-x^{71}+x^{68}-x^{67}+x^{64}-x^{62}-x^{61}-x^{47}-x^{46}+x^{44}))$$

G.f. as rational function with standard denominator.

```
> lgfstandnum:=simplify(numer(lgf)*simplify(standenom/denom(lgf))):
  lgf:=lgfstandnum/standenom;
```

$$lgf := \frac{1}{(x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x+1)\,(1-x^{120})^{5}}\,(x^{6}\,(1+3\,x+13\,x^{3}+7\,x^{2}+23\,x^{4}+37\,x^{5}+118\,x^{8}$$
$$+60\,x^{6}+86\,x^{7}+149\,x^{9}+199\,x^{11}+208\,x^{13}+145\,x^{16}+212\,x^{12}+54\,x^{24}+79\,x^{19}+67\,x^{21}$$
$$+72\,x^{20}+171\,x^{15}+115\,x^{17}+66\,x^{22}+43\,x^{25}+19\,x^{27}+196\,x^{14}+33\,x^{26}+59\,x^{23}+180\,x^{10}$$
$$+96\,x^{18}+9\,x^{28})\,(1-x+x^{3}-x^{4}+x^{8}+x^{6}-x^{7}+x^{11}-x^{13}+x^{24}+x^{30}-x^{25}+x^{27}+x^{14}-x^{10}-x^{31}$$

$$- x^{28} - x^{37} + x^{32} - x^{34} + x^{35} + x^{38} + x^{102} + x^{99} + x^{96} + x^{78} + x^{75} + x^{72} + x^{54} + x^{51} + x^{48} + x^{110}$$
$$- x^{109} + x^{107} - x^{106} + x^{104} - x^{103} - x^{100} - x^{97} + x^{86} - x^{85} + x^{83} + x^{80} - x^{79} - x^{76} - x^{82} - x^{73} + x^{62}$$
$$- x^{61} + x^{59} + x^{56} - x^{55} - x^{52} - x^{49} - x^{58}) (1 + x + x^3 + x^2 + x^4 + x^5 + x^8 + x^6 + x^7 + x^9 + x^{11} + x^{13}$$
$$+ x^{16} + x^{12} + x^{24} + x^{19} + x^{21} + x^{20} + x^{15} + x^{17} + x^{30} + x^{22} + x^{25} + x^{27} + x^{14} + x^{26} + x^{23} + x^{10}$$
$$+ x^{18} + x^{31} + x^{28} + x^{37} + x^{29} + x^{32} + x^{34} + x^{33} + x^{35} + x^{36} + x^{41} + x^{38} + x^{39} + x^{40} + x^{117} + x^{111}$$
$$+ x^{114} + x^{108} + x^{105} + x^{102} + x^{99} + x^{96} + x^{93} + x^{90} + x^{87} + x^{81} + x^{78} + x^{75} + x^{84} + x^{72} + x^{66} + x^{63}$$
$$+ x^{60} + x^{57} + x^{54} + x^{69} + x^{51} + x^{45} + x^{42} + x^{48} + x^{119} + x^{118} + x^{116} + x^{115} + x^{113} + x^{112} + x^{110}$$
$$+ x^{109} + x^{107} + x^{106} + x^{104} + x^{103} + x^{101} + x^{100} + x^{98} + x^{97} + x^{95} + x^{92} + x^{91} + x^{94} + x^{88} + x^{86}$$
$$+ x^{85} + x^{89} + x^{83} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{82} + x^{73} + x^{71} + x^{70} + x^{68} + x^{67} + x^{65} + x^{64}$$
$$+ x^{62} + x^{61} + x^{59} + x^{56} + x^{55} + x^{53} + x^{52} + x^{50} + x^{49} + x^{47} + x^{46} + x^{44} + x^{43} + x^{58})\verb|^|2 \, (1 - x + x^2$$
$$+ x^8 + x^6 - x^7 - x^{13} + x^{12} + x^{24} - x^{19} + x^{20} + x^{30} - x^{25} + x^{14} + x^{26} + x^{18} - x^{31} - x^{37} + x^{32} + x^{36}$$
$$+ x^{38} + x^{114} + x^{108} + x^{102} + x^{96} + x^{90} + x^{78} + x^{84} + x^{72} + x^{66} + x^{60} + x^{54} + x^{42} + x^{48} + x^{116} - x^{115}$$
$$+ x^{110} - x^{109} + x^{104} - x^{103} + x^{98} - x^{97} + x^{92} - x^{91} + x^{86} - x^{85} + x^{80} - x^{79} + x^{74} - x^{73} + x^{68} - x^{67}$$
$$+ x^{62} - x^{61} + x^{56} - x^{55} + x^{50} - x^{49} + x^{44} - x^{43}) (1 - x + x^3 - x^2 + x^4 + x^8 - x^6 - x^7 + x^9 - x^{11} + x^{24}$$
$$+ x^{20} + x^{15} - x^{17} - x^{22} - x^{26} - x^{31} + x^{28} - x^{37} + x^{33} - x^{41} + x^{39} + x^{40} + x^{108} + x^{105} - x^{102} + x^{99}$$
$$+ x^{93} + x^{75} + x^{84} - x^{66} + x^{63} + x^{60} + x^{69} + x^{45} - x^{42} + x^{48} - x^{107} - x^{106} + x^{104} - x^{101} + x^{100} - x^{97}$$
$$- x^{91} + x^{88} - x^{86} + x^{80} - x^{77} - x^{82} - x^{71} + x^{68} - x^{67} + x^{64} - x^{62} - x^{61} - x^{47} - x^{46} + x^{44}))$$

G.f. as rational function with standard denominator.

```
> rgfstandnum:=simplify(numer(rgf)*standenom/denom(rgf)):
  rgf:=rgfstandnum/standenom;
```

$$rgf := - \frac{1}{(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)(1 - x^{120})^5} ((1 + 3x + 13x^3 + 7x^2 + 23x^4 + 37x^5 + 118x^8$$
$$+ 60x^6 + 86x^7 + 149x^9 + 199x^{11} + 208x^{13} + 145x^{16} + 212x^{12} + 54x^{24} + 79x^{19} + 67x^{21}$$
$$+ 72x^{20} + 171x^{15} + 115x^{17} + 66x^{22} + 43x^{25} + 19x^{27} + 196x^{14} + 33x^{26} + 59x^{23} + 180x^{10}$$
$$+ 96x^{18} + 9x^{28}) x^3 (1 - x + x^3 - x^4 + x^8 + x^6 - x^7 + x^{11} - x^{13} + x^{24} + x^{30} - x^{25} + x^{27} + x^{14} - x^{10}$$
$$- x^{31} - x^{28} - x^{37} + x^{32} - x^{34} + x^{35} + x^{38} + x^{102} + x^{99} + x^{96} + x^{78} + x^{75} + x^{72} + x^{54} + x^{51} + x^{48}$$
$$+ x^{110} - x^{109} + x^{107} - x^{106} + x^{104} - x^{103} - x^{100} - x^{97} + x^{86} - x^{85} + x^{83} + x^{80} - x^{79} - x^{76} - x^{82} - x^{73}$$
$$+ x^{62} - x^{61} + x^{59} + x^{56} - x^{55} - x^{52} - x^{49} - x^{58}) (1 + x + x^3 + x^2 + x^4 + x^5 + x^8 + x^6 + x^7 + x^9 + x^{11}$$
$$+ x^{13} + x^{16} + x^{12} + x^{24} + x^{19} + x^{21} + x^{20} + x^{15} + x^{17} + x^{30} + x^{22} + x^{25} + x^{27} + x^{14} + x^{26} + x^{23}$$
$$+ x^{10} + x^{18} + x^{31} + x^{28} + x^{37} + x^{29} + x^{32} + x^{34} + x^{33} + x^{35} + x^{36} + x^{41} + x^{38} + x^{39} + x^{40} + x^{117}$$
$$+ x^{111} + x^{114} + x^{108} + x^{105} + x^{102} + x^{99} + x^{96} + x^{93} + x^{90} + x^{87} + x^{81} + x^{78} + x^{75} + x^{84} + x^{72}$$
$$+ x^{66} + x^{63} + x^{60} + x^{57} + x^{54} + x^{69} + x^{51} + x^{45} + x^{42} + x^{48} + x^{119} + x^{118} + x^{116} + x^{115} + x^{113}$$
$$+ x^{112} + x^{110} + x^{109} + x^{107} + x^{106} + x^{104} + x^{103} + x^{101} + x^{100} + x^{98} + x^{97} + x^{95} + x^{92} + x^{91} + x^{94}$$
$$+ x^{88} + x^{86} + x^{85} + x^{89} + x^{83} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{82} + x^{73} + x^{71} + x^{70} + x^{68} + x^{67}$$
$$+ x^{65} + x^{64} + x^{62} + x^{61} + x^{59} + x^{56} + x^{55} + x^{53} + x^{52} + x^{50} + x^{49} + x^{47} + x^{46} + x^{44} + x^{43} + x^{58})\verb|^|2$$
$$(-1 + x + x^3 - x^2 - x^4 + x^5 - x^8 - x^6 + x^7 + x^9 + x^{11} + x^{13} - x^{16} - x^{12} - x^{24} + x^{19} + x^{21} - x^{20} + x^{15}$$
$$+ x^{17} - x^{30} - x^{22} + x^{25} + x^{27} - x^{14} - x^{26} + x^{23} - x^{10} - x^{18} + x^{31} - x^{28} + x^{37} + x^{29} - x^{32} - x^{34} + x^{33}$$
$$+ x^{35} - x^{36} + x^{41} - x^{38} + x^{39} - x^{40} + x^{117} + x^{111} - x^{114} - x^{108} + x^{105} - x^{102} + x^{99} - x^{96} + x^{93} - x^{90}$$
$$+ x^{87} + x^{81} - x^{78} + x^{75} - x^{84} - x^{72} - x^{66} + x^{63} - x^{60} + x^{57} - x^{54} + x^{69} + x^{51} + x^{45} - x^{42} - x^{48} + x^{119}$$
$$- x^{118} - x^{116} + x^{115} + x^{113} - x^{112} - x^{110} + x^{109} + x^{107} - x^{106} - x^{104} + x^{103} + x^{101} - x^{100} - x^{98} + x^{97}$$
$$+ x^{95} - x^{92} + x^{91} - x^{94} - x^{88} - x^{86} + x^{85} + x^{89} + x^{83} - x^{80} + x^{79} + x^{77} - x^{76} - x^{74} - x^{82} + x^{73} + x^{71}$$
$$- x^{70} - x^{68} + x^{67} + x^{65} - x^{64} - x^{62} + x^{61} + x^{59} - x^{56} + x^{55} + x^{53} - x^{52} - x^{50} + x^{49} + x^{47} - x^{46} - x^{44}$$

$+ x^{43} - x^{58}) (1 - x + x^{3} - x^{2} + x^{4} + x^{8} - x^{6} - x^{7} + x^{9} - x^{11} + x^{24} + x^{20} + x^{15} - x^{17} - x^{22} - x^{26} - x^{31} + x^{28} - x^{37} + x^{33} - x^{41} + x^{39} + x^{40} + x^{108} + x^{105} - x^{102} + x^{99} + x^{93} + x^{75} + x^{84} - x^{66} + x^{63} + x^{60} + x^{69} + x^{45} - x^{42} + x^{48} - x^{107} - x^{106} + x^{104} - x^{101} + x^{100} - x^{97} - x^{91} + x^{88} - x^{86} + x^{80} - x^{77} - x^{82} - x^{71} + x^{68} - x^{67} + x^{64} - x^{62} - x^{61} - x^{47} - x^{46} + x^{44}))$

Expand the series to find the first few values of the number of squares.

```
> Lseries:=series(Lgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

Expand the series to find the first few values of the number of reduced squares.

```
> Rseries:=series(Rgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

Expand the series to find the first few values of the number of symmetry types.

```
> lseries:=series(lgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

```
> rseries:=series(rgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

Find the counting sequences

Generate the labelled sequence of magilatin square numbers of all four kinds. The first step is to compute the degree of the first non-zero term.

```
> Lgfdegree:=ldegree(numer(Lgf),x);
  Rgfdegree:=ldegree(numer(Rgf),x);
  lgfdegree:=ldegree(numer(lgf),x);
  rgfdegree:=ldegree(numer(rgf),x);
```

$Lgfdegree := 6$

$Rgfdegree := 3$

$lgfdegree := 6$

$rgfdegree := 3$

List the coefficients of each series, i.e., the terms of the counting sequences.
The comment symbol # is used for controlling the output. With large "enddegree" the output is huge so it's more convenient to run each sequence's output separately and copy it from the worksheet.

```
> for n from Lgfdegree to enddegree do
   co:=coeff(Lseries,x,n):
   printf("%d  %d \n",n,co);
  od:
  print("Coefficients complete.",n,co);
```

```
6  12
7  12
8  24
```

9	72
10	156
11	240
12	552
13	600
14	1020
15	1548
16	2004
17	2568
18	4008
19	4644
20	6264
21	8136
22	10152
23	12168
24	16284
25	18372
26	22992
27	27972
28	32736
29	37896
30	47352
31	52332
32	62004
33	72288
34	82572
35	93108
36	110280
37	120492
38	138420
39	157428
40	175248
41	193824
42	223428
43	241500
44	270744
45	301716
46	331464
47	362076
48	406956
49	436668
50	482412
51	529596
52	574404
53	620976
54	687576
55	732960
56	798660
57	867204
58	933084
59	1000548
60	1093044
61	1158708
62	1251456

63	1346940
64	1437888
65	1531716
66	1657740
67	1749408
68	1873560
69	2002140
70	2126004
71	2252388
72	2417160
73	2540556
74	2705148
75	2873664
76	3034980
77	3200868
78	3413784
79	3576072
80	3786072
81	4002012
82	4210680
83	4423164
84	4690452
85	4898760
86	5164608
87	5436360
88	5697744
89	5965212
90	6297960
91	6560280
92	6888420
93	7224516
94	7550028
95	7881336
96	8286168
97	8611248
98	9013740
99	9423864
100	9819984
101	10224132
102	10714404
103	11111568
104	11595432
105	12089664
106	12569352
107	13056672
108	13639692
109	14119056
110	14698188
111	15287184
112	15857964
113	16438932
114	17129784
115	17701716
116	18384036

117	19079352
118	19755660
119	20442048
120	21249012
121	21924744
122	22725984
123	23539536
124	24329904
125	25133304
126	26072868
127	26864676
128	27793368
129	28737936
130	29658852
131	30591864
132	31673496
133	32593656
134	33667404
135	34756452
136	35816628
137	36892824
138	38134632
139	39196428
140	40424880
141	41672160
142	42890136
143	44123232
144	45535620
145	46752876
146	48154896
147	49575348
148	50960880
149	52365600
150	53968296
151	55355628
152	56941908
153	58550472
154	60123828
155	61715292
156	63519384
157	65091876
158	66883116
159	68696172
160	70467792
161	72261840
162	74289252
163	76062564
164	78070248
165	80104212
166	82095768
167	84109140
168	86371572
169	88362372

170	90609036
171	92880900
172	95104380
173	97353960
174	99874920
175	102100416
176	104598084
177	107126028
178	109604508
179	112108404
180	114900708
181	117378252
182	120151512
183	122954004
184	125700192
185	128476620
186	131565396
187	134313672
188	137375304
189	140471676
190	143510628
191	146578812
192	149977704
193	153015612
194	156391932
195	159801768
196	163146876
197	166526652
198	170262408
199	173609856
200	177314688
201	181058436
202	184736664
203	188448300
204	192535716
205	196212864
206	200273712
207	204372936
208	208398288
209	212462916
210	216930576
211	220958304
212	225390324
213	229866300
214	234267684
215	238707120
216	243570072
217	247970304
218	252802764
219	257678208
220	262470528
221	267306948
222	272595828
223	277390632

224	282639408
225	287937288
226	293150856
227	298407336
228	304138308
229	309350760
230	315046812
231	320790960
232	326441868
233	332141916
234	338347128
235	344000772
236	350160660
237	356375376
238	362495556
239	368663832
240	375360276
241	381479088
242	388136040
243	394846200
244	401452608
245	408113880
246	415335252
247	421944540
248	429115512
249	436347072
250	443473476
251	450653088
252	458417712
253	465542640
254	473262588
255	481041756
256	488705724
257	496430208
258	504773256
259	512440284
260	520727424
261	529080984
262	537318120
263	545614368
264	554554884
265	562790580
266	571681272
267	580637052
268	589465824
269	598361304
270	607936512
271	616768452
272	626281812
273	635867928
274	645325812
275	654848676
276	665078088
277	674534532

278	684708540
279	694954116
280	705060408
281	715239384
282	726162276
283	736271628
284	747126624
285	758061036
286	768854472
287	779719476
288	791356116
289	802148076
290	813723660
291	825377076
292	836878284
293	848458944
294	860850336
295	872355000
296	884671908
297	897075252
298	909324804
299	921652188
300	934819572
301	947067468
302	960167496
303	973352268
304	986371296
305	999476796
306	1013462580
307	1026485136
308	1040389920
309	1054388340
310	1068219252
311	1082134836
312	1096961520
313	1110790596
314	1125543516
315	1140388344
316	1155053100
317	1169811636
318	1185523032
319	1200191712
320	1215815232
321	1231539660
322	1247082720
323	1262717364
324	1279336980
325	1294878168
326	1311417288
327	1328055840
328	1344499632
329	1361044692
330	1378618320
331	1395065928

332	1412544228
333	1430131356
334	1447522068
335	1465012104
336	1483563648
337	1500952488
338	1519416588
339	1537987752
340	1556349144
341	1574819292
342	1594396452
343	1612761768
344	1632236112
345	1651827312
346	1671206760
347	1690693272
348	1711320852
349	1730698464
350	1751231508
351	1771879464
352	1792301772
353	1812840900
354	1834568544
355	1854994956
356	1876611684
357	1898353872
358	1919868180
359	1941497616
360	1964350740
361	1985863104
362	2008614024
363	2031488064
364	2054120184
365	2076877584
366	2100907236
367	2123543604
368	2147454528
369	2171499336
370	2195300100
371	2219223984
372	2244457056
373	2268255624
374	2293378572
375	2318633532
376	2343629148
377	2368758792
378	2395247952
379	2420248068
380	2446609968
381	2473114608
382	2499358176
383	2525733432
384	2553506148
385	2579747556

386	2607401976
387	2635197156
388	2662715568
389	2690377080
390	2719487856
391	2747010876
392	2775985788
393	2805112512
394	2833960596
395	2862949260
396	2893426464
397	2922272316
398	2952622764
399	2983123332
400	3013328952
401	3043686528
402	3075586500
403	3105796764
404	3137551728
405	3169468260
406	3201087648
407	3232856940
408	3266208660
409	3297825780
410	3331042356
411	3364417980
412	3397480188
413	3430704000
414	3465565680
415	3498632784
416	3533340132
417	3568218948
418	3602781828
419	3637503972
420	3673905708
421	3708466212
422	3744723480
423	3781149732
424	3817243272
425	3853508100
426	3891509364
427	3927607872
428	3965445264
429	4003464204
430	4041147876
431	4079000532
432	4118632464
433	4156313580
434	4195791972
435	4235449248
436	4274753724
437	4314239820
438	4355563728

439	4394873496
440	4436023776
441	4477365756
442	4518352704
443	4559518428
444	4602566244
445	4643550744
446	4686435192
447	4729509144
448	4772209848
449	4815102396
450	4859937264
451	4902643152
452	4947294204
453	4992147540
454	5036625252
455	5081292360
456	5127946752
457	5172421872
458	5218903212
459	5265584568
460	5311871688
461	5358361236
462	5406900348
463	5453192832
464	5501537616
465	5550095736
466	5598257136
467	5646618336
468	5697075396
469	5745234240
470	5795508132
471	5845992768
472	5896061676
473	5946343956
474	5998785888
475	6048860340
476	6101097180
477	6153558696
478	6205601604
479	6257855400
480	6312316476
481	6364356648
482	6418624008
483	6473113200
484	6527164488
485	6581440488
486	6637988820
487	6692045940
488	6748378272
489	6804946800
490	6861074796
491	6917424408
492	6976095600

```
493  7032220608
494  7090687428
495  7149387636
496  7207626972
497  7266102648
498  7326966576
499  7385211852
500  7445848512
```

"Coefficients complete.", 501, 7445848512

```
> for n from Rgfdegree to enddegree do
   co:=coeff(Rseries,x,n):
   printf("%d  %d \n",n,co);
  od:
  print("Coefficients complete.",n,co);
```

```
3  12
4  12
5  24
6  60
7  144
8  216
9  480
10  444
11  780
12  996
13  1404
14  1548
15  2460
16  2640
17  3696
18  4128
19  5508
20  5904
21  8148
22  8220
23  10824
24  11688
25  14364
26  14904
27  19380
28  19596
29  24108
30  24936
31  30240
32  31104
33  37992
34  37920
35  45312
36  47148
37  54756
38  55404
39  66000
40  66252
```

41	76920
42	78288
43	89964
44	91332
45	105240
46	105204
47	120336
48	122640
49	137736
50	138564
51	157980
52	158556
53	177684
54	179628
55	200124
56	201888
57	225840
58	225624
59	250908
60	253896
61	279180
62	280260
63	310800
64	311520
65	341844
66	344400
67	376596
68	378828
69	415020
70	414552
71	452760
72	456504
73	494424
74	495720
75	540120
76	541092
77	585204
78	588228
79	634608
80	637092
81	688440
82	688080
83	741444
84	745908
85	798984
86	800604
87	861600
88	862536
89	923208
90	926556
91	989748
92	992916
93	1061652
94	1061220

95	1132404
96	1137696
97	1208736
98	1210392
99	1290540
100	1291584
101	1371300
102	1375260
103	1457784
104	1461240
105	1550028
106	1549704
107	1641516
108	1647492
109	1738908
110	1740744
111	1842600
112	1843752
113	1945104
114	1949568
115	2053944
116	2058012
117	2169660
118	2169084
119	2283936
120	2290524
121	2405160
122	2407320
123	2533332
124	2534772
125	2660064
126	2665068
127	2794176
128	2798496
129	2935560
130	2934804
131	3075540
132	3082956
133	3222972
134	3225420
135	3378180
136	3379800
137	3532056
138	3537528
139	3693708
140	3698352
141	3863460
142	3862740
143	4031664
144	4039728
145	4208004
146	4210704
147	4392948
148	4394748

149	4576308
150	4582176
151	4768200
152	4773384
153	4968912
154	4968048
155	5167824
156	5176788
157	5375916
158	5378724
159	5593080
160	5594772
161	5808408
162	5814960
163	6033204
164	6038892
165	6267360
166	6266604
167	6499896
168	6509328
169	6742008
170	6744924
171	6994020
172	6996036
173	7244124
174	7251108
175	7504092
176	7510320
177	7774680
178	7773744
179	8043108
180	8053296
181	8321940
182	8325108
183	8611392
184	8613480
185	8898684
186	8906280
187	9196956
188	9203508
189	9506028
190	9504984
191	9813120
192	9824064
193	10131264
194	10134720
195	10460640
196	10462980
197	10788036
198	10796028
199	11126808
200	11133612
201	11477280

202	11476200
203	11825412
204	11837220
205	12185424
206	12189204
207	12557640
208	12560016
209	12927408
210	12935724
211	13309380
212	13316796
213	13703772
214	13702620
215	14095644
216	14108136
217	14500224
218	14504184
219	14917620
220	14920104
221	15332460
222	15341460
223	15760224
224	15767928
225	16201020
226	16199904
227	16639476
228	16652652
229	17091108
230	17095104
231	17556168
232	17558904
233	18018744
234	18028248
235	18494784
236	18503172
237	18984900
238	18983532
239	19472208
240	19485924
241	19973520
242	19977840
243	20489052
244	20491932
245	21001632
246	21011820
247	21528936
248	21537576
249	22070640
250	22069164
251	22609500
252	22624044
253	23163084
254	23167620
255	23731500

256	23734560
257	24297216
258	24307728
259	24877836
260	24886944
261	25473900
262	25472460
263	26066904
264	26082168
265	26675244
266	26680032
267	27299460
268	27302628
269	27920508
270	27931416
271	28557360
272	28566864
273	29210160
274	29208720
275	29859864
276	29876028
277	30525876
278	30530844
279	31208160
280	31211220
281	31887240
282	31898760
283	32582844
284	32592852
285	33295080
286	33293604
287	34004184
288	34020960
289	34730208
290	34735284
291	35473260
292	35476716
293	36212964
294	36224916
295	36969804
296	36980280
297	37744320
298	37742664
299	38515308
300	38532696
301	39303828
302	39309300
303	40110312
304	40113840
305	40913124
306	40925760
307	41734116
308	41744916
309	42573180

310	42571344
311	43408680
312	43426824
313	44262504
314	44268120
315	45134688
316	45138612
317	46003596
318	46016628
319	46891008
320	46902132
321	47797320
322	47795448
323	48699924
324	48718860
325	49621464
326	49627404
327	50562480
328	50566296
329	51499536
330	51513036
331	52456140
332	52467876
333	53432292
334	53430420
335	54404484
336	54424104
337	55396656
338	55402704
339	56408700
340	56412624
341	57416820
342	57430860
343	58444992
344	58457160
345	59493540
346	59491704
347	60538236
348	60558612
349	61603308
350	61609392
351	62689080
352	62693184
353	63770784
354	63785328
355	64873224
356	64885932
357	65996868
358	65994924
359	67116408
360	67137324
361	68257080
362	68263560
363	69419172

364	69423420
365	70576944
366	70592100
367	71756496
368	71769456
369	72957720
370	72955524
371	74154588
372	74176476
373	75373524
374	75380220
375	76614420
376	76618920
377	77851176
378	77866656
379	79110108
380	79123464
381	80391540
382	80389380
383	81668544
384	81691008
385	82968012
386	82975104
387	84290700
388	84295308
389	85608708
390	85624656
391	86949720
392	86963472
393	88313952
394	88311720
395	89673504
396	89696868
397	91056636
398	91063764
399	92463168
400	92467812
401	93865200
402	93881760
403	95290884
404	95305212
405	96740400
406	96738132
407	98185416
408	98209320
409	99654408
410	99661644
411	101147700
412	101152596
413	102636132
414	102653268
415	104149044
416	104163840
417	105686760

418	105684384
419	107219508
420	107244024
421	108777060
422	108784620
423	110359632
424	110364600
425	111937164
426	111954840
427	113540004
428	113555268
429	115168260
430	115165704
431	116791440
432	116816784
433	118440144
434	118447848
435	120114480
436	120119772
437	121783956
438	121802028
439	123479208
440	123494652
441	125200488
442	125198040
443	126916764
444	126942900
445	128659104
446	128667204
447	130428120
448	130433304
449	132191808
450	132210276
451	133982100
452	133998156
453	135799212
454	135796620
455	137610852
456	137637816
457	139449816
458	139458024
459	141315780
460	141321144
461	143176380
462	143195388
463	145064304
464	145080720
465	146979660
466	146977104
467	148889796
468	148917372
469	150827436
470	150835824
471	152793120

```
472  152798664
473  154753224
474  154772808
475  156741264

476  156758220
477  158757780
478  158755044
479  160768608
480  160796724
481  162807840
482  162816480
483  164875620
484  164881452
485  166937784
486  166957980
487  169028856
488  169046136
489  171148800
490  171145812
491  173263020
492  173292036
493  175406364
494  175415220
495  177578940
496  177584880
497  179745864
498  179766528
499  181942308
500  181959984
```

"Coefficients complete.", 501, 181959984

```
> for n from lgfdegree to enddegree do
    co:=coeff(lseries,x,n):
    printf("%d  %d \n",n,co);
  od:
  print("Coefficients complete.",n,co);
```

```
6  1
7  1
8  2
9  4
10  7
11  10
12  20
13  22
14  35
15  50
16  63
17  78
18  116
19  131
20  170
```

21	215
22	260
23	306
24	395
25	440
26	537
27	640
28	737
29	841
30	1025
31	1125
32	1310
33	1507
34	1700
35	1898
36	2213
37	2404
38	2729
39	3071
40	3391
41	3725
42	4242
43	4566
44	5075
45	5612
46	6127
47	6656
48	7418
49	7931
50	8703
51	9499
52	10254
53	11038
54	12140
55	12903
56	13989
57	15119
58	16205
59	17316
60	18819
61	19901
62	21405
63	22952
64	24426
65	25945
66	27962
67	29446
68	31432
69	33485
70	35463
71	37481
72	40086
73	42057
74	44656

75	47315
76	49863
77	52480
78	55811
79	58372
80	61656
81	65030
82	68291
83	71611
84	75756
85	79011
86	83132
87	87341
88	91393
89	95536
90	100656
91	104721
92	109770
93	114938
94	119945
95	125039
96	131229
97	136229
98	142381
99	148646
100	154701
101	160876
102	168327
103	174397
104	181751
105	189258
106	196548
107	203951
108	212768
109	220052
110	228808
111	237710
112	246341
113	255123
114	265521
115	274168
116	284439
117	294900
118	305080
119	315408
120	327505
121	337676
122	349686
123	361877
124	373726
125	385766
126	399798
127	411667
128	425538

129	439640
130	453394
131	467326
132	483426
133	497169
134	513151
135	529356
136	545139
137	561155
138	579582
139	595387
140	613617
141	632121
142	650196
143	668492
144	689392
145	707456
146	728203
147	749217
148	769723
149	790507
150	814161
151	834692
152	858106
153	881843
154	905067
155	928554
156	955119
157	978330
158	1004706
159	1031397
160	1057487
161	1083902
162	1113688
163	1139802
164	1169301
165	1199179
166	1228443
167	1258022
168	1291195
169	1320447
170	1353389
171	1386695
172	1419301
173	1452284
174	1489176
175	1521810
176	1558365
177	1595355
178	1631631
179	1668273
180	1709065
181	1745327
182	1785842

183	1826778
184	1866902
185	1907461
186	1952509
187	1992662
188	2037318
189	2082472
190	2126799
191	2171547
192	2221042
193	2265354
194	2314522
195	2364171
196	2412890
197	2462106
198	2516427
199	2565178
200	2619053
201	2673486
202	2726977
203	2780948
204	2840302
205	2893777
206	2952748
207	3012268
208	3070729
209	3129752
210	3194543
211	3253037
212	3317316
213	3382224
214	3446062
215	3510445
216	3580885
217	3644706
218	3714707
219	3785322
220	3854747
221	3924803
222	4001323
223	4070783
224	4146728
225	4223374
226	4298814
227	4374867
228	4457695
229	4533118
230	4615444
231	4698457
232	4780137
233	4862519
234	4952107
235	5033825

236	5122765
237	5212486
238	5300857
239	5389914
240	5486501
241	5574852
242	5670873
243	5767653
244	5862952
245	5959033
246	6063094
247	6158433
248	6261774
249	6365977
250	6468680
251	6572142
252	6683933
253	6786615
254	6897767
255	7009762
256	7120116
257	7231331
258	7351348
259	7461744
260	7580963
261	7701127
262	7819632
263	7938979
264	8067488
265	8185972
266	8313770
267	8442493
268	8569409
269	8697273
270	8834798
271	8961758
272	9098402
273	9236080
274	9371933
275	9508710
276	9655525
277	9791357
278	9937382
279	10084423
280	10229484
281	10375578
282	10532234
283	10677338
284	10833028
285	10989845
286	11144659
287	11300489
288	11467271
289	11622063

290	11787975
291	11954992
292	12119847
293	12285830
294	12463313
295	12628216
296	12804641
297	12982291
298	13157758
299	13334329
300	13522811
301	13698254
302	13885778
303	14074504
304	14260878
305	14448478
306	14648555
307	14834978
308	15033905
309	15234158
310	15432035
311	15631113
312	15843099
313	16040950
314	16251888
315	16464128
316	16673816
317	16884832
318	17109343
319	17319085
320	17542349
321	17767042
322	17989164
323	18212584
324	18449948
325	18672043
326	18908265
327	19145894
328	19380765
329	19617069
330	19867929
331	20102853
332	20352362
333	20603411
334	20851678
335	21101351
336	21366042
337	21614282
338	21877733
339	22142698
340	22404694
341	22668229
342	22947419
343	23209470

344	23487204
345	23766590
346	24042980
347	24320884
348	24614921
349	24891284
350	25183981
351	25478303
352	25769433
353	26062215
354	26371794
355	26662981
356	26970991
357	27280773
358	27587333
359	27895520
360	28220997
361	28527529
362	28851559
363	29177329
364	29499679
365	29823799
366	30165890
367	30488299
368	30828711
369	31171013
370	31509866
371	31850459
372	32209539
373	32548361
374	32905883
375	33265269
376	33620992
377	33978607
378	34355415
379	34711200
380	35086209
381	35463233
382	35836569
383	36211765
384	36606684
385	36979989
386	37373236
387	37768469
388	38159795
389	38553140
390	38966934
391	39358324
392	39770199
393	40184216
394	40594299
395	41006366
396	41439432
397	41849483

398	42280758
399	42714150
400	43143380
401	43574754
402	44027880
403	44457175
404	44908254
405	45361611
406	45810776
407	46262055
408	46735647
409	47184779
410	47656462
411	48130388
412	48599893
413	49071677
414	49566549
415	50036122
416	50528817
417	51023928
418	51514584
419	52007485
420	52524058
421	53014680
422	53529214
423	54046130
424	54558355
425	55072994
426	55612101
427	56124395
428	56661191
429	57200544
430	57735171
431	58272180
432	58834255
433	59368846
434	59928755
435	60491184
436	61048642
437	61608658
438	62194560
439	62752091
440	63335545
441	63921699
442	64502850
443	65086520
444	65696694
445	66277810
446	66885681
447	67496220
448	68101502
449	68709485
450	69344815

```
451  69950169
452  70582909
453  71218497
454  71848794
455  72481758
456  73142698
457  73772958
458  74431459
459  75092775
460  75748540
461  76407155
462  77094616
463  77750456
464  78435180
465  79122906
466  79805047
467  80490000
468  81204447
469  81886551
470  82598417
471  83313249
472  84022229
473  84734212
474  85476580
475  86185637
476  86925118
477  87667759
478  88404509
479  89144226
480  89914994
481  90651705
482  91419745
483  92190906
484  92955905
485  93724065
486  94524186
487  95289266
488  96086347
489  96886749
490  97680953
491  98478275
492  99308245
493  100102407
494  100929500
495  101759875
496  102583768
497  103410984
498  104271781
499  105095756
500  105953355
```

"Coefficients complete.", 501, 105953355

```
> for n from rgfdegree to enddegree do
```

```
    co:=coeff(rseries,x,n):
    printf("%d  %d \n",n,co);
  od:
  print("Coefficients complete.",n,co);
```

```
3  1
4  1
5  2
6  3
7  6
8  8
9  16
10  15
11  25
12  30
13  41
14  43
15  66
16  68
17  92
18  99
19  129
20  136
21  180
22  180
23  231
24  245
25  297
26  304
27  385
28  388
29  469
30  482
31  575
32  588
33  706
34  704
35  831
36  858
37  987
38  996
39  1171
40  1175
41  1350
42  1370
43  1561
44  1581
45  1806
46  1804
47  2047
48  2081
49  2323
50  2335
51  2641
52  2649
```

```
53  2951
54  2979
55  3302
56  3327
57  3700
58  3696
59  4089
60  4133
61  4525
62  4540
63  5010
64  5020
65  5487
66  5523
67  6017
68  6049
69  6601
70  6594
71  7175
72  7229
73  7806
74  7824
75  8496
76  8509
77  9176
78  9219
79  9919
80  9955
81  10726
82  10720
83  11521
84  11585
85  12382
86  12404
87  13315
88  13328
89  14234
90  14282
91  15224
92  15269
93  16291
94  16284
95  17342
96  17417
97  18472
98  18495
99  19681
100  19696
101  20875
102  20931
103  22151
104  22200
105  23510
106  23504
```

107	24857
108	24942
109	26289
110	26315
111	27811
112	27827
113	29316
114	29379
115	30912
116	30969
117	32605
118	32596
119	34278
120	34372
121	36050
122	36080
123	37921
124	37941
125	39772
126	39842
127	41727
128	41788
129	43786
130	43775
131	45825
132	45930
133	47970
134	48004
135	50226
136	50248
137	52462
138	52539
139	54809
140	54875
141	57271
142	57260
143	59711
144	59825
145	62267
146	62304
147	64944
148	64969
149	67599
150	67682
151	70375
152	70448
153	73276
154	73263
155	76152
156	76278
157	79157
158	79196
159	82291
160	82315

161	85399
162	85491
163	88641
164	88721
165	92016
166	92004
167	95367
168	95500
169	98854
170	98895
171	102481
172	102509
173	106081
174	106179
175	109821
176	109908
177	113710
178	113696
179	117569
180	117713
181	121575
182	121619
183	125731
184	125760
185	129857
186	129963
187	134137
188	134229
189	138570
190	138555
191	142975
192	143129
193	147536
194	147584
195	152256
196	152288
197	156947
198	157059
199	161799
200	161895
201	166816
202	166800
203	171800
204	171966
205	176952
206	177004
207	182275
208	182308
209	187564
210	187681
211	193025
212	193129
213	198661
214	198644

215	204262
216	204437
217	210041
218	210096
219	216001
220	216036
221	221925
222	222051
223	228031
224	228139
225	234321
226	234304
227	240577
228	240762
229	247019
230	247075
231	253650
232	253688
233	260246
234	260379
235	267032
236	267149
237	274015
238	273995
239	280959
240	281152
241	288100
242	288160
243	295441
244	295481
245	302741
246	302883
247	310247
248	310368
249	317956
250	317935
251	325625
252	325829
253	333501
254	333564
255	341586
256	341628
257	349632
258	349779
259	357888
260	358016
261	366361
262	366340
263	374791
264	375005
265	383437
266	383503
267	392305
268	392349

269	401129
270	401282
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272	410308
273	419445
274	419424
275	428672
276	428898
277	438127
278	438196
279	447811
280	447854
281	457450
282	457611
283	467321
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286	477404
287	487486
288	487721
289	497784
290	497855
291	508321
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293	518811
294	518978
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296	529688
297	540520
298	540496
299	551449
300	551693
301	562624
302	562700
303	574051
304	574100
305	585427
306	585603
307	597057
308	597208
309	608941
310	608915
311	620775
312	621029
313	632866
314	632944
315	645215
316	645269
317	657517
318	657699
319	670079
320	670235
321	682906

322	682879
323	695681
324	695946
325	708722
326	708804
327	722035
328	722088
329	735293
330	735482
331	748825
332	748989
333	762631
334	762604
335	776382
336	776656
337	790412
338	790496
339	804721
340	804776
341	818975
342	819171
343	833510
344	833680
345	848331
346	848304
347	863097
348	863382
349	878149
350	878234
351	893491
352	893548
353	908776
354	908979
355	924352
356	924529
357	940224
358	940196
359	956039
360	956332
361	972150
362	972240
363	988561
364	988620
365	1004912
366	1005123
367	1021567
368	1021748
369	1038526
370	1038495
371	1055424
372	1055730
373	1072631
374	1072724
375	1090146

376	1090208
377	1107602
378	1107818
379	1125369
380	1125556
381	1143451
382	1143420
383	1161471
384	1161785
385	1179806
386	1179904
387	1198465
388	1198529
389	1217059
390	1217282
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396	1274718
397	1293897
398	1293996
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404	1353801
405	1374036
406	1374003
407	1394407
408	1394741
409	1415114
410	1415215
411	1436161
412	1436229
413	1457140
414	1457379
415	1478462
416	1478668
417	1500130
418	1500096
419	1521729
420	1522072
421	1543675
422	1543780
423	1565971
424	1566040
425	1588197
426	1588443
427	1610776
428	1610989
429	1633711

430	1633675
431	1656575
432	1656929
433	1679796
434	1679903
435	1703376
436	1703449
437	1726887
438	1727139
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440	1750975
441	1774995
442	1774960
443	1799161
444	1799526
445	1823692
446	1823804
447	1848595
448	1848667
449	1873424
450	1873682
451	1898625
452	1898849
453	1924201
454	1924164
455	1949701
456	1950077
457	1975582
458	1975696
459	2001841
460	2001916
461	2028025
462	2028290
463	2054591
464	2054820
465	2081541
466	2081504
467	2108417
468	2108802
469	2135678
470	2135795
471	2163331
472	2163408
473	2190906
474	2191179
475	2218872
476	2219108
477	2247235
478	2247196
479	2275519
480	2275912
481	2304200
482	2304320
483	2333280

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485  2362282
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487  2391687
488  2391928
489  2421496
490  2421454
491  2451225
492  2451630
493  2481361
494  2481484
495  2511906
496  2511988
497  2542371
498  2542659
499  2573249
500  2573496
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"Coefficients complete.", 501, 2573496

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